Spatial phase-shifting interferometry with compensation of geometric errors based on genetic algorithm

Invited Paper

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We propose a novel spatial phase-shifting interferometry that exploits a genetic algorithm to compensate for geometric errors. Spatial phase-shifting interferometry is more suitable for measuring objects with properties that change rapidly in time than the temporal phase-shifting interferometry. However, it is more susceptible to the geometric errors since the positions at which interferograms are collected are different. In this letter, we propose a spatial phase-shifting interferometry with separate paths for object and reference waves. Also, the object wave estimate is parameterized in terms of geometric errors, and the error is compensated by using a genetic algorithm.

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In digital holography, phase-shifting interferometry is one of the most widely used techniques in biological and environmental sensing applications because of its beneficial capability of measuring phase profiles that remove the ambiguity in the interference pattern between object and reference waves[1,2]. The phase-shifting interferometry is typically categorized into temporal and spatial measurement methods. The temporal phase-shifting method uses a single detector for obtaining the interferograms of the object wave with a reference wave with shifted phase. In the temporal method, the positions of detection for interferograms with different phase shifts are the same, which makes the method robust to geometric errors (the definition of geometric errors can be found in Ref. [3]). But it is more difficult to collect multiple interferograms of an object whose properties rapidly change in real time with the temporal method because it sequentially measures the interferograms.

Meanwhile, the spatial phase-shifting interferometry has been investigated for collecting dynamic measurements. It simultaneously collects multiple interferograms at different collection positions, each corresponding to a phase shift[4,5]. For obtaining multiple spatial phase shifts simultaneously, the optical systems with polarization optics[6,7] and diffraction gratings[8] have been proposed. Takeda considered a similar method utilizing a spatial carrier in the Fourier domain[9]. However, in these methods, precise positional alignment of detectors is typically very difficult. Meanwhile, the invention of detectors with pixelated masks (e.g., micro polarizer and retarder) overcomes such a disadvantage by removing the needs for alignment[10−12]. On the other hand, the pixelated mask phase-shifting interferometry is inevitably subject to a mismatch between the sets of pixels corresponding to different phase shifts within a frame. Such a mismatch has been addressed by interpolation. This aspect of the pixilated mask approach places an important restriction that the method can only measure the object wave with spatial frequencies relatively smaller than the frequency determined by the pixel pitch.

In this letter, we propose a spatial phase-shift interferometry utilizing three charge-coupled devices (CCDs). When using the multiple CCDs, the geometric (i.e., positional) errors of optics cause degradation of reconstructions since the object wave is computed on a pixel

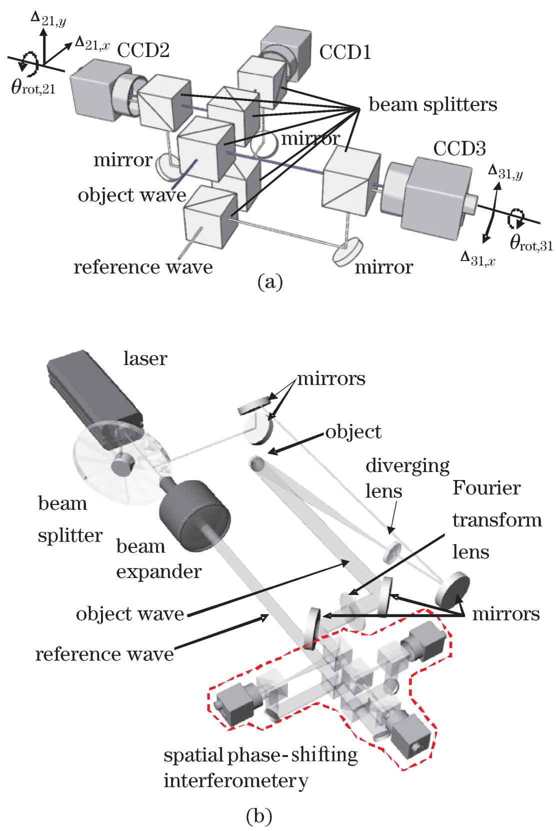

Fig. 1. Schematics of (a) the proposed spatial phase-shifting interferometry and (b) the spatial measurement system with Fourier transform optics.

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basis from the interferograms. In most cases, the geometric errors can be modeled as a function of longitudinal and transverse displacements of CCDs[3]. Therefore, the function in terms of the displacements can serve as a cost function to evaluate the quality of reconstructions, which effectively compensates for the degradation of the reconstructions due to the geometric errors. Since the function is likely to be non-convex in general, a genetic algorithm is used to maximize the cost function in our method.

