Effect of the third-order filter term on soliton interactions in soliton transmission systems with filters

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The effect of the third-order filter term on soliton interactions in optical fibers with guiding filters is theoretically analyzed. We find that this term causes a significant difference to the interaction of solitons through filters among the regimes of without sliding, with up- and down-sliding frequencies of filters. It is shown that the interaction between solitons can be more effectively suppressed by up-sliding filters than that by down-sliding filters in the presence of the third-order filter term. Moreover, the third-order filter term is found to play a positive role in suppressing soliton interactions in the case of non-sliding.

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The interaction between adjacent pulses is one of the main limitations of soliton-based telecommunications. Therefore, reduction of soliton interaction is a promising proposal and a lasting topic. The interactions of solitons may be controlled by use of various methods, such as the temporal synchronous active modulation scheme[1], randomly varying birefringence[2], photonic crystal fibers(PCFs)[3], and modified group velocity dispersion(GVD)[4]. More recently, the interactions of chirped and chirp-free similarities in optical fiber amplifiers have been researched[5]. Another method to control the interaction of solitons is the use of filters[6,7]. A more dramatic increase of the distance of soliton collision may be achieved by sliding the center frequency of the filters along the transmission line[8,9]. The filters with/without sliding peak frequency can also effectively reduce soliton timing jitter and overcome the self-frequency-shift, because through moderate sliding rate it can create a transmission line that is opaque to noise and transparent to solitons[10−16].

However, all previous reports neglected the third-order filter term in studying the interaction of solitons by use of filters. The effect of the third-order filter term on soliton transmission and timing jitter in optical fibers has been analyzed[16,17]. It has been clearly shown that this term produces a significant difference, in particular, between the regimes of up- and down-sliding frequencies of filters.

In this letter, we derive the theoretical results of soliton interactions using guiding filters, taking into account the third-order filter term. We find that this term greatly affects the interaction of solitons. The interaction between solitons can be suppressed more effectively by the filters with up sliding than that with down sliding in the presence of the third-order filter term. In particular, for a very small separation of two soliotns (up to 4.6 times of the solitons width, which is never found for maintaining such a small separation of solitons upon propagation in the known reports to the best of our knowledge), the case of up-sliding frequency can greatly overcome the soliton interactions. In addition, for the case of non-sliding, the third-order filter term plays a positive role in suppressing the interaction of solitons.

In a transmission line that uses Fabry-Perot (FP) etalon filters with a mirror spacing d and a reflectivity R, the distributed filter function is

$$F(\omega)=\frac{1}{l_{\mathrm{f}}}\ln\left\{\frac{1-R}{1-R\exp[\mathrm{i}(\omega-\omega_{\mathrm{f}})2d/c]}\right\}, \tag{1}$$

where l_{f} is separation between the filters; ω_{f} is the peak frequency of the filters; c is the light velocity in vacuum. We assume that the filters are inserted into the fiber periodically, and the average effect of the filters in the transmission line is considered. In addition, to compensate for the loss produced by filters, an excess gain α is added to the system. Thus, keeping terms through the third order in the Taylor expansion of Eq. (1), we obtain the propagation equation in soliton units[17]:

$$\begin{aligned}\frac{\partial u}{\partial z}=&\frac{\mathrm{i}}{2}\frac{\partial^2 u}{\partial t^2}+\mathrm{i}\,|u|^2\,u+\alpha u\\&+\beta_2\left[\frac{\partial}{\partial t}+\mathrm{i}\omega_{\mathrm{f}}(z)\right]^2 u-\beta_3\left[\frac{\partial}{\partial t}+\mathrm{i}\omega_{\mathrm{f}}(z)\right]^3 u,\end{aligned} \tag{2}$$

where

$$\begin{aligned}\beta_2&=\frac{1}{2}\frac{R}{(1-R)^2}\frac{8\pi}{Dl_{\mathrm{f}}c}\left(\frac{d}{\lambda}\right)^2,\\ \beta_3&=\frac{2d}{c}\frac{(1+R)}{(1-R)}\frac{1}{3t_0}\beta_2,\\ \omega_{\mathrm{f}}'&=\frac{\mathrm{d}\omega_{\mathrm{f}}}{\mathrm{d}z}=\frac{4\pi^2 f' c t_0^3}{\lambda^2 D},\\ \alpha&=\frac{\alpha_{\mathrm{R}} t_0^2 2\pi c}{\lambda^2 D},\end{aligned} \tag{3}$$

