Image registration based on matrix perturbation analysis using spectral graph

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We present a novel perspective on characterizing the spectral correspondence between nodes of the weighted graph with application to image registration. It is based on matrix perturbation analysis on the spectral graph. The contribution may be divided into three parts. Firstly, the perturbation matrix is obtained by perturbing the matrix of graph model. Secondly, an orthogonal matrix is obtained based on an optimal parameter, which can better capture correspondence features. Thirdly, the optimal matching matrix is proposed by adjusting signs of orthogonal matrix for image registration. Experiments on both synthetic images and real-world images demonstrate the effectiveness and accuracy of the proposed method.

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Spectral graph theory is a powerful tool that aims to characterize the structural properties of graphs using the eigenvalues and eigenvectors of either the adjacency matrix or the closely related normalized Laplacian matrix[1], which is used in computer vision fields, such as graph matching[2−9], image segmentation[10−12] and clustering[13−15]. In recent years, there have been a number of interests in the application of using spectral properties for graph matching. Among the graph matching is Umeyama's formulation for the same-size graph matching which derives the minimum difference permutation matrix by singular value decomposition techniques[2]. According to the ideas of structural chemistry, Scott *et al.* first used a Gaussian weighted function to build an inter-image proximity matrix between feature points in different images being matched and then performed singular value decomposition on the obtained matrix in order to get correspondences from the strength matrix[3], which can cope with point sets of different sizes but is sensitive to the degree of rotation. To overcome this disadvantage, Shapiro *et al.* constructed the intra-image proximity matrix for the individual point sets being matched, which aims to capture relational image structure[4]. Wang *et al.* proposed the feature matching method based on Laplacian spectra of graphs which cannot find correspondence under bigger affine and projective transform[6]. Caelli *et al.* have extended the method of seeking correspondences in Ref. [4] by searching for matching that maximizes the inner product of the truncated and renormalized eignevactors[9]. The work of Shokoufandeh *et al.* on indexing hierarchical structures with topological signature vectors was obtained from the sums of adjacency matrix eigenvalues[16]. Some other methods have also been proposed[17−21]. Wang *et al.* proposed that kernel methods could solve the point correspondence matching problem[17,18]. A convex-concave programming approach achieved graph matching for the labelled weighted graph matching problem[20], and a robust image registration algorithm was proposed based on the false feature point pairs rejected by the random sample consensus (RANSAC) algorithm[21].

Considering the problems of the graph matching method proposed by Umeyama[2] which only deals with the same-size graph matching, when the weighted graphs are far different from the isomorphic cases, the method may not work well as in the nearly isomorphic cases. In order to overcome the drawbacks of Umeyama's method, we exploit the structure of a graph model (matrix) changed by a small perturbation according to matrix perturbation analysis[22], that is, eigenvalues are relatively stable to a small perturbation, but eigenvectors are not stable to a small perturbation. Borrowing the ideas of matrix perturbation analysis, we give a small perturbation on eigenvalues to get the needed matching matrix based on eigenvectors in order to achieve better correspondence. In view of these, we propose a novel image registration algorithm by exploiting a small parameter of matrix perturbation analysis which can adjust eigenvectors of sensed image or point sets to seek better correspondence with the reference image or reference point sets, so as to adapt bigger rotation and different weighted graphs and optimization problem. The optimal matching matrix is proposed by adjusting signs of orthogonal matrix, which can also capture correspondence robustly and swiftly without iteration. Experimental results show that the matching results with the proposed method are robust and accurate compared with Wang's method (Lapace method)[6].

Given an $m \times l$ image I, let $N = m \times l$, and $V = \{v_1, v_2, v_3, \cdots, v_N\}$ denote the full set of pixels in the image I, where v_i denotes the ith pixel of I. Based on the set V, we construct a weighted undirected graph $G(V, E, W)$ with V being the node set and $E = V \times V$ being the edge set. For simplicity, an edge $e \in E$ spanning two nodes v_i and v_j, is denoted by e_{ij}. $W = (w_{ij})$ is a weighed function which gives a real nonnegative value $w(v_i, v_j)$ to each pair of nodes v_i and v_j, whose element is the weight of an edge denoted by w_{ij}, where the weight w_{ij} on edge e_{ij} is a measure of the similarity between nodes v_i and

v_j. The weighted undirected graph is called undirected graph when its weighted functioning is symmetric, i.e., $w(v_i, v_j) = w(v_j, v_i)$ for all v_i and v_j, $v_i \neq v_j$. Otherwise, it is called directed graph. The degree of a node is $d_i = \sum w_{ij}$ for all edges e_{ij} incident on v_i, and the degree matrix D is an $N \times N$ diagonal matrix with $d_1, d_2, \cdots, d_N$ on its diagonal, i.e., $D = diag(d_1, d_2, \cdots, d_N)$.

