

# High-quality three-dimensional holographic display with use of multiple fractional Fourier transform

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In order to realize holographic display of three-dimensional (3D) objects and suppress zero-order light, conjugate image, and speckle noise, a novel method is proposed based on multiple fractional Fourier transform (M-FrFT) for calculating holograms of 3D objects. A series of kinoforms are generated by adding pseudorandom phase factor (PPF) to object planes in calculating each kinoform, and generating the PPF randomly again in the next kinoform calculation. The reconstructed images from kinoform sequence are superposed together in order to suppress the speckle noise of reconstructed image and improve the contrast and detail resolution of the reconstructed images. The qualities of reconstructed images from single amplitude hologram, single kinoform, and kinoform sequence calculated by M-FrFT are compared. The effects of suppressing speckle noise are analyzed by calculating the speckle index of numerical reconstructed images. The analytical results illustrate that, with the proposed method for 3D holographic display, the zero-order light, conjugate image, and speckle noise can be suppressed, and the qualities of reconstructed images can be improved significantly.

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Computer holography can be used to generate holograms of virtual objects without need of complicated optical recording setups. In addition, computer-generated hologram (CGH) has some other merits, such as good repeatability and flexibility. In recent years, some computational approaches have been proposed to generate holograms of three-dimensional (3D) objects, and they are mainly based on ray-tracing<sup>[1]</sup>, look-up table (LUT)<sup>[2,3]</sup>, angular spectrum propagation<sup>[4]</sup>, multi-view images synthesizing<sup>[5]</sup>, and tomographic Fresnel transform<sup>[6]</sup>. The complexity and time-consumption of the approach based on ray-tracing depend on the number of sampling points. The feature of the approach based on angular spectrum propagation is that the target objects should be composed of tilted planes. For multi-view images synthesizing method, the pixel number of hologram is determined by the number of multi-view images, and the resolution of the reconstructed image is determined by the interval of the adjacent viewing angles. Tomographic Fresnel transform is a promising method to generate hologram of 3D objects, and this method can also be accelerated by use of discrete fast Fourier transform (DFFT).

Recently, an improved method based on tomographic Fresnel transform is proposed to generate multiple kinoforms of 3D objects<sup>[7]</sup>, which can suppress the speckle noise, zero-order light, and conjugate image in the numerical and opto-electronic reconstruction processes. But this method is only feasible to calculate holograms in the Fresnel and Fraunhofer diffraction zone. However, the fractional Fourier transform (FrFT) can be used for uniformly describing the whole diffraction phenomena from object plane to far-field diffraction zones. The concept of FrFT was firstly proposed and defined by Namias in 1980<sup>[8]</sup>. In 1993, Lohmann proposed two kinds of optical FrFT systems<sup>[9]</sup>. FrFT has been widely used in the fields

of optics and information processing, such as wavefront transform analysis<sup>[10,11]</sup>, pattern recognition<sup>[12]</sup>, image encryption, and holographic reconstruction<sup>[13–15]</sup>. Some fast algorithms for numerical calculation of FrFT have also been proposed recently<sup>[16,17]</sup>. In this letter, a novel method based on multiple fractional Fourier transform (M-FrFT) is proposed to generate kinoforms of 3D objects in order to suppress the zero-order light, conjugate image, and speckle noise in the reconstruction process.

Figure 1 illustrates the FrFT optical system based on single lens, and its one-dimensional simplified mathematical model is described as<sup>[9]</sup>

$$\begin{aligned} Fr(u) &= F^\alpha[g(x)] \\ &= C_\alpha \int_{-\infty}^{+\infty} \exp[j\pi(x^2 \cot \varphi - 2ux \csc \varphi + u^2 \cot \varphi)] \\ &\quad \times g(x) du, \end{aligned} \quad (1)$$

where  $g(x)$  denotes the information in object plane,  $F^\alpha$  denotes the  $\alpha$ th FrFT of  $g(x)$ ,  $\varphi = \alpha\pi/2$  ( $0 < |\alpha| < 2$ ), and  $C_\alpha = \{\exp[-j\pi \cdot \text{sgn}(\sin \varphi)/4 + j\varphi/2]\}/\sqrt{|\sin \varphi|}$ .