Figure 1 shows the schematics of our proposed spatial phase-shifting interferometry. We use the Fourier transform optics for measuring the object wave. The Fourier transform optics is appropriate for detecting the scattered wave from the object with diffusive surfaces since there is relatively small contrast in the interferogram within the detectable range of CCD. The proposed phase-shifting interferometry is composed of beam splitters, mirrors, and CCDs. The object wave enters the interferometry and is split into three CCDs, and the reference wave is also split into three in the interferometry. They are then folded by mirrors and combined respectively with the object waves at the beam splitters in front of the individual CCDs.

In phase-shifting interferometry, the ith-step interferogram is represented as

$$I_i = |U_{\rm O} + U_{{\rm R}i}|^2 = A_{\rm O}^2 + A_{\rm R}^2 + 2A_{\rm O}A_{\rm R}\cos(\varphi - \alpha_i) \quad \text{for } i = 1,\ 2,\ \text{and } 3, \tag{1}$$

where the object wave and reference wave are respectively given by

$$U_{\rm O} = A_{\rm O}\exp(\mathrm{j}\varphi), \tag{2a}$$

$$U_{{\rm R}i} = A_{\rm R}\exp(\mathrm{j}\alpha_i). \tag{2b}$$

Using these interferograms, the object wave can be extracted by using

$$U'_{\rm O} = \sum\nolimits_{i\neq 1} a_{i1} I_{i1}. \tag{3}$$

Here, prime means the estimated value calculated from experimental data. I_{ij}, a phase-shifting interferometry base, is defined by

$$I_{ij} \equiv (I_i - I_j)/A_{\rm R}, \tag{4}$$

and a_{i1} is the complex coefficient which means the contribution of each I_{i1} for estimating the object wave[2].

The geometric errors may be decomposed into longitudinal and transverse displacement components at the planes in which CCDs are positioned. The longitudinal displacements of CCDs result in the change in a_{i1} in Eq. (3). To consider the effect of transverse displacements, we assume that both rotational and translational shifts are the first-order mapping. The interferograms and the intensities of reference wave can then be computed by using

$$I_{i,t} = R(\theta_{{\rm rot},i1})\, I_i(x - \Delta_{i1,x}, y - \Delta_{i1,y}), \tag{5a}$$

$$I_{{\rm R}i,t} = R(\theta_{{\rm rot},i1})\, I_{{\rm R}i}(x - \Delta_{i1,x}, y - \Delta_{i1,y}), \tag{5b}$$

where subscript 't' means the values calculated by considering transverse displacements and $R(\theta_{{\rm rot},i1})$ represents a matrix for rotation by the angle $\theta_{{\rm rot},i1}$, $\Delta_{i1,x}$ and $\Delta_{i1,y}$ denote shifts along the x- and y-axis, respectively. Therefore, the object wave in the proposed spatial phase-shifting interferometry can be estimated using

$$\begin{aligned} U'_{\rm O} = {} & [c_{21}(I_{2,t} - I_{{\rm R}2,t}) - (I_1 - I_{{\rm R}1})] \\ & + a_{31}[c_{31}(I_{3,t} - I_{{\rm R}3,t}) - (I_1 - I_{{\rm R}1})], \end{aligned} \tag{6}$$

where c_{21} and c_{31} represent the coefficients to take into account the sensitivity differences between the second and the third CCDs relative to the first CCD. The coefficient a_{21} is set as a unity without loss of generality, because the multiplication of estimated object wave by a constant does not affect the profile of reconstruction image.

Figure 2 shows the flow of the genetic algorithm for compensating the geometric errors. In this letter, the mutation operations are applied for optimization without the crossover operations. The chromosome is defined by

$$\begin{aligned} x_i = \Big\{ & \theta_{{\rm rot},21}, \theta_{{\rm rot},31}, \Delta_{21,x}, \Delta_{21,y}, \Delta_{31,x}, \Delta_{31,y}, \\ & c_{21}, c_{31}, |a_{31}|, \arg(a_{31}) \Big\}. \end{aligned} \tag{7}$$

