# Estimation and optimization of computer-generated hologram in null test of freeform surface

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Freeform surfaces are increasingly used in the design of compact optical systems. Interferometric null test with computer generated hologram (CGH), which has been successfully used in highly accurate test of aspheric surfaces, is adopted to test the freeform surfaces. The best fitting sphere of the freeform surface under the test is firstly calculated to quickly estimate the possibility of null test. To decrease the maximum spatial frequency of the null CGH, the position of the CGH and the direction of optical axis are optimized. The estimated maximum spatial frequency of the CGH is 7.8% apart from the optimized one, which shows the validity of the best fitting sphere.

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Freeform optical elements can reduce the number of elements and light path length as well as improve the quality of optical systems. With the development of nanofabrication and testing technology[1], freeform surfaces are increasingly used in the design of compact optical systems[2]. Freeform optical surfaces with aperture of 1−100 mm, surface roughness of sub-micro or nanometer level, and surface vertical range above 100-$\mu$m level, can still hardly be tested online or in workshop economically, simply, and quickly.

The interferometric method with computer generated hologram (CGH) has been used to test aspheric optical elements for more than 30 years[3], and is now widely used in the precise testing of aspheric optical elements [4−10]. A commercial phase shifted Fizeau or Twyman-Green wavefront measuring interferometer, which can provide a reference spherical wave, is often used in null test to simplify the experiment. The null CGH is designed to produce a wavefront that compensates the departure of the tested surface from a spherical one. The CGH used as a null corrector is usually located in the convergence or divergence beam of the gage beam of the interferometer, as shown in Fig. 1.

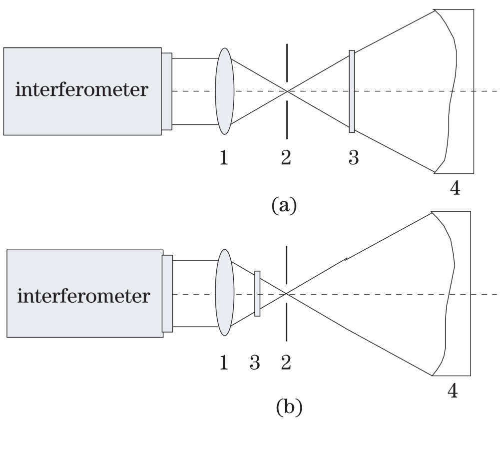


Fig. 1. Optical layouts of null CGH test. 1: Transmission sphere; 2: Pinhole; 3: Null CGH; 4: Tested surface.

The difficulty of aspheric surface testing depends on the asphericity of the tested surface[11]. A novel method for calculating the asphericity of high-order optical aspheric surface is to find a best fitting sphere and define the maximum deviation with the sphere as the asphericity[12]. The best fitting sphere of a specified surface has the minimum surface deviation.

Unsymmetrical aspherical cubic phase plate has been tested by using the CGH[13]. But a freeform optical surface may have a large vertical range and partial steep slope, corresponding to large line density of the null corrector CGH, so it might not be null tested. For freeform surfaces testing, a fast estimation of the testing feasibility and optimization of the optical arrangement to get an easier-fabricated CGH are needed. The method of the best fitting sphere is adopted and developed here to quickly and easily estimate the maximum line density, i.e., the fabrication feasibility of the null CGH. To decrease the maximum spatial frequency of the null CGH, the position of the CGH and the direction of optical axis are optimized.

The wavefront reflected from the freeform surface has high-order aberrations, and the CGH with the tested surface is preferred to be positioned in the same side of the cat's eye, as shown in Fig. 1(a). For easier calculation

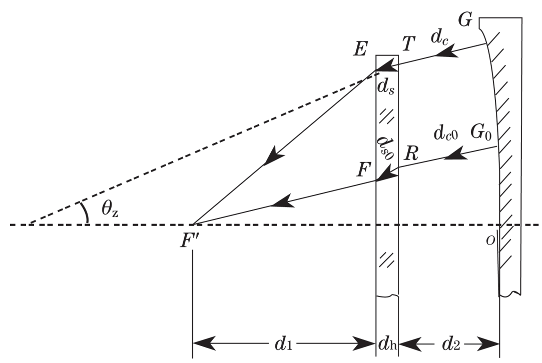


Fig. 2. Geometry for defining the CGH phase distribution in CGH null test of a freeform surface.