The adjacency matrix of a weighted graph $G = (V, E, W)$ is an $N \times N$ matrix A_{G} defined as

$$A_G = [a_{ij}] = \begin{cases} w(v_i, v_j) & i \neq j \\ 0 & \text{else} \end{cases}. \tag{1}$$

When G is a weighted undirected graph, A_G becomes a symmetric matrix.

Let us consider the above given adjacency matrix A_{G} and diagonal matrix D. When the adjacency matrix A_G is symmetric, the Laplacian matrix L of the graph is defined in the usual manner:

$$L = D - A_G. \tag{2}$$

The normalized Laplacian matrix is defined to be

$$\widehat{L}(v_i, v_j) = \begin{cases} 1 & \text{if} \quad v_i = v_j \quad \text{and} \quad d_i \neq 0 \\ -\dfrac{1}{\sqrt{d_{v_i} d_{v_j}}} & \text{if} \quad v_i \quad \text{and} \quad v_j \quad \text{are} \quad \text{adjacent} \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

In this letter, the Laplacian matrix is defined as

$$L = [L_{ij}] = \begin{cases} - \parallel v_i - v_j \parallel^2 & i \neq j \\ -\sum\limits_{k \neq i} w_{ik} & i = j \end{cases} \quad i, j = 1, 2, \cdots, N. \tag{4}$$

Let $D = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_N)$, we can also write it as $\widehat{L} = D^{-\frac{1}{2}} L D^{-\frac{1}{2}} = D^{-\frac{1}{2}}(D - A_G) D^{-\frac{1}{2}}$. The normalized Laplacian matrix $\widehat{L}$ is positive semidefinite and thus has positive or zero eigenvalues. The normalization factor means that the largest eigenvalue is less than or equal to 2, with equality only when G is bipartite. Again, the matrix has at least one zero eigenvalue. Hence all the eigenvalues are in the range $0 \leq \lambda \leq 2$.

Given two graphs G and H with the same number of vertices N, the problem of matching G and H consists in finding a correspondence between the nodes and edges of G and H, respectively. However, Laplacian matrix is invariant to rotation, scaling, and translation of the image, the graph matching is transformed to the Laplacian matrix matching. The correspondence between the eigenvectors of Laplacian matrix can be defined as follows by using permutation matrix P[2] :

$$J(P) = \min_P \left\| P L_G P^{\mathrm{T}} - L_H \right\|^2, \tag{5}$$

where P represents the correspondence nodes of G and H. The rows of P, like those of G, index the features in the reference image, and its columns index those in the sensed image. The element P_{ij} indicates the extent of pairing between nodes of G_i and H_j. The correspondence between the two nodes is strong only if P_{ij} is the largest element both in its row and in its column, then we regard those two different nodes G_i and H_j as being in 1:1 correspondence with one another; if this is not the case, it means that the node G_i competes unsuccessfully with other nodes for partnership with H_j. However, the problem of finding P cannot let Eq. (5) be minimized to zero, that is to say, P does not represent the good correspondence of nodes and when the weighted graphs are far different from the isomorphic cases, the method may not work well as in the nearly isomorphic cases.

In order to improve the performance of matching, we focus on matrix perturbation analysis to obtain a matching matrix R which can measure the correspondence of feature points of the Laplacian matrix by the optimal perturbation parameter. We consider the Laplacian matrix L_G, and the perturbation matrix L_{HH}, where L_{HH} can be obtained from L_H with some anisotropic perturbation, the perturbation Laplacian matrix is presented as

$$L_{HH} = L_H + \omega G_a * L_H, \tag{6}$$

where $G_a = G_a(x, y, \sigma) = \exp\left(-(x^2 + y^2)/2\sigma^2\right)/\sqrt{2\pi\sigma^2}$ is a Gaussian function, and ω is a perturbation parameter, which can adjust the matching performance by optimization problem. The purpose of perturbing is that the better correspondence features are captured for point sets G and H. In other words, the L_{HH} is very similar to L_G by perturbing L_H with the optimization problem based on the perturbation parameter ω.

