An enhanced method for fast generation of hologram sub-lines

P. W. M. Tsang[1*], J. P. Liu[2], K. W. K. Cheung[1], and T.-C. Poon[3]

[1] *Department of Electronic Engineering, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong SAR, China*

[2] *Department of Photonics, Feng Chia University, Taiwan, No. 100 Wenhwa Rd., Seatwen, Taichung, 40724, China*

[3] *Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, Virginia 24061, USA*

[*] *E-mail: eewmtsan@cityu.edu.hk*

Received November 2, 2009

Sub-lines are one-dimensional diffraction patterns representing the light beams emerging from horizontal planes of an object image. Past research has demonstrated that the sub-lines can be encapsulated as a multi-bank filtering process, and implemented with a field programmable gate array (FPGA) device. As the complexity of the filters is high, their length and the number of input pins have to be reduced substantially, hence leading to degradation on the reconstructed images. We propose an enhanced method to overcome the problem by binarizing the filters' coefficients, and half-toning the pixel intensities of the object image. Experimental evaluation reveals that our method results in reconstructed images are superior to that obtained with the parent method.

OCIS codes: 090.0090, 090.1995.

doi: 10.3788/COL20090712.0000.

An effective means to generate holograms of digital three-dimensional (3D) models is to superimpose the optical waves of individual object points onto the hologram plane[1]. At present, such approach has become one of the de facto in producing both white light and Fresnel holograms. The major disadvantage of this method lies in the enormous amount of arithmetic operations involving in the formation of the hologram. In the past, numerous research works such as Refs. [2−4] have been conducted to overcome this problem from a fundamental nature, or through the use of high-speed processing units[5−7]. An alternative approach has been adopted by Yoshikawa[8] who suggested that the generation of rainbow holograms can be decomposed into a pair of one dimensional (1D) processes. In the first stage, a 3D object scene is sectioned into a uniformly spaced stack of horizontal scan-planes. A 1D diffraction pattern known as sub-line is then generated for each of the scan-plane. Next, a holo-line is generated by adding a reference wave to each sub-line. Finally, a collection of the holo-lines along the vertical direction forms a two-dimensional (2D) white light (rainbow) hologram. The method is computationally efficient, and also portrays the framework of a holographic video system as shown in Fig. 1, where the low data-rate sub-lines can be transmitted to the receiving end through the existing distribution media.

Recently, Poon *et al.*[9] proposed an optical method to generate holo-lines from real objects to form a horizontal-parallax-only (HPO) hologram using optical scanning holography. Most recently, Tsang *et al.*[10] proposed a method to generate Fresnel holograms from sub-lines of a 3D object image which is centered around a depth $z = z_o$. A brief outline of this approach is given as follows. To begin with, the following terminology is adopted. Let X and Y be the horizontal and vertical extents of the hologram to be generated. Suppose there are $N(n)$ object points on the nth scan-plane, and the distance of the object points is sufficiently far away from the hologram, Fresnel approximation is taken and the sub-line at the hologram plane is given by

$$O\left(m,n\right)_{0\leq m<X-1} = \sum_{j=0}^{N(n)-1} \frac{a\left(n\right)_j \exp\left(\mathrm{i}k(m\delta x - m_j\delta x)^2\Big/2z_j\right)}{\sqrt{\left(m\delta x - m_j\delta x\right)^2 + z_j^2}} = \sum_{j=0}^{N(n)-1} a\left(n\right)_j G\left(m-m_j\right), \quad (1)$$

where $G\left(m-m_j\right) = \dfrac{\exp\left(\mathrm{i}k(m\delta x - m_j\delta x)^2\Big/2z_j\right)}{\sqrt{\left(m\delta x - m_j\delta x\right)^2 + z_j^2}}$, λ is the wavelength of the optical beam, and $k = 2\pi/\lambda$ is the wave number. $a\left(n\right)_j$ and m_j denote the amplitude and the horizontal position of the jth object point on the nth scan-plane, respectively. δx is the pixel width of the object scene and z_j is the perpendicular distance (i.e., the depth) of the object point from the hologram plane. The quantity z_j is quantized into discrete value hereafter referring to as the "depth plane". The vertical position

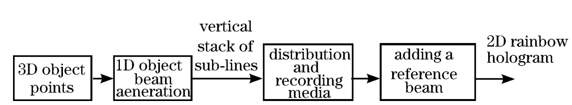

Fig. 1. A holographic video system based on Ref. [8].