where β_2 and β_3 are the coefficients of the second and third-order filter terms, respectively; ω_f' is the sliding rate of peak frequency of filters; D is the fiber dispersion; $t_0 = T/1.763$, T is the soliton pulse full-width at half-maximum (FWHM); λ is the wavelength; α_R is the loss coefficient of reflective mirror. It should be noted that the third-order filter term is inversely proportional to the pulse duration (Eq. (3)), so this term becomes more important as the soliton width decreases. Set

$$u(t,z) = v(t+\omega_f' z^2/2, z) \times \exp(-i\omega_f' zt - i\omega_f'^2 z^3/3), \quad (4)$$

then v obeys

$$\frac{\partial v}{\partial z} = \frac{i}{2}\frac{\partial^2 v}{\partial \tau^2} + i|v|^2 v + \alpha v + \beta_2 \frac{\partial^2 v}{\partial \tau^2} - \beta_3 \frac{\partial^3 v}{\partial \tau^3} + i\omega_f' \tau v, \quad (5)$$

where $\tau = t + \omega_f' z^2/2$. It is remarkable that in Eq. (5), one can separate the effect of sliding from that of the filter. Introducing the usual ansatz for the soliton $u = A\text{sech}[A(\tau - q)]\exp\{(i\Omega\tau + iA^2 z/2 + 3i\beta_3 A\tanh[A(\tau - q) + i\sigma]\}$ (Ω is the soliton frequency) into Eq. (5), we obtain the following equations for the amplitude A, mean frequency ω_0, soliton peak position q, and phase σ :

$$\frac{dA}{dz} = 2\alpha A - 2\beta_2 A(\omega_0^2 + \frac{1}{3}A^2), \quad (6a)$$

$$\frac{d\omega_0}{dz} = \omega_f' - \frac{4}{3}\beta_2 A^2(\omega_0 - \frac{6}{5}\beta_3 A^2), \quad (6b)$$

$$\frac{dq}{dz} = -\omega_0 + \beta_3(A^2 - 3\omega_0^2), \quad (6c)$$

$$\frac{d\sigma}{dz} = \frac{A^2 - \omega_0^2}{2} + \beta_3(3\omega_0 A^2 - \omega_0^3). \quad (6d)$$

The mean frequency ω_0 is related to Ω by $\omega_0 = \Omega - 2\beta_3 A^2$ and A is normalized to 1. From Eq. (6), it requires that

$$\alpha = \beta_2(\omega_0^2 + \frac{1}{3}), \quad (7a)$$

$$\omega_f' = \frac{4}{3}\beta_2(\omega_0 - \frac{6}{5}\beta_3). \quad (7b)$$

From Eq. (7b), the mean frequency of soliton is $\omega_0 = 3\omega_f'/(4\beta_2) + 6/5\beta_3$.

By means of two-soliton perturbation theory, we consider the two pulses:

$$v_j(z,\tau) = A_j \text{sech}[A_j(\tau - q_j)] \exp\{(i\Omega_j(\tau - q_j) + iA_j^2 z/2 + 3i\beta_3 A_j \tanh[A_j(\tau - q_j) + i\sigma_j]\}, j = 1, 2. \quad (8)$$

Note that the pulse separation $q_1 - q_2$ cannot be smaller than 3 times of the pulse width. Otherwise, the perturbation that arises from the superposition of two pulses is too strong to fit the perturbation theory, which leads to the inaccurate analytic results. By use of the method of analyzing two-soliton interaction in Ref. [18] to solve Eq. (6), we obtain the following coupled equations:

$$\frac{dA}{dz} = 2\alpha A - 2\beta_2 A(\omega_0^2 + \frac{1}{3}A^2), \quad (9a)$$

$$\frac{dk}{dz} = \omega_f' - \frac{4}{3}\beta_2 A^2(\omega_0 - \frac{6}{5}\beta_3 A^2), \quad (9b)$$

$$\frac{dp}{dz} = 4A^3\exp(-A\Delta)\sin(\delta) + 2p[\alpha - \beta_2(A^2 + k^2)] - 4\beta_2 Akf, \quad (9c)$$