The problem of matching two Laplacian matrices is to find a one-to-one correspondence between the two correspondence sets of nodes that minimizes the distance between L_G and L_{HH}, which are Laplacian matrix of weighted graph of A_G and A_H. In this letter, we use the following criterion for a measure of difference:

$$J(R) = \min_R \parallel R L_G R^{\mathrm{T}} - L_{HH} \parallel^2. \tag{7}$$

The matching matrix (permutation matrix) R obtained by perturbing L_H represents the correspondence of the nodes of weighted graph. Thus, R is also an optimal matching matrix by the optimization problem based on matrix perturbation analysis.

Firstly, a lemma is given[22].

Lemma: Let A and B be Hermitian matrices with N distinct eigenvalues $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_N$ and $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_N$, respectively, then

$$\|A - B\|^2 \geq \sum_{i=1}^{N} (\alpha_i - \beta_i)^2. \tag{8}$$

The proof is omitted.

We give the following theorem according to the theorem in Ref. [2].

Theorem 1: Let L_G and L_H be $N \times N$ (real) symmetric matrices with N distinct eigenvalues $\alpha_1 > \alpha_2 > \cdots > \alpha_N$ and $\beta_1 > \beta_2 > \cdots > \beta_N$, respectively, and $L_{HH} = L_H + \omega G_a * L_H$ attained by perturbing L_H, $G_a * L_H$ with N distinct eigenvalues $\gamma_1 > \gamma_2 > \cdots > \gamma_N$, namely, the L_{HH} with N distinct eigenvalues $\beta_1 + \omega\gamma_1 > \beta_2 + \omega\gamma_2 > \cdots > \beta_N + \omega\gamma_N$, then

$$\sum_{i=1}^{N} (\alpha_i - (\beta_i + \omega\gamma_i))^2 = \|L_G - L_{HH}\|^2 . \tag{9}$$

Proof: Since L_G and L_H are (real) symmetric matrices with N distinct eigenvalues, then $L_G = U_G \Lambda_G U_G^{\mathrm{T}}, L_H = V_H \Lambda_H V_H^{\mathrm{T}}, G_a * L_H = V'_H \Lambda'_H V'^{\mathrm{T}}_H$, where U_G, V_H, and V'_H are orthogonal matrices, and $\Lambda_G = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$, $\Lambda_H = \mathrm{diag}(\beta_1, \beta_2, \cdots, \beta_N)$, $\Lambda'_H = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_N)$. So there is $L_{HH} = V_{HH} \Lambda_{HH} V_{HH}^{\mathrm{T}}$, where V_{HH} is an orthogonal matrix, and $\Lambda_{HH} = \mathrm{diag}(\beta_1 + \omega\gamma_1, \beta_2 + \omega\gamma_2, \cdots, \beta_N + \omega\gamma_N)$.
There is

$$\begin{aligned} \|L_G - L_{HH}\|^2 &= \left\|U_G \Lambda_G U_G^{\mathrm{T}} - V_{HH} \Lambda_{HH} V_{HH}^{\mathrm{T}}\right\|^2 = \left\|V_{HH}^{\mathrm{T}} U_G \Lambda_G U_G^{\mathrm{T}} V_{HH} - \Lambda_{HH}\right\|^2 \\ &= \left\|Z \Lambda_G Z^{\mathrm{T}} - \Lambda_{HH}\right\|^2 = \mathrm{tr}(Z \Lambda_G Z^{\mathrm{T}} - \Lambda_{HH})(Z \Lambda_G Z^{\mathrm{T}} - \Lambda_{HH})^{\mathrm{T}} \\ &= \mathrm{tr}(\Lambda_G \Lambda_G^{\mathrm{T}} + \Lambda_{HH} \Lambda_{HH}^{\mathrm{T}}) - \mathrm{tr}(Z \Lambda_G Z^{\mathrm{T}} \Lambda_{HH}^{\mathrm{T}} + \Lambda_{HH} Z \Lambda_G^{\mathrm{T}} Z^{\mathrm{T}}) \\ &= \sum_{i=1}^{N} (|\alpha_i|^2 + |\beta_i + \omega\gamma_i|^2) - 2\mathrm{Re}\ \mathrm{tr}(Z \Lambda_G Z^{\mathrm{T}} \Lambda_{HH}^{\mathrm{T}}) \\ &= \sum_{i=1}^{N} \left[|\alpha_i|^2 + |\beta_i + \omega\gamma_i|^2 - 2\mathrm{Re}\left(\alpha_i(\beta_i + \omega\gamma_i)\right)\right] \\ &= \sum_{i=1}^{N} (\alpha_i - (\beta_i + \omega\gamma_i))^2, \end{aligned} \tag{10}$$

