Synchronization of chaotic VCSELs by external chaotic signal parameter modulation

Wei Wang (王 伟)[1,2*], Shenghai Zhang (张胜海)[1], Xingzhong Qian (钱兴中)[1], and Yanbin Wang (王彦斌)[3]

[1] *Department of Mathematics and Physics, Institute of Science, Information Engineering University, Zhengzhou 450001, China*

[2] *Sergeant College of the Second Artillery, Weifang 262500, China*

[3] *College of Optoelectric Science and Engineering, National University of Defense Technology, Changsha 410073, China*

* *E-mail: wdwldl2002@163.com*

Received December 16, 2008

Synchronization of chaotic vertical-cavity surface-emitting lasers (VCSELs) is achieved by external chaotic signal modulation successfully. Simulation indicates that we can get chaos synchronization if the intensity of external chaotic signal is large enough. First of all, we use direct current modulation to achieve the chaos of VCSELs, and determine the laser's chaotic state by analyzing time series of the output and the corresponding power spectrum. And then we achieve synchronization of the two chaotic systems by external chaotic signal parameter modulation. We also find that the larger the modulation intensity is, the easier it is to achieve synchronization for chaotic VCSELs. This approach can also be applied to systems with a number of modulated lasers.

OCIS codes: 190.0190, 190.3100, 140.7260.

doi: 10.3788/COL20090710.0945.

In 1990, Pecora *et al*. proposed the principle of chaos self-synchronization and achieved the synchronization of electro-circuit chaos for the first time[1]. Since then, chaotic synchronization became the focus of nonlinear science and the cross field of science[2,3], and its application to secure communication showed a broad prospect[4,5]. As a new type of micro-cavity laser, vertical-cavity surface-emitting laser (VCSEL) is an ideal choice for the optical interconnection, parallel optical signal processing, and chaotic communications, because they have many advantages compared with edge-emitting semiconductor lasers such as low threshold current, single-longitudinal-mode operation, circular output beam profile, and wafer-scale integrability[6−8]. Therefore, the study on chaos synchronization of VCSELs is of great significance.

Synchronization of chaotic edge emitting semiconductor laser[9] has made brilliant achievements presently[10], with coupled synchronization program and driver synchronization program. Experimental and theoretical researches on the synchronization of chaotic VCSEL have also made great progress in recent years and synchronization of chaos has been achieved experimentally in unidirectionally coupled external-cavity semiconductor VCSELs operating in an open-loop regime[11−14]. In this letter, we investigate the chaos and synchronization of chaotic VCSELs by external chaotic signal parameter modulation. According to the dynamic model of VCSELs, we give the time series and the corresponding power spectrum of the lasers. Then we determine the laser's chaotic state. By external chaos signal parameter modulation, we get the synchronization of the two systems. By numerical simulation about the correlation coefficient of modulated lasers' photon density versus modulation intensity, we determine the range of modulation intensity in which the precise synchronization of modulated lasers is achieved. Finally, we analyze the effect of synchronization at different modulation intensities.

VCSEL consists of mirrors, active layer, and the composition of metal contacts. Two mirrors are made up of the n-type and p-type disrtibuted Bragg reflectors (DBRs). As the active layer is constituted by multiple quantum wells (MQWs), the active layer thickness is equal to the sum of the quantum well thicknesses. On the p-type DBR, there is a circle light aperture and its radius ω is also the radius of the active layer[15]. In order to achieve the chaotic state of lasers, we use current modulation to increase the degrees of freedom of lasers. The dynamic model of VCSELs subject to current modulation can be defined by[16−18]

$$\frac{\partial P(t)}{\partial t} = v_{\mathrm{g}}(\Gamma G(t) - \alpha)P(t) + \beta B_{\mathrm{sp}}N(t)^2, \quad (1)$$

$$\frac{\partial N(t)}{\partial t} = \frac{I(t)}{qV} - v_g \Gamma G(t)P(t) - \frac{N(t)}{\tau_{\mathrm{c}}}, \quad (2)$$

$$I(t) = I_{\mathrm{d}} + I_{\mathrm{m}}\sin(2\pi f_{\mathrm{m}}t), \quad (3)$$

$$G(t) = \Gamma_z a_N \frac{\ln[N(t)/N_0]}{1 + \varepsilon P(t)}, \quad (4)$$

where $P(t)$ is the photon density, $N(t)$ is the carrier concentration, V_{g} is the group velocity ($V_{\mathrm{g}} = 8.3\times10^9$ cm/s), q is the unit charge. The injection currrent $I(t)$ is the sum of the bais current I_{d} and sinusoidal modulation current $I_{\mathrm{m}}\sin(2\pi f_{\mathrm{m}}t)$, where I_{m}