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and experiments and higher utility rate of luminous energy, the optical axis is better through or near the central point of the tested surface. The direction of optical axis should be set to ensure that the deviation between the tested surface and the best fitting sphere approximates to central symmetry.

Supposing a freeform surface is descried in a set coordinate as $z = f(x, y)$ and the optimized optical axis is $z$-axis, the deviation $\delta$ in the normal direction of spherical surface between the freeform surface and a spherical surface with the radius of $R$ and center of $(x_0 = 0, y_0 = 0, z_0)$ is

$$\delta(x,y,z) = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} - R. \quad (1)$$

The gradient modulus of $\delta$ is

$$\|\nabla\delta\| = \left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right]^{-\frac{1}{2}} \times \sqrt{\left[x - x_0 + (z-z_0)\frac{\partial z}{\partial x}\right]^2 + \left[y - y_0 + (z-z_0)\frac{\partial z}{\partial y}\right]^2}, \quad (2)$$

whose maximum value determines the maximum line density of the null corrector CGH.

According to Eq. (2), the gradient modulus has nothing to do with $R$. So firstly we can find a sphere center $(x_0 = 0, y_0 = 0, z_0)$ along the optical axis, where the maximum $\Delta\delta$ all over the tested surface is the minimal, namely

$$\min_{(x_0,y_0,z_0)\in\text{optic axis}} \left( \max_{(x,y,z)\in\text{tested surface}} \|\nabla\delta\| \right). \quad (3)$$

As the spherical central point is determined, the deviation $\delta$ is only related to $R$ according to Eq. (1). The radius value of the best fitting sphere can be obtained according to

$$\min_{R} \left( \max_{(x,y,z)\in\text{tested surface}} \delta \right). \quad (4)$$

If another direction of optical axis is chosen, it is necessary to transform the coordinate system to make the optical axis as $z$-axis first. The gradient modulus is corresponding to the spatial frequency of the null CGH, and

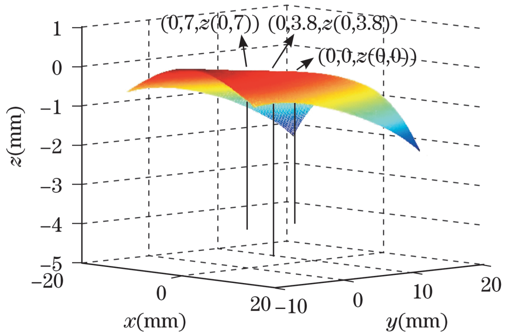


Fig. 3. A freeform surface with the expression of Eq. (7). The line with $(0, 0, z(0, 0))$ is $z$-axis; the line with the center point $(0, 3.8, z(0, 3.8))$ is the normal for the point; the line with the point $(0, 7, z(0, 7))$ is perpendicular to the surface determined by four edge points.

then the maximum spatial frequency of the null CGH can be estimated. At present 5-$\mu$m line spacing can be realized by several fabricating technologies, for example, single point diamond cutting, laser or e-beam direct writing, and so on. If the maximum spatial frequency of the CGH is less than or around 200 lp/mm, the freeform surfaces can be null test. Here it should be emphasized that, in Eq. (1), the deviation between the freeform surface and the best fitting sphere is calculated along the normal direction of the best fitting sphere, but actually in the design of the null CGH, rays should propagate along the normal direction of the freeform surface. Hence the practical maximum spatial frequency of the null CGH will be apart from the estimated one, but as the surface to be tested is usually very near to its best fitting sphere, the difference between the practical and the estimated maximum spatial frequencies is very small.

The partial optical configuration from the cat's eye point to the tested surface is redrawn in Fig. 2. A spherical wavefront from the cat's eye point $F'$ is transformed into a freeform wavefront by the CGH to compensate the tested freeform surface. An arbitrary ray in the gage beam comes from point $G(x, y, z)$ on the tested freeform surface in the normal of the tested surface at point $G$, and then goes to $T(x_c, y_c, z_c)$ on one side of the CGH positioned $d_2$ from the central point of the tested surface. The ray goes out from the point $E(x_s, y_s, z_s)$ on the other side of the CGH, and focuses to cat's eye point $F'$ in the optical axis, where $F'$ is at the distance of $d_1$ from the CGH.