where $Z = V_{HH}^{\mathrm{T}} U_G$.

Theorem 2: Let L_G and L_H be $N \times N$ (real) symmetric matrices with N distinct eigenvalues $\alpha_1 > \alpha_2 > \cdots > \alpha_N$ and $\beta_1 > \beta_2 > \cdots > \beta_N$, respectively, and $L_{HH} = L_H + \omega G_a * L_H$ attained by perturbing L_H, their singular value decomposition can be given by

$$L_G = U_G \Lambda_G U_G^{\mathrm{T}}, \tag{11}$$

$$L_H = V_H \Lambda_H V_H^{\mathrm{T}}, \tag{12}$$

$$G_a * L_H = V'_H \Lambda'_H V'^{\mathrm{T}}_H, \tag{13}$$

where U_G, V_H, and V'_H are orthogonal matrices, $\Lambda_G = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$, $\Lambda_H = \mathrm{diag}(\beta_1, \beta_2, \cdots, \beta_N)$, $\Lambda'_H = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_N)$. So there is $L_{HH} = V_{HH} \Lambda_{HH} V_{HH}^{\mathrm{T}}$, where V_{HH} is an orthogonal matrix, and $\Lambda_{HH} = \mathrm{diag}(\beta_1 + \omega\gamma_1, \beta_2 + \omega\gamma_2, \cdots, \beta_N + \omega\gamma_N)$. Then

$$\begin{aligned} J(P) &= \min_P \left\|P L_G P^{\mathrm{T}} - L_H\right\|^2 \\ &\Leftrightarrow J(R) = \min_R \left\|R L_G R^{\mathrm{T}} - L_{HH}\right\|^2 \\ &\Leftrightarrow J(\omega) = \min_\omega \sum_{i=1}^{n} (\alpha_i - (\beta_i + \omega\gamma_i))^2. \end{aligned} \tag{14}$$

where $R = V_{HH} S U_G^{\mathrm{T}} = \widehat{V}_{HH} U_G^{\mathrm{T}}$ is an optimal matching matrix, and S is a sign matrix.

Proof: Starting with the optimal problem Eq. (14), we have

$$\begin{aligned} J(R) &= \min_R \left\|R L_G R^{\mathrm{T}} - L_{HH}\right\|^2 = \left\|R U_G \Lambda_G U_G^{\mathrm{T}} R^{\mathrm{T}} - V_{HH} \Lambda_{HH} V_{HH}^{\mathrm{T}}\right\|^2 \\ &= \left\|V_{HH}^{\mathrm{T}} R U_G \Lambda_G U_G^{\mathrm{T}} R^{\mathrm{T}} V_{HH} - \Lambda_{HH}\right\|^2 = \left\|V_{HH}^{\mathrm{T}} V_{HH} S U_G^{\mathrm{T}} U_G \Lambda_G U_G^{\mathrm{T}} U_G S^{\mathrm{T}} V_{HH}^{\mathrm{T}} V_{HH} - \Lambda_{HH}\right\|^2 \\ &= \left\|S \Lambda_G S^{\mathrm{T}} - \Lambda_{HH}\right\|^2 = \|\Lambda_G - \Lambda_{HH}\|^2 = \sum_{i=1}^{n} (\alpha_i - (\beta_i + \omega\gamma_i))^2. \end{aligned}$$

So

$$\begin{aligned} J(R) &= \min_R \left\|R L_G R^{\mathrm{T}} - L_{HH}\right\|^2 \\ &\Leftrightarrow J(\omega) = \min_\omega \sum_{i=1}^{n} (\alpha_i - (\beta_i + \omega\gamma_i))^2. \end{aligned} \tag{15}$$

The optimal parameter ω is obtained by minimizing $J(\omega)$ to zero, and the orthogonal matrix V_{HH} is also obtained by decomposing L_{HH}. Revise the signs of columns of V_{HH}, and acquire the rectified matrix $\widehat{V}_{HH} = V_{HH} S$, where S is sign matrix. So the optimal matching matrix is given by $R = \widehat{V}_{HH} U_G^{\mathrm{T}}$.