'y' of the scan-plane is given by $y = n\delta y$, where δy is the separation between an adjacent pair of scan-planes. We have also assumed that δx and δy are identical to the width and the height of the pixels in the hologram, respectively.

If we assume that z_j is close to a fixed depth at z_o, it has been shown that a Fresnel hologram $H(m,n)$ can be generated from the holo-lines as[10]

$$H(m,n)_{0\leq n<Y} = \mathrm{Re}\left\{[O(m,n) * B(n)]\Re(n)\right\}, \qquad (2)$$

where $*$ denotes 1D convolution, $B(n) = \exp\left(\mathrm{i}k\frac{(n\delta y)^2}{2z_o}\right)$, and $\Re(n) = \exp(\mathrm{i}n\delta y)$ is a 1D reference beam. As it turns out, the algorithm derived in Eq. (2) has shown to save a tremendous amount of computation and yet without much undesirable effects upon the reconstruction of such hologram[10].

At present, there are several ways of generating sub-lines or holo-lines. The first approach is through numerical calculation based on Eq. (1), but the speed is relatively slow even with modern computers. Secondly, holo-lines can be captured directly with optical scanning[9]. However the method is only applicable to real objects, and requires precise optical setup. The third way encapsulates sub-line generation process into the juxtaposition of finite impulse response (FIR) filters which can be realized with field programmable gate array (FPGA) device, and operate at a rate of over 100M pixels per second[11]. Due to the high complexity of the FIR filters, two restrictions have been imposed so that the sub-line generator can be housed in a single FPGA device. First, the number of input pins of each FIR filter is reduced to 2 bits. Consequently, the intensities of the object image have to be re-quantized to 2 bits to match the input of the filters, leading to heavy degradation on the source image. Second, the length of the filter (i.e., the number of coefficients), which is originally equal to the extent of sub-line, is shortened to 256. The reduction jeopardizes the fidelity of sub-lines, as well as the Fresnel holograms generated from them, in representing the object image.

In this letter, we noted that the complexity of an FIR filter can be lowered by reducing the size of the input data bus, number of bits employed to quantize the tap coefficients, as well as the length of the filter. On this basis, we propose an enhanced sub-lines generation scheme so that the abovementioned problems can be alleviated. The solution can be realized with a single FPGA device with smaller amount of logic elements. Following the introduction, the implementation of the sub-line generator based on multi-bank filtering[11] is outlined. Then, we analyze the factors governing the tradeoff between the complexity of the sub-line generator and its capability in preserving the fidelity of the source images. Next we propose a scheme to provide a good compromise between these two factors, and demonstrate its merits over the parent method. Subsequently, a conclusion is given with summarizing the essential findings.

For the sake of clarity, a brief outline on Ref. [11] is presented. To begin with the following assumptions have been made. First, at each horizontal position there is at most a single object point. In other words, the complete collection of the scan-planes represents a 3D surface of the scene. Second, the horizontal extent of a sub-plane is the same as that of a sub-line (i.e., X'). Third, δx is the width of a pixel on the sub-plane. Fourth, δy is the separation between a pair of adjacent sub-planes. Fifth, the depth "z" is quantized in L levels and denoted by $z(q)|_{1\leq q<L}$.