Specially, when  $\alpha = 1$ , the integrated form of FrFT is the same as the normal Fourier transform. That is, the value of  $\alpha$  changing from 0 to 1 means that the wavefronts propagate from object plane to infinite far-field diffraction zone. So the FrFT can be used for uniformly describing the whole diffraction phenomena from near-field to far-field diffraction zones<sup>[9]</sup>.

The relationship between the fraction order and the recording distance of FrFT optical system shown in Fig. 1 is described as<sup>[9]</sup>

$$f = f_1/\sin \varphi, \quad z = f_1 \tan(\varphi/2), \quad (2)$$

where  $f$  denotes the physical focal length of the lens used

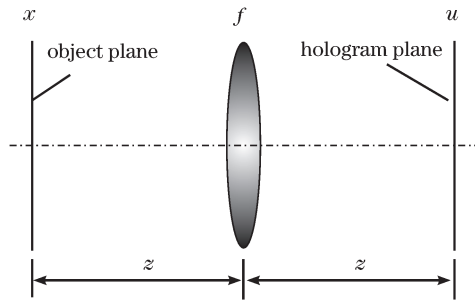


Fig. 1. Schematic of FrFT optical system based on a single lens.

in the system, and  $f_1$  denotes the standard focal length of the system.

According to the characteristics of additivity and periodicity of FrFT, we have

$$F^{\alpha'+\alpha}[g(x)] = F^{\alpha'}\{F^{\alpha}[g(x)]\}. \quad (3)$$

If  $\alpha'+\alpha = 4k$  ( $k$  is an integer), then

$$F^{\alpha'+\alpha}[g(x)] = g(x). \quad (4)$$

In the process of reconstruction, another FrFT with the fractional order  $\alpha'$  ( $\alpha' = 4k - \alpha$ ) is needed to obtain the reconstructed image of objects. Note that if  $k = 0$ , then  $\alpha' = -\alpha$ .

According to the characteristics of FrFT, for different fractional orders, the diffraction distances are also different. Thus, we can calculate holograms of 3D objects with the use of FrFT. Here we suppose that the 3D objects are composed of a certain number of discretized planes. Suppose that  $L$  is the total number of discretized planes, the FrFT of the  $i$ th plane ( $1 \leq i \leq L$ ) can be described as

$$\begin{aligned} Fr(u)_i &= F^{\alpha_i}[g(x_i)] \\ &= C_{\alpha_i} \int_{-\infty}^{+\infty} \exp[j\pi(x_i^2 \cot\varphi_i - 2ux_i \csc\varphi_i + u^2 \cot\varphi_i)] \\ &\quad \times g(x_i) du, \end{aligned} \quad (5)$$

where  $\alpha_i$  is the fractional order in the  $i$ th plane, which is associated with the recording distance of the  $i$ th object plane,  $\varphi_i = \alpha_i\pi/2$ , and  $g(x_i)$  is the intensity distribution of the  $i$ th plane. Then the whole complex amplitude distribution in the hologram plane can be described as

$$Fr(u) = \sum_{i=1}^L Fr(u)_i. \quad (6)$$

The amplitude hologram can be obtained by

$$H(u) = |Fr(u) + R(u)|^2, \quad (7)$$

where  $R(u)$  denotes the reference wave, and in this letter,  $R(u)$  is an in-line plane wave with constant intensity distribution.

In order to improve the diffraction efficiency, suppress the zero-order light, and remove the conjugate image, we

generate kinoforms by calculating only the phase distribution of  $Fr(u)$ , and it can be described as

$$H(u) = \arg[Fr(u)]. \quad (8)$$

In this letter, the kinoform generated by M-FrFT is named as M-FrFT kinoform. However, the reconstruction error of intensity distribution in the image plane is introduced inevitably, because kinoform ignores the amplitude component of wavefront in the hologram plane. This error distribution is often irregular, and it is shown as speckle noise generally, which damages the details of image and decreases the image contrast. A novel method is proposed to suppress the speckle noise and realize high-quality 3D holographic display of 3D objects with the use of FrFT.