Based on the above theory discussion, now we detail the steps of our algorithm for matching algorithm based on perturbation analysis.

1) Given two sets of points, construct normalized Laplacian matrices L_G and L_H on point sets G and H, respectively.

2) Given perturbation matrix $L_{HH} = L_H + \omega G_a * L_H$ by perturbing L_H.

3) Perform singular value decomposition on L_G, L_H, and $G_a * L_H$, respectively, and get the orthogonal matrices U_G, V_H, and V_H' of point sets G and H.

4) Use Eq. (15) to seek an optimal parameter ω, namely let Eq. (15) be minimized to zero.

5) Perform singular value decomposition on perturbation matrix L_{HH}, and get the orthogonal matrix V_{HH}.

6) Revise the signs of columns of V_{HH}, and acquire the rectified matrix $\widehat{V}_{HH} = V_{HH}S$, where S is a sign matrix.

7) Attain the optimal matching matrix $R = \widehat{V}_{HH}U_G^{\mathrm{T}}$ by the optimal parameter ω. According to the fact that the element of R_{ij} is the greatest both in its row and in its column, we regard those two different features G_i and H_j as being in 1:1 correspondence with one another.

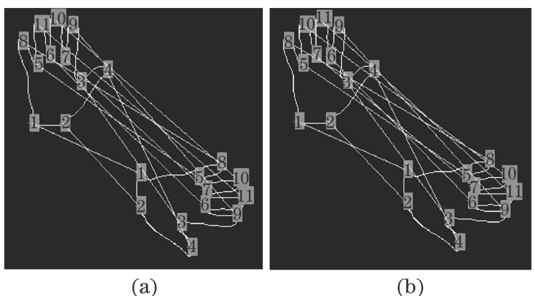

Fig. 1. Matching results of the hand images. (a) Laplace method; (b) our method.

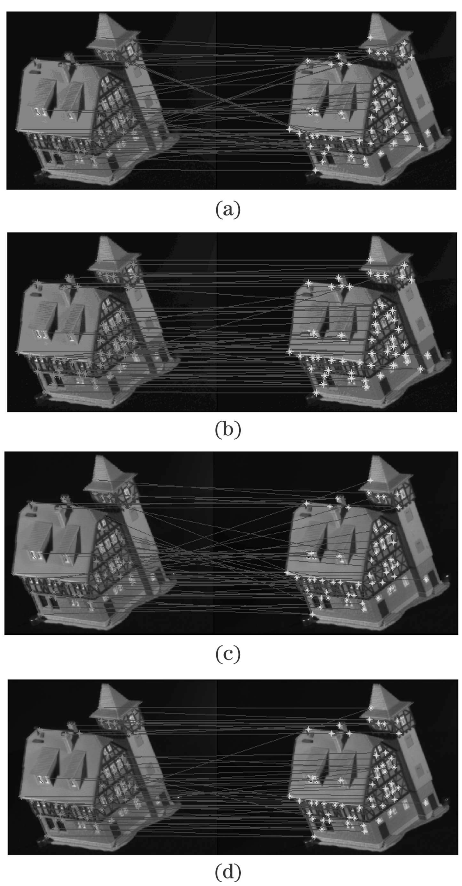

Fig. 2. Matching results on the house images. (a) Laplace method (the 0th, 1st frames); (b) our method, $\omega = 4.5715 \times 10^{-4}$ (the 0th, 1st frames); (c) Laplace method (the 0th, 5th frames); (d) our method, $\omega = 4.3973 \times 10^{-4}$ (the 0th, 5th frames).

To test our algorithm, we applied it to some synthetic images, the CMU/VASC houses (from the image database of Vision and Autonomous Systems Center, Carnegie Mellon University, USA) and the synthetic aperture radar (SAR) images. Figure 1 gives experimental results of synthetic hand image to test the ability to cope with rotation and translation of Laplace method[6] and our method respectively. The synthetic data is the character hand consisting of eleven feature points. The optimal parameter $\omega = 0.0021$ controls how can the matching matrix R measure the correspondence between two feature point sets. Our algorithm can cope with rotation and translation better than Laplace method. The matching results are robust and accurate from experiments.