At each depth level 'q' on a scan-plane, a sequence $p_q(m,n)|_{0<m<X-1}$ (where m and n denote the horizontal and vertical positions, respectively, of the object points) is generated as

$$p_q(m,n) = \begin{cases} \text{intensity of an object point if exists} \\ 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{cases}. \qquad (3)$$

With this arrangement, the length of the sequence $p_q(m,n)$ will be fixed at M and independent on the number of object points. The horizontal position of an object point, or a pixel on a sub-line, is equal to the product of its index 'm' and the spatial distance between adjacent samples. Applying Eq. (1), a sub-line for the nth scan-plane is generated by summing the contributions from all the L sequences (each representing a depth level) as

$$\begin{aligned} O(m,n) &= \sum_{q=1}^{L}\sum_{j=0}^{X-1}\frac{p_q(m,n)\exp\left(\mathrm{i}k(m\delta x - j\delta x)^2\big/2z_q\right)}{\sqrt{(m\delta x - j\delta x)^2 + z_q^2}} \\ &= \sum_{q=0}^{L-1}\sum_{j=0}^{X-1} p_q(m,n)\,G_q(m-j) \\ &\Rightarrow O(m,n) = \sum_{q=0}^{L-1} p_q(m,n) * G_q(m). \end{aligned} \qquad (4)$$

However based on the assumptions at the start of this section, at most one object point is present for each value of m, and $p_{\mathrm{q}}(m,n)|_{1\leq q\leq L}$ can be grouped into a single sequence of order pairs as $P(m,n) = [(p_{\mathrm{a}}(0,n), p_{\mathrm{g}}(0,n)), (p_{\mathrm{a}}(1,n), p_{\mathrm{g}}(1,n)), \cdots, (p_{\mathrm{a}}(X-1,n), p_{\mathrm{g}}(X-1,n))]$, where $p_{\mathrm{a}}(m,n)$ and $p_{\mathrm{g}}(m,n)$ are the amplitude and depth of the 'mth' object point. Eqatiov (4) can be rewritten as

$$O(m,n) = \sum_{q=0}^{L-1}\left[p_{\mathrm{a}}(m,n)\,s_q(p_{\mathrm{g}}(m,n))\right] * G_q(m). \qquad (5)$$

Noted that in Eqs. (4) and (5) the variable 'n' denotes the vertical position of the scan-plane of interest, and is not directly involved in the derivation of the sub-line, where $s_q(p_{\mathrm{g}}(m,n)) = 1$ if $p_{\mathrm{g}}(m,n) = q$, and zero otherwise. Eqatiov (5) can be further encapsulated as a multi-bank FIR. filtering process as shown in Fig. 2, where the implementation of Eq. (5) makes use of the AND gates.

In Fig. 2, the multiplexer plays the role of $s_q(\bullet)$ and directs object points at different depth levels to their corresponding FIR filter inputs. $G_q(m)$ is the unit impulse response of the filter for depth level q as given by

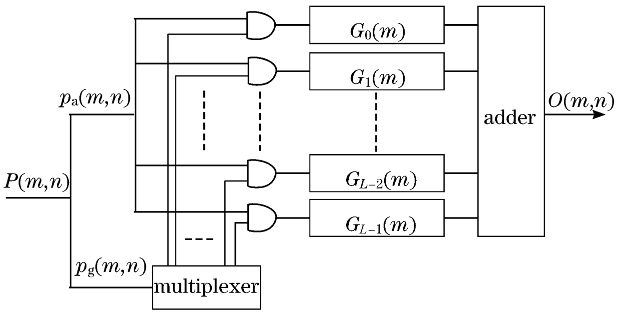

Fig. 2. Sub-line generation encapsulated as a multi-bank filtering process.

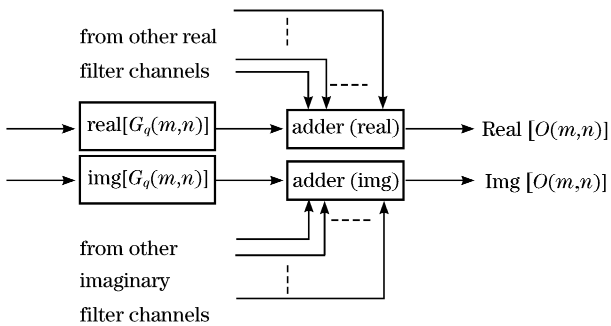

Fig. 3. Structure of the complex FIR filter.