1671-7694/2009/100945-04

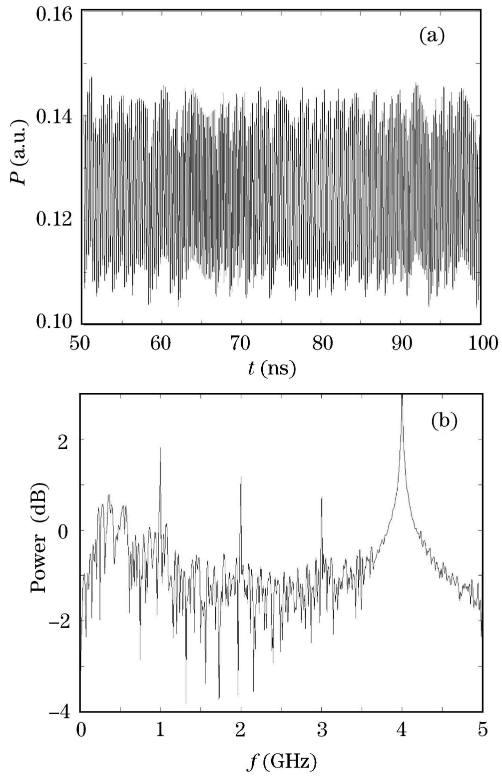

Fig. 1. (a) Time series of normalized photon density and (b) the corresponding power spectrum. $f_{\mathrm{m}} = 4$ GHz, $m = 0.6$.

Table 1. Typical Parameters of VCSELs

Symbol	Parameter	Value
τ_{c}	Carrier Lifetime	2.7×10^{-9}s
β	Spontaneous Emission Factor	1×10^{-6}
Γ	Lateral Confinement Factor	1
Γ_z	Longitudinal Confinement Factor	0.07
α	Equivalent Cavity Loss	5.03×10^{3} m^{-1}
a_N	Gain Coefficient	1.4×10^{5} m^{-1}
ε	Gain Suppression	1×10^{-23} m^{3}
N_0	Transparent Carrier Density	1.3×10^{24} m^{-3}
B_{sp}	Bimolecular Recombination Coefficient	1×10^{-22} m^{6}/s
f_{m}	Modulation Frequency	4 GHz
ω	Current Aperture	5×10^{-6} m
d	Thickness of Active Region	0.1 μm

and f_{m} are the amplitude and modulation frequency. Γ_z is the longitudinal confinement, Γ is the lateral confinement factor, α is the equivalent cavity loss, a_N is the gain coefficient, ε is the gain compression factor, N_0 is the transparent carrier density, B_{sp} is the bimolecular recombination coefficient, β is the spontaneous emission factor. $V = \pi\omega^2 d$ is the size of the active layer, where ω is the current aperture and d is the thickness of the active layer.

In this letter, we use the fourth-order Runge-Kutta numerical simulation method and the parameters are listed in Table 1. In order to facilitate the discussion, we define $m = I_{\mathrm{m}}/I_{\mathrm{d}}$ and use m as the modulation parameter.

According to the power spectral theory of time series, if there is a single peak or several peaks in the power spectrum, the time series is a cycle sequence and the systems are in a cycle state. And if the power spectrum is a continuum, the time series is a chaotic sequence and the systems are in a chaotic state. Therefore, the power spectral analysis is an important method to observe bifurcation and distinguish chaos.

In order to display the state of lasers clearly, we draw time series of normalized photon density at $m = 0.6$ and the corresponding power spectrum in Fig. 1. It is observed that the laser output is the confusion of random and the power spectrum is a continuum. According to the time series and corresponding power spectrum, we make sure that the systems are in a state of chaos at $m = 0.6$. But there is a pinnacle at 4 GHz in the power spectrum and 4 GHz is even the modulation frequency of injected current. It means that there is a central frequency at any modulation intensity, no matter in a cycle state or a chaotic state, if we use the injected current modulation method to increase the degrees of freedom of lasers. According to different modulation depths, the modulation frequency brings different resonance frequencies. As a result, the lasers produce some different state cycles, and then produce continuum spectrum with continuous increase of the harmonic components and give birth to chaos at last.