Figure 2 shows the comparison of matching results of the house images using Laplace method and our method. Experimental results indicate that our algorithm is feasible for real house images. We selected 52 and 53 feature points to match the (0th, 1st) and (0th, 5th) frames of CMU/VASC houses, respectively. It is illustrated that the feature matching ability of the proposed method is better than the Laplace method.

Figure 3 shows the performances of Laplace method

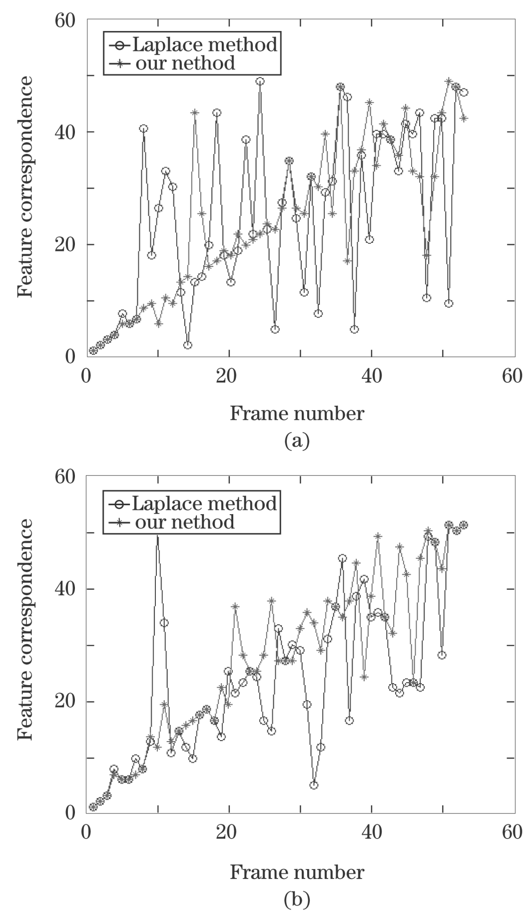

Fig. 3. Performance comparison of feature correspondences on the house images of Laplace method and our method. (a) Results of Figs. 2(a) and (b); (b) results of Figs. 2(c) and (d).

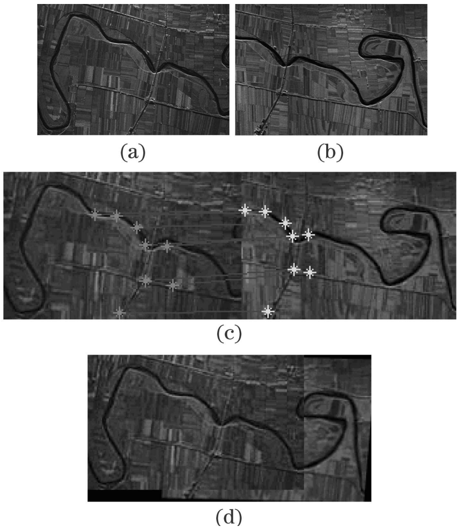

Fig. 4. Matching and registration results on the SAR images. (a) Reference image; (b) sensed image; (c) matching result using our method; (d) registration result using our method, $\omega = 0.0023$.

and our proposed method. From the experimental results, we can see that the feature points are one-to-one corresponded except a few points, so our method is robust and better than the Laplace method.

Figure 4 shows the SAR images registration result using our method. Thirty-nine feature points were selected in each image respectively on 3-m resolution of the SAR images which were shot by an unpiloted airplane of Northwestern Polytechnical University. And eight feature points were exploited by eliminating the other feature points.

In conclusion, we propose a general procedure for computing the correspondence between nodes of a weighted undirected graph. Specifically, we generalize the feature matching method based on matrix perturbation analysis by optimizing the optimal parameter for image registration, and present an efficient computation technique. The proposed method can capture accurate correspondence in different images and under rotation, scaling, and translation of the image. Experimental results are very encouraging and illustrate that the proposed method is indeed a good feature matching algorithm for registration. We will further research the perturbation problem and give general perturbation or adaptive perturbation based on intensity to achieve better correspondence of the nodes.

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