$$\frac{df}{dz} = 4A^3\exp(-A\Delta)\cos(\delta) - \frac{4}{3}\beta_2 fA^2 - \frac{8}{3}\beta_2 Akp + \frac{32}{5}\beta_2\beta_3 A^3 p, \quad (9d)$$

$$\frac{d\Delta}{dz} = -2f + 4\beta_3(Ap - 3\omega_0 f), \quad (9e)$$

$$\frac{d\delta}{dz} = 2Ap - \frac{4}{3}\beta_2 A^2(k - \frac{6}{5}\beta_3 A^2)\Delta + 2\beta_3(3A^2 f + 6\omega_0 Ap - 3\omega_0^2 f), \quad (9f)$$

where $A = (A_1 + A_2)/2$, $k = (\omega_{01} + \omega_{02})/2$, $p = (A_1 - A_2)/2$ is the amplitude difference, $f = (\omega_{01} - \omega_{02})/2$ is the frequency difference, $\Delta = q_1 - q_2 > 0$ is the pulse separation, and $\delta = k\Delta + \sigma$ with $\sigma = \sigma_1 - \sigma_2$ is the phase difference.

By numerically solving Eq. (9), we obtain the two-soliton separation versus propagation distance z with $\Delta_0 = 6$ and 7 for $\beta_2 = 0.15$, as shown in Fig. 1. Three cases are shown in Fig. 1: $\omega_f' = -0.05$ and $\beta_3 = \beta_2/2$[16,17], $\omega_f' = \pm 0.05$ and $\beta_3 = 0$, $\omega_f' = 0.05$ and $\beta_3 = \beta_2/2$. From Fig. 1, we fine that the case of up-sliding frequency is more effective in suppressing soliton interaction than that of down-sliding frequency in the presence of the third-order term β_3. The numerical simulations with $\Delta_0 = 6$ based on Eq. (2) are shown in Fig. 2 for $\omega_f' = -0.05$ and (b) 0.05, which further demonstrates the results indicated in Fig. 1.

In addition, when the initial two-soliton separation is very small, e.g., $\Delta_0 = 4.6$, the advantage of up sliding is more obvious than that of down sliding in the presence of β_3 (Fig. 3). It is surprising that the case of up sliding can greatly reduce the soliton interaction in such small separation with $\Delta_0 = 4.6$, which is hardly found in previous results of studying the soliton interaction. This is different from the results of Ref. [8] in the absence

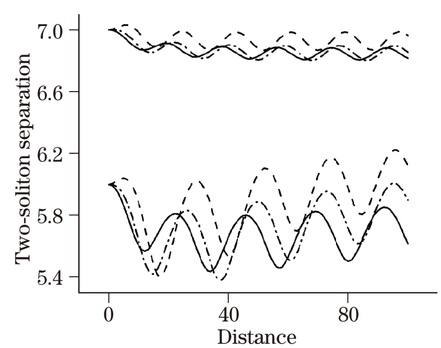

Fig. 1. Two-soliton separation versus propagation distance with $\Delta_0 = 6$ and 7 for $\beta_2 = 0.15$. Solid curves: $\omega_f' = -0.05$ and $\beta_3 = \beta_2/2$; dash-dotted curves: $\omega_f' = \pm 0.05$ and $\beta_3 = 0$; dashed curves: $\omega_f' = 0.05$ and $\beta_3 = \beta_2/2$.

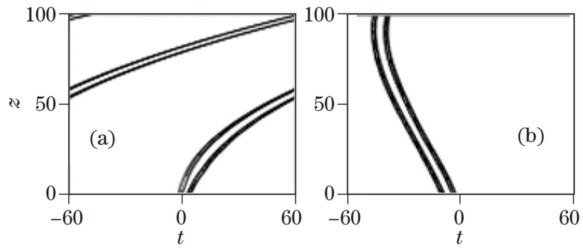

Fig. 2. Numerical simulations with $\Delta_0 = 6$ and $\beta_3 = \beta_2/2$ for (a) $\omega_f' = -0.05$ and (b) $\omega_f' = 0.05$. t is the tempoval cordinate.