In order to suppress the speckle noise in M-FrFT kinoform reconstruction, we add a pseudorandom phase factor (PPF) in object planes during the M-FrFT kinoform calculation, thus Eq. (5) is rewritten as

$$\begin{aligned} Fr(u)_{i,t} &= F^{\alpha_i}\{g(x_i)\exp[j\varphi_t(x_i)]\} \\ &= C_{\alpha_i} \int_{-\infty}^{+\infty} \exp[j\pi(x_i^2 \cot\varphi_i - 2ux_i \csc\varphi_i + u^2 \cot\varphi_i)] \\ &\quad \times g(x_i)\exp[j\varphi_t(x_i)] du, \end{aligned} \quad (9)$$

where  $\varphi_t(x_i)$  is the PPF in the  $t$ th M-FrFT kinoform calculation process, and  $t = 1, 2, 3, \dots, T$  ( $T$  is the total number of times used in kinoform calculation). Note that each element of  $\varphi_t(x_i)$  is randomly generated between 0 and  $2\pi$ , and it is invariant in calculating each M-FrFT kinoform, but generated randomly again in the next kinoform calculation.

With this process, the complex amplitude distribution of a 3D object in hologram plane with the  $t$ th M-FrFT can be described as

$$Fr(u)_t = \sum_{i=1}^L Fr(u)_{i,t}. \quad (10)$$

Thus, the  $t$ th FrFT kinoform of a 3D object is described as

$$H(u)_t = \arg[Fr(u)_t]. \quad (11)$$

As mentioned above, in the reconstruction of M-FrFT holograms (both amplitude-type and phase-type), FrFT process with fractional order  $\alpha' = 4k - \alpha$  is needed to obtain the reconstructed results. For M-FrFT amplitude hologram generated by Eq. (7), the reconstructed complex amplitude at the distance  $z_i$  (that is, reconstructed with fractional order  $\alpha'_i = 4k - \alpha_i$ ) can be described as

$$\begin{aligned} O(x_i) &= F^{\alpha'_i}[H(u)] \\ &= C_{\alpha'_i} \int_{-\infty}^{+\infty} \exp[j\pi(x_i^2 \cot\varphi_i - 2ux_i \csc\varphi_i + u^2 \cot\varphi_i)] \\ &\quad \times H(u) dx_i, \end{aligned} \quad (12)$$

However, for M-FrFT kinoforms generated by Eq. (11), it should be put back to the argument of the complex amplitude in hologram plane, so the reconstructed complex amplitude distribution at the distance  $z_i$  should be

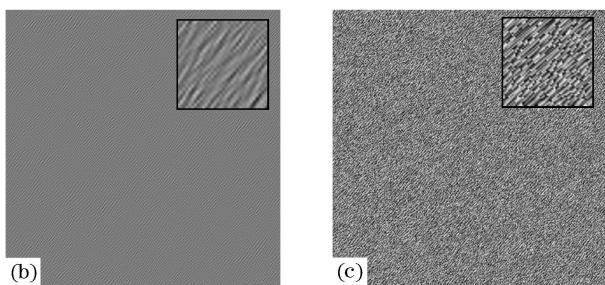
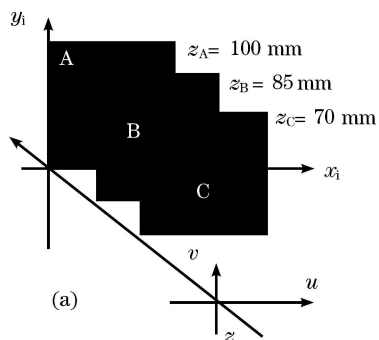


Fig. 2. Holograms calculation of 3D objects by M-FrFT. (a) Original 3D image; (b) M-FrFT amplitude hologram; (c) M-FrFT kinoform.