We use a modulating system which can bring a chaotic signal to modulate the other two chaotic systems (known as the modulated systems). By this means, the output of the modulated systems can achieve synchronization. Figure 2 shows the program for the realization of parameter modulation chaotic synchronization. VCSEL1 is a modulating laser, while VCSEL2 and VCSEL3 are modulated lasers. The process is as follows. Firstly, chaos signals from the modulating laser VCSEL1 is put into the photoelectric converter and transformed into electrical signal. The electrical signal is amplified by the amplifier, and then injected into the injection currents of two modulated lasers by certain intensity from the adders C1 and C2 respectively. So the injection currents of two lasers are modulated by the modulating laser and the output of the modulated lasers VCSEL2 and VCSEL3 can achieve synchronization at certain modulation intensity lastly.

Now the dynamic model of VCSEL systems can be defined by

$$\frac{\partial P_1}{\partial t} = v_{\mathrm{g}}(\Gamma G_1 - \alpha)P_1 + \beta B_{\mathrm{sp}} N_1^2, \tag{5}$$

$$\frac{\partial N_1}{\partial t} = \frac{I_1}{qV} - v_{\mathrm{g}}\Gamma G_1 P_1 - \frac{N_1}{\tau_{\mathrm{c}}}, \tag{6}$$

$$I_1 = I_{\mathrm{d}} + I_{\mathrm{m}}\sin(2\pi f_{\mathrm{m}} t), \tag{7}$$

$$\frac{\partial P_2}{\partial t} = v_{\mathrm{g}}(\Gamma G_2 - \alpha)P_2 + \beta B_{\mathrm{sp}} N_2^2, \tag{8}$$

$$\frac{\partial N_2}{\partial t} = \frac{I_2}{qV} - v_{\mathrm{g}}\Gamma G_2 P_2 - \frac{N_2}{\tau_{\mathrm{c}}}, \tag{9}$$

$$I_2 = I_{\mathrm{d}} + I_{\mathrm{m}}\sin(2\pi f_{\mathrm{m}} t) + kP_1, \tag{10}$$

$$\frac{\partial P_3}{\partial t} = v_{\mathrm{g}}(\Gamma G_3 - \alpha)P_1 + \beta B_{\mathrm{sp}} N_3^2, \tag{11}$$

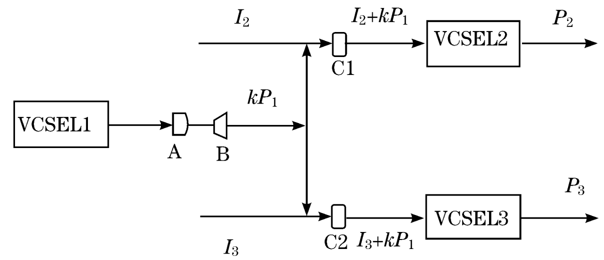

Fig. 2. External chaotic signal modulation synchronization. A: photoelectric converter; B: amplifer; C1, C2: adders.

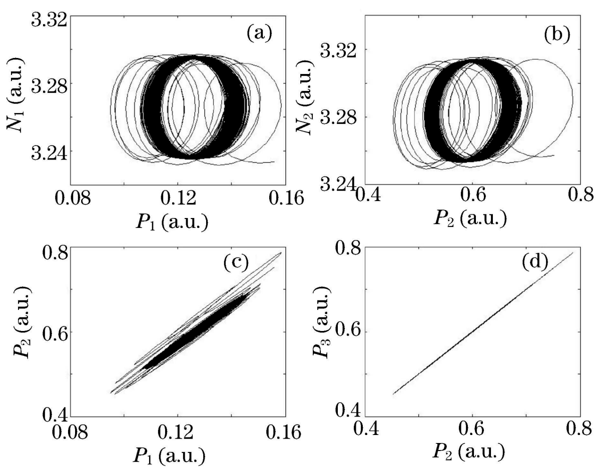

Fig. 3. Chaos synchronization of lasers, k = 0.002. (a) Chaotic attractor of VCSEL1; (b) chaotic attractor of VCSEL2; (c) broad synchronization of P_1 and P_2; (d) precise synchronization of P_2 and P_3.