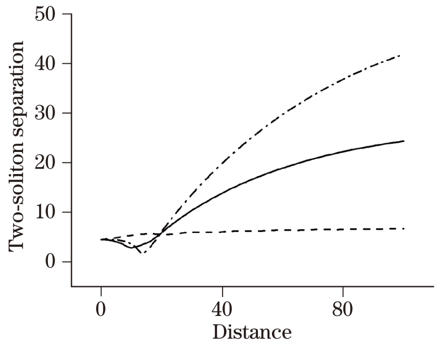

Fig. 3. Two-soliton separation versus propagation distance with $\Delta_0 = 4.6$ and $\beta_2 = 0.15$. Solid curve: $\omega_f' = -0.075$ and $\beta_3 = \beta_2/2$; dash-dotted curve: $\omega_f' = \pm 0.075$ and $\beta_3 = 0$; dashed curve: $\omega_f' = 0.075$ and $\beta_3 = \beta_2/2$.

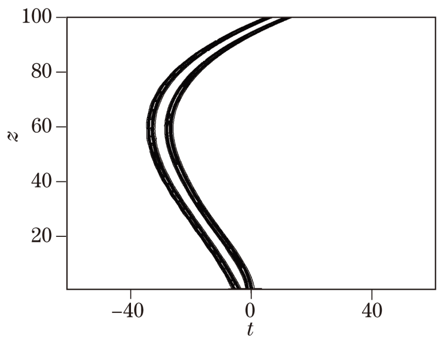

Fig. 4. Numerical simulation corresponding to Fig. 3 for the case of up-sliding frequency with $\omega_f' = 0.075$ and $\beta_3 = \beta_2/2$.

of β_3, in which the soliton separation of bound states is $\Delta_0 = 8$. Numerical simulation corresponding to Fig. 3 for the case of up-sliding frequency is shown in Fig. 4.

For the case of without sliding frequency, two-soliton separation versus propagation distance z is achieved (Fig. 5). The results indicate that as the third-order term β_3 increases, the suppression of soliton interaction is more effective, and two solitons with $\Delta_0 = 7$ do not collide. In contrast, the results of Ref. [6] in the absence of β_3 showed the collision of two solitons with $\Delta_0 = 8$ at $z = 80$. Numerical simulations are given in Fig. 6, which are in agreement with the results shown in Fig. 5. Therefore, from the results above, we find that the third-order term β_3 plays an important role in overcoming soliton interaction for two cases of up-sliding and without sliding frequency.

In conclusion, we derive the theoretical results of soliton interactions using guiding filters taking into account the third-order filter term. We find that this term causes

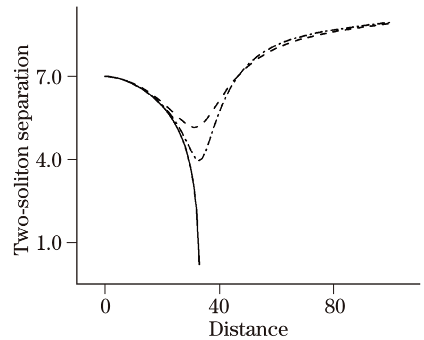

Fig. 5. Two-soliton separation versus propagation distance with $\Delta_0 = 7$, $\omega_f' = 0$, $\beta_2 = 0.075$, and $\beta_3 = 0$ (solid curve), $\beta_3 = \beta_2/4$ (dash-dotted curve), $\beta_3 = \beta_2/2$ (dashed curve).

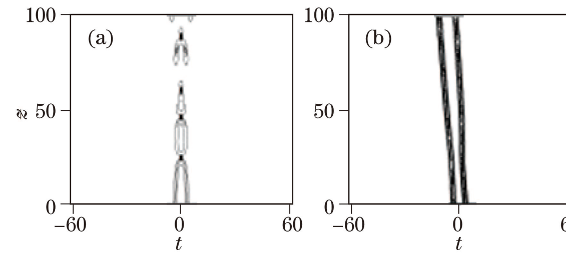

Fig. 6. Numerical simulations corresponding to Fig. 5 for (a) $\beta_3 = 0$ and (b) $\beta_3 = \beta_2/2$.

a significant difference in controlling interaction of solitons through guiding filters among the regimes of without sliding, with up- and down-sliding filter frequencies. The interaction between solitons can be suppressed more effectively in the system with up sliding than that with down sliding in the presence of the third-order filter term. In particular, for up-sliding filter, the interaction between solitons can be greatly overcome. We also reveal that the third-order filter term also contributes to suppress soliton interaction in the case of without sliding. Our results may be useful to the other solitons, such as chirped soliton and squeezed soliton[19,20].

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