According to the aplanatic principle, all optical paths from $F'$ to the tested surface are equal. Here the ray through the point $G_0$ on the surface is supposed to be the reference ray, and considering arbitrary ray launches onto $G$ on the tested surface along the corresponding normal direction, the optical path difference is

$$\omega(x_s, y_s, z_s) = |F'E| + n_2|ET| + |TG| - |FF'| - n_2|FR| - |RG_0|, \quad (5)$$

where $n_2$ is the refractive index of the substrate of the CGH. $|F'E|$, $|ET|$, $|TG|$, $|FF'|$, $|FR|$, and $|RG_0|$ can be calculated by ray tracing. The phase distribution of the CGH is

$$\phi(x_s, y_s, z_s) = 2\pi \cdot \omega(x_s, y_s, z_s)/\lambda, \quad (6)$$

where $\lambda$ is the working wavelength.

For example, a freeform surface under test is described as

$$z = \frac{c\left(x^2+y^2\right)}{1+\sqrt{\left[1-c^2\left(x^2+y^2\right)\right]}} + C_3 y + C_4 x^2 + C_6 y^2, \quad (-13 \le x \le 13, \quad -8.3 \le y \le 15.9), \quad (7)$$

where $c = -2.85 \times 10^{-4}$, $C_3 = -0.046$, $C_4 = -0.00953$, and $C_6 = 8.88 \times 10^{-4}$. The units of $x$, $y$, and $z$ are millimeters. $\lambda$ is 632.8 nm, the substrate of the CGH is fused quartz, and the refractive index $n_2$ is 1.45726. The thickness of the CGH substrate $d_h$ is 3 mm. In practice, $d_2$ must be chosen by considering the interference between the holding tools of the CGH and the tested surface, and to be as small as possible to avoid intersection of rays. We finally choose $d_2$ to be 20 mm.

Firstly we choose the optical axis as $z$-axis, namely,

$x_0 = 0$ and $y_0 = 0$ for the sphere center, as shown in Fig. 3. The maximum of the gradient values of the deviation all over the tested surface varying with $z$-coordinate value of the sphere center is calculated, as shown in Fig. 4(a). The center of the best fitting sphere is (0, 0, −90.4). After optimizing the sphere center, the maximum deviation varying with sphere radii is calculated, as shown in Fig. 4(b). The radius of the best fitting sphere is 90.8 mm. For the best fitting sphere, the maximum departure is 410 $\mu$m and the maximum gradient modulus is 0.12 corresponding to the spatial frequency of 190 lp/mm, which ensures the feasibility of the null test for the specified freeform surface.

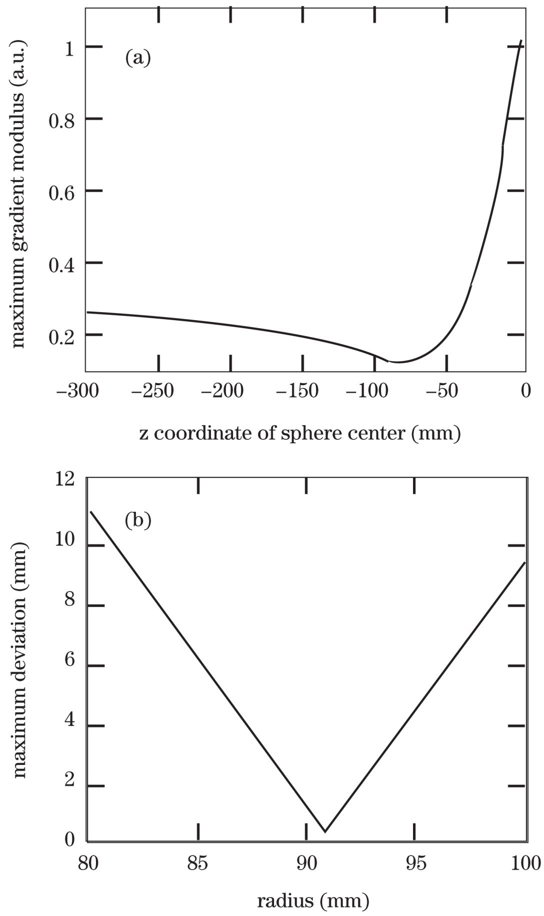


Fig. 4. (a) Maximum versus $z$-coordinates of sphere center; (b) maximum deviation versus sphere radius.