In the bases of phase-shifting interferometry, the transverse geometric errors increase the non-diffracted terms because the autocorrelation terms in Eq. (1) are not eliminated. The longitudinal geometric errors are related to the twin image noise. Hence, the cost function of the genetic algorithm is defined as

$$\begin{aligned} \text{cost function} = {} & \int_{\Sigma_{\rm OBJ}} \left(|Fr\{F[U'_{\rm O}(x,y)], z_{\rm O}\}|^2 - |Fr\{F[U'_{\rm O}(-x,-y)], -z_{\rm O}\}|^2 \right) \mathrm{d}x\mathrm{d}y \\ & -2\times 10^{-3} \int_{\Sigma_{\rm DC}} |Fr\{F[U'_{\rm O}(x,y)], z_{\rm O}\}|^2 \,\mathrm{d}x\mathrm{d}y, \end{aligned} \tag{8}$$

where F means the Fourier transform and $Fr(U'_{\rm O}, z)$ denotes the Fresnel transform of $U'_{\rm O}(x,y)$ with propagation distance z. Since the original object image and its twin image in the Fourier optics are centrosymmetric at focal plane, the object is located at $+z_{\rm O}$, and the twin image noise appears at $-z_{\rm O}$. In Eq. (8), the first term quantifies the degree of twin image noise. The integral over the region $\Sigma_{\rm OBJ}$ in which the object is placed becomes small when the twin image noise dominates the original object image. The second term represents the amount of the non-diffracted term by the integral of the object wave over the region $\Sigma_{\rm DC}$ over which the non-diffracted term is distributed. For achieving the desirable results with genetic algorithm, the weights of terms in the cost

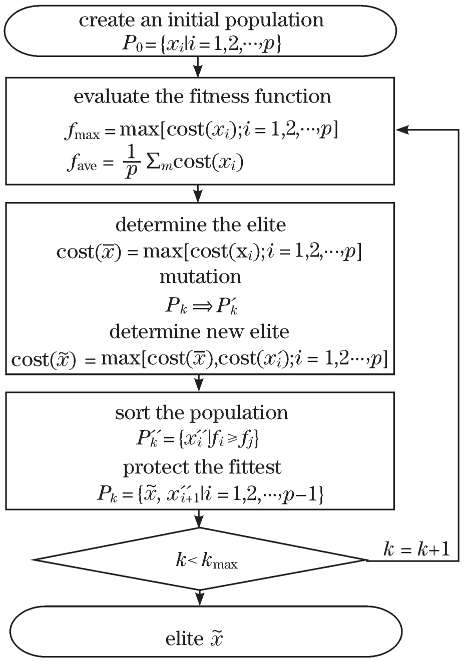

Fig. 2. Flow of genetic algorithm with mutation operation.

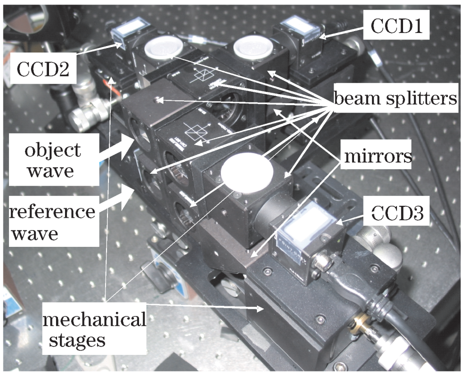

Fig. 3. Setup of the proposed spatial phase-shifting interferometry.

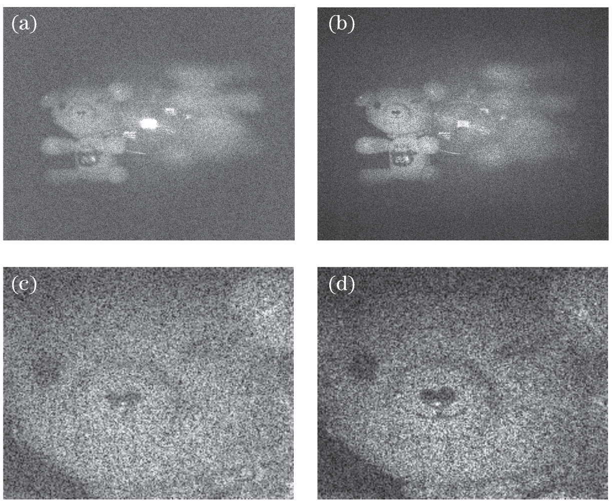

Fig. 4. Numerically reconstructed image (a) before and (b) after compensation of geometric errors. (c) and (d) are the zoom-in images of parts (a) and (b), respectively.

Table 1. Results of Compensation Parameters After Genetic Algorithm.