$$G_q(m) = \frac{\exp\left(\mathrm{i}k(m\delta x)^2 \Big/ 2z_q\right)}{\sqrt{(m\delta x)^2 + z_q^2}} = \frac{\cos\left(k(m\delta x)^2 \Big/ 2z_q\right)}{\sqrt{(m\delta x)^2 + z_q^2}} + \mathrm{i}\frac{\sin\left(k(m\delta x)^2 \Big/ 2z_q\right)}{\sqrt{(m\delta x)^2 + z_q^2}}, \quad (6)$$

where $G_q(m)$ is a complex function which can be implemented with a pair of FIR filters as shown in Fig. 3. However for the sake of simplicity in description, we shall treat $G_q(m)$ as a single FIR filter with complex tap coefficients.

To implement the sub-line generator with FPGA device, the number of logic elements (LE) is approximately governed by the follow relationship:

$$\mathrm{LE} \propto (I_{\mathrm{bit}} \times T_{\mathrm{len}} \times C_{\mathrm{q}} \times L), \quad (7)$$

where I_{bit} and C_{q} are the number of bits representing the input data (i.e., the intensity of the object points) and the coefficients of the FIR filter. T_{len} is the tap length of the latter, and L is the number of depth plane.

Without loss of generality, the sub-line generator is assumed to be implemented based on the Altera Cyclone II FPGA chip which contains a total of 68416 LEs. The method can be extended to other FPGA devices with only minor modification. In Ref. [11], the optical setting and the FIR filter specification in Table 1 are adopted to allow sub-lines representing 8 depth levels to be generated on a single FPGA chip. The design, developed under the Quartus II 6.0 design environment, consumed 58256 out of a total of 68416 LEs (around 85%), and the throughput is 100M pixels per second. As the intention of the letter is to establish a functional framework suitable for hardware implementation, the choice of parameters are rather ad hoc and only serves to reflect the feasibility of the approach.

Based on the above configuration, the sub-line can only record a crude approximation of the object image. The latter is distorted from the inadequate number of bits to represent the gray intensities of the pixels. As an example the original image 'Lenna' with pixels represented in 8 bits, and its appearance after re-quantization to 2 bits, are shown in Figs. 4(a) and (b), respectively.

It can be seen that the degradation is rather prominent with a reduction to 2 bits. On top of that the shortened filter length also decreases the fidelity of the sub-lines in representing the object images. To illustrate the effects caused by these two kinds of distortions, the sub-lines representing the 2-bits 'Lenna' image in Fig. 4(b) are generated with the setting in Table 1, and converted into a Fresnel hologram with the method reported in Ref. [10]. The image is reconstructed from the hologram with Holovision[12] and shown in Fig. 4(c). It can be seen that the image is rather blurred and the overall visual quality is poor. Although the quality of the sub-lines can be enhanced by increasing the values of I_{bit} and T_{len}, the number of LEs will far exceed the limit of the FPGA chip.

In this letter, we proposed to simplify the scheme in Ref. [11] so that the sub-line generation process can be realized with FIR filters of lower complexity. Essentially we are preserving the multi-bank filter architecture as depicted in Fig. 2, but exploring measures to reduce the parameters I_{bit}, C_q, and T_{len}, while at the same time improving the quality of the reconstructed images. Our method can be divided into three parts and outline as follows.

According to Fig. 2, each FIR filter accepts the input data $p_{\mathrm{a}}(m,n)$ which is quantized into certain number of bit-planes. As the latter determines the number of input pins I_{bit}, a reduction of which will also decreases the complexity of the filter. In the first part of simplification, we propose to reduce the intensity of the object points to a one-bit representation. However as shown in Fig. 4(b), direct re-quantization of $p_{\mathrm{a}}(m,n)$ will lead to heavy distortion. To overcome this problem, the object is first decomposed into a depth map $p_{\mathrm{g}}(m,n)$, and a texture map $T_{\mathrm{X}}(m,n)$ as

$$S = \{p_{\mathrm{g}}(m,n), T_{\mathrm{X}}(m,n)\}, \quad (8)$$

Table 1. Details of Optical Setting and FIR Filter Specification

I_{bit}	C_{q}	T_{len}	λ	δx
2 bits	8	256	650 nm	10.583 μm
δy	X	Y	z_o	L
10.583 μm	1024	1024	0.4 m	8

Fig. 4. (a) Original image 'Lenna' (8 bits); (b) re-quantization of image 'Lenna' with 2 bits; (c) reconstruction result of a hologram representing the image in (b). The hologram is derived from the sub-lines generated with the setting in Table 1, followed by applying Eq. (2).