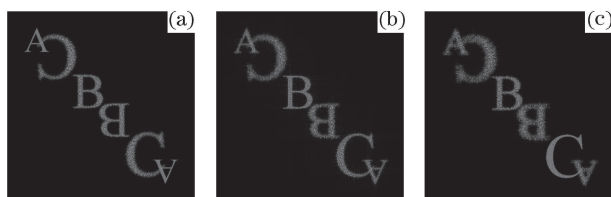


Fig. 3. Numerically reconstructed images from single M-FrFT amplitude hologram at different distances. (a)  $z = 100$  mm; (b)  $z = 85$  mm; (c)  $z = 70$  mm.

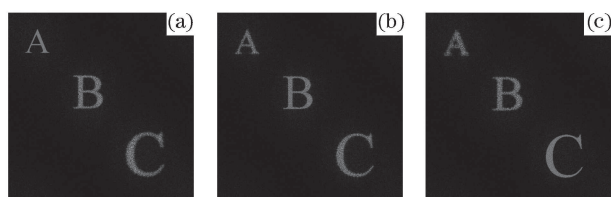


Fig. 4. Numerically reconstructed images from single M-FrFT kinoform at different distances. (a)  $z = 100$  mm; (b)  $z = 85$  mm; (c)  $z = 70$  mm.

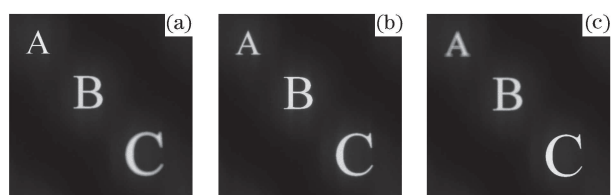


Fig. 5. Results of superposing images reconstructed from 100 M-FrFT kinoforms at different distances. (a)  $z = 100$  mm; (b)  $z = 85$  mm; (c)  $z = 70$  mm.

described as

$$\begin{aligned}
 O_t(x_i) &= F^{\alpha'_i} \{ \exp[jH(u)_t] \} \\
 &= C_{\alpha'_i} \int_{-\infty}^{+\infty} \exp[j\pi(x_i^2 \cot \varphi_i - 2ux_i \csc \varphi_i + u^2 \cot \varphi_i)] \\
 &\quad \times \exp[jH(u)_t] dx_i,
 \end{aligned}
 \tag{13}$$

The complex amplitude distribution reconstructed from all the kinoforms are then superposed in the  $i$ th reconstructed plane according to

$$O(x_i) = \sum_{t=1}^T O_t(x_i).
 \tag{14}$$

In order to verify the feasibility of the proposed method, we take it to calculate M-FrFT holograms of three-plane objects and then reconstruct images from the M-FrFT holograms. As shown in Fig. 2(a), we suppose that there are three different characters (“A”, “B”, and “C”) in different distances, and the distance from plane A, plane B, and plane C to the hologram plane are  $z_A = 100$  mm,  $z_B = 85$  mm, and  $z_C = 70$  mm, respectively. The number of sampling points in the object plane and hologram is  $512 \times 512$ . Supposing that the standard focal length  $f_1 = 100$  mm, according to the relationship shown in Eq. (2), the fraction orders of planes A, B, and C are  $\alpha_A = 1.0$ ,  $\alpha_B = 0.897$ , and  $\alpha_C = 0.778$ , respectively.

Figures 2(b) and (c) are respectively the amplitude hologram and kinoform of Fig. 2(a) with use of M-FrFT. The top-right insets in Figs. 2(b) and (c) are the partial enlarged views of the corresponding holograms. Note that the fringe patterns of the amplitude hologram and the kinoform are different. The computing time for each amplitude hologram or kinoform with pixel number of  $512 \times 512$  is about 1 s using a personal computer with a central processing unit (CPU) operating at 3.2 GHz (Intel Pentium IV) and memory of 512 MB. It should be noted that the computing time will increase almost linearly with the increasing number of layers.

Figure 3 illustrates the numerically reconstructed images from single M-FrFT amplitude hologram at different distances with zero-order light suppressed by the use of mean-value subtraction in the process of the M-FrFT amplitude hologram reconstruction. Figure 4 illustrates the numerical reconstructions from single M-FrFT kinoform. Note that the results of Fig. 4 are directly reconstructed from M-FrFT kinoform without any pre- and post-filtering operation in reconstruction. The reconstructed images in Fig. 3 are disturbed seriously by conjugate images. However, no conjugate image exists in Fig. 4, which improves the quality of the reconstructed images.