Parameter	Optimized Value	Parameter	Optimized Value
$\theta_{\text{rot},21}$	$-1.41°$	$\Delta_{31,y}$	-14.66 μm
$\theta_{\text{rot},31}$	$0.09°$	c_{21}	1.08
$\Delta_{21,x}$	-18.66 μm	c_{31}	0.93
$\Delta_{21,y}$	-31.75 μm	$\|a_{31}\|$	1.65
$\Delta_{31,x}$	-4.93 μm	arg (a_{31})	1.52 rad

function should be balanced according to the experimental data. In Eq. (8), the value of the first term is about 2×10^{-3} of that of the second term for the following experiment, so the coefficient of the second term is set as 2×10^{-3}.

For the experiment, a Coherent diode-pumped solid-state (DPSS) Nd:YAG laser with the wavelength of 532 nm is used as a light source, and the focal length of the Fourier lens is 500 mm. We use a Flea2 -14S3M CCD manufactured by Point Grey Research Inc. with IEEE 1394b interface, and the CCD can be synchronized with other detectors in 0.125-μs accuracy. The proposed interferometry acquires three interferograms with the frame rate of 15 frames per second through peripheral component interconnect-express (PCIE) boards. Figure 3 presents a photograph of our actual spatial phase-shifting interferometry instrumentation. The positions of the CCDs are controlled by three-axis mechanical stages, and the relative angles between the reference waves and the combined object waves are also controllable by the mechanical stage holding the folding mirrors.

The object is the bear doll hanging by a fine thread. The spatial phase-shifting method is adopted for capturing rapid variations of the object in real time. The interferograms are acquired with the shutter speed 0.34 ms, and the numerical reconstructions are shown in Fig. 4. The reconstruction images without and with compensation of the geometric errors are compared in Figs. 4(a) and (b), respectively. Figures 4(c) and (d) show the enlarged versions of the reconstructions. After the compensation, the high-frequency details of the object become more prominent and hence more distinguishable. By the compensation using the genetic algorithm, the magnitude of the cost function remarkably increases from -7760 to 60.3 and the twin image noise and non-diffracted term are reduced. It is understood that it is hard to perfectly get rid of them since the geometric errors bring about wrong sampling of the interferograms. The resultant compensated parameters are summarized in Table 1. These results are expected to be applied for adjusting the positions of CCDs.

In summary, we have proposed a method of compensating the geometric errors using a genetic algorithm in a spatial phase-shifting interferometry. This method implements separate paths for object and reference waves, which makes it easier to identify the effects of geometric errors. In our method, the object wave is expressed in terms of transverse and longitudinal geometric parameters, and the cost function is defined as a function of these positional parameters for quantifying twin image noise and the effect of non-diffracted term in terms of the parameters. The effectiveness of our compensation method using the genetic algorithm has been experimentally demonstrated.

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References

1. I. Yamaguchi, "Phase-shifting digital holography" in T.-C. Poon, (ed.) *Digital Holography and Three-Dimensional Display* (Springer, Berlin, 2006) pp.145−171.
2. J. Hahn, H. Kim, S.-W. Cho, and B. Lee, Appl. Opt. **47,** 4068 (2008).
3. J. Hahn, H. Kim, E.-H. Kim, J. Park, and B. Lee, Proc. SPIE **7056,** 70561N (2008).
4. M. Kujawinska, "Spatial phase measurement methods" in D. W. Robinson and G. T. Reid, (eds.) *Interferogram Analysis, Digital Fringe Pattern Measurement Techniques* (Taylor & Francis, London, 1993) pp.141−193.
5. J. C. Wyant, Opt. Photon. News **14,** (4) 36 (2003).
6. R. Smythe and R. Moore, Opt. Eng. **23,** 361 (1984).
7. H. Kihm and S.-W. Kim, Opt. Lett. **30,** 2059 (2005).
8. O. Y. Kwon, Opt. Lett. **9,** 59 (1984).
9. M. Takeda, Ind. Metrol. **1,** 79 (1990).
10. J. P. Guo and D. J. Brady, Opt. Eng. **36,** 2268 (1997).
11. J. Millerd, N. Brock, J. Hayes, M. North-Morris, M. Novak, and J. C. Wyant, Proc. SPIE **5531,** 304 (2004).
12. Y. Awatsuji, M. Sasada, and T. Kubota, Appl. Phys. Lett. **85,** 1069 (2004).