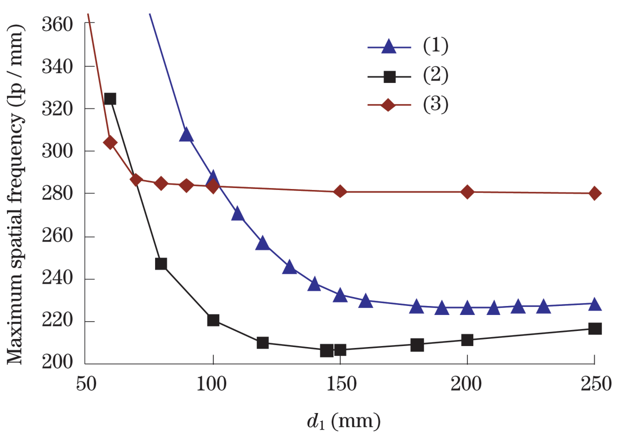


Fig. 5. Relationship between the distance $d_1$ and the maximum spatial frequency, for the optical axis chosen (1) as z axis; (2) as normal for the center point; (3) to be perpendicular to the surface determined by four edge points.

According to the fast estimation with the method of the best fitting sphere, the freeform surface expressed by Eq. (7) can be null tested owing to the estimated maximum spatial frequency. Then the phase distribution of the practical CGH is calculated based on Eqs. (5) and (6). When $d_2$ is 20 mm, the maximum spatial frequencies of the CGH varying with $d_1$ are calculated as shown in Fig. 5. We choose three different directions of optical axis shown in Fig. 3, which are the line with (0, 0, $z$(0, 0)) being $z$-axis, the line with the center point (0, 3.8, $z$(0, 3.8)) being the normal of the point, and the line with the point (0, 7, $z$(0, 7)) being perpendicular to the surface determined by four edge points. To get easier-fabricated CGH, $d_1$ should correspondingly be chosen in the flat part of the curves, namely, >160, >145, and >90 mm, respectively. The maximum spatial frequencies are 228, 206, and 280 lp/mm, respectively. The three directions of optical axes are in a range of less than 5°, but the corresponding maximum spatial frequency of the CGH varies about 26%. Therefore, the phase distribution of the null CGH is very sensitive to the direction of optical axis. Unfortunately, there is no common method to optimize the direction of optical axis for all freeform surfaces.

Obviously, the CGH with the smallest maximum spatial frequency will be used as the null CGH. The optimized optical axis being the normal of the center point (0, 3.8, $z$(0, 3.8)), the value of $d_1$ >145 mm, and $d_2 = 20$ mm will be adopted in future experiment. The practical maximum spatial frequency of 206 lp/mm is very near to the estimated one of 190 lp/mm, which shows the validity of the method of the best fitting sphere.

In conclusion, interferometric null test with CGH is used to test the freeform surface. The method of the best fitting sphere is developed to quickly estimate the possibility of null test for a freeform surface. Subsequently the phase distribution of null CGH is optimized by changing the direction of the optical axis and the position of the CGH to decrease the maximum line density of the CGH. For a specified freeform surface with 1.89-mm PV value over a 26×24.2 mm$^2$ area, the phase distribution and position of null corrector CGH is optimized, and the maximum line density is 206 lp/mm, which ensures the possibility of null test for the freeform surface. The estimated maximum spatial frequency of the CGH is 7.8% apart from the optimized one, which shows the validity of the method of the best fitting sphere. According to intuition, three directions of optical axis are used to design the CGH. The phase distribution of the CGH is very sensitive to the direction of optical axis. Unfortunately, the optimization of the optical axis, which is very important and complicated, is not solved in this letter. New method should be developed to optimize the optical axis.

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