Fig. 5. (a) The half-toned 'Lenna' image; (b) reconstruction of a hologram representing (a). The hologram is derived from sub-lines generated with 8-bit filter coefficients based on the optical setting in Table 1, followed by applying Eq. (2). (c) Reconstruction of a hologram representing (a). The hologram is derived from sub-lines generated with 1-bit filter coefficients based on the optical setting in Table 1, followed by applying Eq. (2).

where S is the set of object points that forms the 3D surface. $p_{\mathrm{g}}(m,n)$ and $T_{\mathrm{X}}(m,n)$ are 2D images containing the depth (i.e., the index 'q') and intensity of each object point, respectively.

Next, the texture $T_{\mathrm{X}}(m,n)$ is converted into a binary image $p_{\mathrm{a}}(m,n)$ with the half-toning technique capable of retaining reasonable visual quality in gray images with a one-bit representation. To avoid complicating the design of the sub-line generation process, the classical Bayer's matrix[13] is employed in the half-toning process, although the quality of the binarized image can be improved with more sophisticated algorithms. The original texture map is approximated by the binary image $p_{\mathrm{a}}(m,n)$, i.e.,

$$S' = \{p_{\mathrm{g}}(m,n), p_{\mathrm{a}}(m,n)\}, \tag{9}$$

where $p_{\mathrm{a}}(m,n)$ and $p_{\mathrm{g}}(m,n)$ are input to the filter banks in Fig. 2 to generate the sub-lines. As $p_{\mathrm{a}}(m,n)$ is represented with 1 bit, a single input pin for each filter (i.e., $I_{bit} = 1$) will suffix, leading to the reduction in its complexity.

In the second part of simplification, we explore the relation between the fidelity of the hologram in representing the object image, and the quantization of the impulse response function (i.e., the filter tap coefficients) of $G_q(m)$. Evaluation reflects that the latter can be quantized coarsely and still results in reconstructed images of reasonable quality. To illustrate this, two groups of sub-lines are generated for the half-toned image 'Lenna' shown in Fig. 5(a) with filter coefficients quantized with 8 bits and 1 bit, as shown in Figs. 5(b) and (c), respectively. For the latter, each coefficient in the real or imaginary part of the filter is converted to +1 and −1 according to its polarity. The sub-lines in each test case are converted into a Fresnel hologram with Eq. (2), and the object image is reconstructed with Holovision. It can be seen that the difference in appearance of the pair of reconstructed images is not prominent. As a result of this finding, the FIR filters in the sub-line generator are designed with 1-bit tap coefficients, leading to the reduction in the number of LEs.

Similar to the strategy adopted in Ref. [11], further reduction in the complexity of the sub-line generator is achieved by reducing the length of the FIR filters. As explained before, although shorter filter length is preferred for reducing the complexity of the filters, it will increase the degradation on the holograms. We have evaluated the quality of the reconstructed images based on different values of T_{len}, and deduced that a good compromise can be made with filter length of 360 coefficients, which is longer than that in Table 1.

Based on the above analysis, the optical settings and the FIR filter specification listed in Tables 1 and 2, respectively, are adopted to generate sub-lines for the half-toned 'Lenna' image in Fig. 5(a). Subsequently, the sub-lines are converted to a Fresnel hologram with Eq. (4), and the image reconstructed with Holovision is shown in Fig. 6. Comparing with Fig. 4(c) which is obtained with the parent method in Ref. [11], the visual quality of the image derived from our method is apparently clearer and more appealing. In addition, a reduction of over 50% in the usage of LEs is noted when our scheme is implemented on the same FPGA device adopted in Ref. [11].