It is also important to note that, the reconstructed images in Figs. 3 and 4 are both seriously disturbed by speckle noise, which decreases the contrast and detail resolution of reconstructed images. However, as shown in Fig. 5, the speckle noise is well suppressed after superposing the images reconstructed from 100 M-FrFT kinoforms, and the contrast and detail resolution of reconstructed images are improved significantly. In addition,

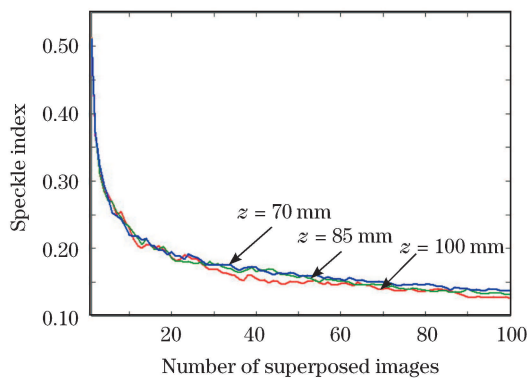


Fig. 6. Variation trends of speckle index according to the superposed number of reconstructed images from M-FrFT kinoforms at different distances.

at the reconstruction distance  $z = 100$  mm, the image of the character “A” is in sharp focus, but the characters “B” and “C” are out of focus. The phenomenon is similar at the reconstruction distances  $z = 85$  or  $70$  mm. It means that the reconstructed image is a 3D image.

In order to evaluate the feasibility and effect of speckle noise reduction with the proposed method in FrFT kinoforms reconstruction, speckle index (SI) is introduced, which is defined as

$$SI = \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N \frac{\sigma(m, n)}{\mu(m, n)}, \quad (15)$$

where  $M \times N$  is the total pixel number of the reconstructed image,  $\sigma(m, n)$  and  $\mu(m, n)$  are respectively the standard deviation and mean value of the  $3 \times 3$  neighborhood of a reconstructed image point  $P(m, n)$ .

Figure 6 illustrates the variation trends of speckle index according to the superposed number of reconstructed images from M-FrFT kinoforms in the reconstruction distance  $z = 70, 85,$  and  $100$  mm, respectively. The SIs of the reconstructed images from single kinoform are 0.512, 0.513, 0.514 at distance  $z = 70, 85,$  and  $100$  mm, respectively. However, by superposing all the images reconstructed from 100 kinoforms at the three distances, the SIs of the reconstructed images decrease to 0.138, 0.133, 0.126, respectively. We also note that the SI decreases slower and slower along with the increasing number of superposed images.

We have also demonstrated that the kinoforms generated by M-FrFT can be reconstructed in normal diffractive electro-holographic display systems based on phase-type spatial light modulator (SLM), but the factors influencing the image scaling and reconstruction distance should be researched in the future work. Compared with other digital filtering methods for suppressing speckle noise of digital images, the advantage of the proposed method is that it is effective to suppress speckle noise in both numerical and optoelectronic reconstructions. In order to suppress the speckle noise of optoelectronic reconstructed image from M-FrFT kinoforms, phase-type SLM with high refresh rate is needed in the

opto-electronic reconstruction process of kinoforms calculated by the proposed method. Now this kind of phase-type SLM with a refresh rate of 60 Hz or faster is available commercially. The improvement of refresh rate of phase-type SLM is urgent and helpful to realize high-quality holographic dynamic 3D display with the use of M-FrFT kinoform sequence.

In conclusion, a novel method based on M-FrFT is proposed to calculate amplitude hologram and kinoform sequence for 3D holographic display. The effects of suppressing speckle noise are analyzed by calculating the speckle index of numerical reconstructed images. It is demonstrated that, by calculating M-FrFT kinoform sequence of 3D objects and superposing the reconstructed images from the kinoform sequence, the zero-order light, conjugate image, and speckle noise can be suppressed, and the contrast and detail resolution of reconstructed images are improved. The proposed method will be useful for high-quality optoelectronic 3D display based on phase-type SLM.

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