Similar evaluation is conducted in applying the proposed method on the half-toned image shown in Fig. 7(a). The latter contains a logo (upper half of the image) and a text message (lower half of the image), located at 51.5 and 48.5 cm from the hologram, respectively. The images reconstructed from the hologram at these two distances are shown in Figs. 7(b) and (c). It can be seen that the logo and the text message are reconstructed with reasonable quality.

In conclusion, the generation of white light and Fresnel holograms can be speeded up with the use of sub-lines which are 1D diffraction patterns each representing a horizontal scan-plane of a 3D object scene. An effective method of deriving the sub-lines is to encapsulate the process as a multi-bank FIR filter architecture which can be directly realized with FPGA devices. Despite the success of this approach, implementation of the FIR filters involves large amount of LEs, hence jeopardizing the value of the method in practical applications. In this letter, we have proposed an enhanced method which enables sub-lines to be generated with FIR filters of lower complexity. Our method can be divided into three parts. First, the object scene image is converted into a halftone image. Second,

Table 2. FIR Filter Setting in Proposed Method

I_{bit}	C_{q}	T_{len}	L
1 bit	1 bit	360	8
Number of LEs			
27568 out of a total of 68416 LEs (40%)			

Fig. 6. Reconstruction of the 1-bit, half-toned 'Lenna' image from hologram derived from sub-lines generated with the FIR filter setting in Table 2.

Fig. 7. (a) A half-toned image containing a logo (upper half of the image) and a text message (lower half of the image) located at 51.5 and 48.5 cm from the hologram, respectively. (b) Reconstruction of the hologram, generated with the proposed method, representing the half-toned image in (a). The reconstruction distance is 48.5 cm. (c) Reconstruction of the hologram, generated with the proposed method, representing the half-toned image in (a). The reconstruction distance is 51.5 cm.

the coefficients of the FIR filters are binarized to one bit. Finally, we adjusted the length of the filter so that a good compromise is made between the complexity of the latter and the visual quality of the reconstructed images. The proposed solution is implemented on a FPGA device and two improvements are noted as compared with the existing method reported in Ref. [11]. On the hardware aspect, a reduction of over 50% in the usage of LEs is noted, implying that the sub-line generator can be realized with more simple and economical FPGA devices. In the generation of Fresnel holograms based on the sub-lines derived with our scheme, the visual quality of the reconstructed images is enhanced as compared with the parent method.

References

1. T.-C. Poon, *Digital holography and three-dimensional display* (Springer, Berlin, 2006).
2. M. Lucente and J. Electron. Imaging **2,** 28 (1993).
3. S. C. Kim and E.-S. Kim, Appl. Opt. **47,** D55 (2008).
4. S. C. Kim and E.-S. Kim, Appl. Opt. **48,** 1030 (2009).
5. T. Shimobaba, Y. Sato, J. Miura, M. Takenouchi, and T. Ito, Opt. Express **16,** 11776 (2008).
6. L. Ahrenberg, Opt. Express **14,** 7636 (2006).
7. Y.-H. Seo, H.-J. Cho, and D.-W. Kim, *in Digital Holography and Three-Dimensional Imaging,* OSA Techmical Digest (CD) (OSA, 2008), JMA9.
8. H. Yoshikawa, *in Digital holography and three-dimensional display: principles and applications* T.-C. Poon, (ed.) (Springer Verlag, 2006).
9. T.-C. Poon T. Akin, G. ndebetouw, and T. Kim, Opt. Express **13,** 2427 (2005).
10. P. Tsang, J.-P. Liu, W.-K. Cheung, and T.-C. Poon, Appl. Opt. **48,** H23 (2009).
11. P. Tsang, J.-P. Liu, T.-C. Poon, and W. K. Cheung, *in Digital Holography and Three-Dimensional Imaging*, OSA Technical Digest (CD) (OSA, 2009), DWC2.
12. V. L. Tuft, "Holovision 2.2 user's manual" (http://www2.edge.no/projects /holovision/doc/holovision_221_manual.pdf, 2001).
13. B. E. Bayer, *in Proceedings of the IEEE Intemational Conference on Communications* **1,** 26-11 (1973).