$$\frac{\partial N_3}{\partial t} = \frac{I_3}{qV} - v_{\rm g}\Gamma G_3 P_3 - \frac{N_3}{\tau_{\rm c}}, \quad (12)$$

$$I_3 = I_{\rm d} + I_{\rm m}\sin(2\pi f_{\rm m} t) + kP_1, \quad (13)$$

$$G_1 = \Gamma_z a_N \frac{\ln[N_1/N_0]}{1+\varepsilon P_1}, \quad (14)$$

$$G_2 = \Gamma_z a_N \frac{\ln[N_2/N_0]}{1+\varepsilon P_2}, \quad (15)$$

$$G_3 = \Gamma_z a_N \frac{\ln[N_3/N_0]}{1+\varepsilon P_3}. \quad (16)$$

Equations (5)−(7) correspond to the modulating laser VCSEL1, Eqs. (8)−(10) correspond to the modulated laser VCSEL2, and Eqs. (11)−(13) correspond to the modulated laser VCSEL3. P_1, P_2, P_3 are the photon densities of lasers, and N_1, N_2, N_3 are the carrier concentrations of lasers, respectively. kP_1 is the chaotic signal used to achieve synchronization and k is the modulation intensity.

It should be noted that the lasers in the model are assumed to be in the same place, and the distances among them can be neglected, so the time delay of systems for transmitting signal is equal to 0.

For the chaotic attractors of VCSEL1 and VCSEL2, as well as the relation diagrams of P_1-P_2 and P_2-P_3 at k = 0.002 with the initial conditions of P_1 = 2, N_1 = 0.1, P_2 = 0.5, N_2 = 1, P_3 = 0.1, N_3 = 0.4 are given in Fig. 3. It is observed from Figs. 3(a) and (b) that the attractors are strange ones and the modulating system and modulated systems are in states of chaos, but the chaos attractors of the two systems are different. So VCSEL1 and VCSEL2 are not in the same chaotic state. Figure 3(c) shows that the relationship between P_1 and P_2 is a complex function rather than a simple function. It is observed from Fig. 3(d) that the relation diagram of P_2-P_3 is a straight line with an inclination of 45°. We can get a conclusion that at this modulation intensity, the modulated lasers VCSEL2 and VCSEL3 can achieve precise synchronization.

The reason for VCSEL2 and VCSEL3 achieving precise synchronization is that they can reach a broad synchronization with VCSEL1 by modulation from VCSEL1 to VCSEL2 and VCSEL3 respectively[19,20]. At this point, $P_2 = F$ (P_1, φ), $P_3 = F$ (P_1, φ), where F is a complex function and φ is the parameter that influences P_2 and P_3 in addition to P_1. As a result, under the condition that other parameters of VCSEL2 and VCSEL3 are equal, if P_2 and P_3 reach a broad synchronization with VCSEL1, the states of VCSEL2 and VCSEL3 are only determined by VCSEL1. Thereby, VCSEL2 and VCSEL3 can reach precise synchronization if VCSEL1 is in the chaotic state.

External chaotic signal parameter modulation synchronization used in this letter is based on chaos-driven synchronization[21], namely using a driving system which can produce a chaotic signal to drive the other two chaotic systems directly to achieve synchronization. The difference of our method from the conventional chaos-driven synchronization program is that chaos signal is transformed into electrical signal and then injected into the modulation currents of modulated lasers. The two identical lasers modulated by the chaotic signal from the same laser finally reach the synchronization automatically through a transient process.

The correlation coefficient is a valuable mathematical tool to determine whether the two systems achieve synchronization or not. So we introduce the correlation coefficient to determine whether the two modulated lasers can achieve precise synchronization and to determine the range of parameters in which two modulated lasers achieve precise synchronization. The formula for calculating the correlation coefficient is[22]

$$\rho = \frac{\langle [X(t) - \langle X(t)\rangle][Y(t) - \langle Y(t)\rangle]\rangle}{\langle |X(t) - \langle X(t)\rangle|^2\rangle^{1/2} \langle |Y(t) - \langle Y(t)\rangle|^2\rangle^{1/2}}, \quad (17)$$

where $X(t)$ and $Y(t)$ are the outputs of the two systems; the terms $\langle\cdot\rangle$ mean the average versus time. We set $X(t) = P_2(t)$, $Y(t) = P_3(t)$ and put them into the formula. The photon density correlation coefficient of the two modulated lasers VCSEL2 and VCSEL3 can be calculated. The two modulated laser systems can achieve precise synchronization if the correlation coefficient is 1.00.

Figure 4 shows the photon density correlation coefficient of VCSEL2 and VCSEL3 versus mudulation

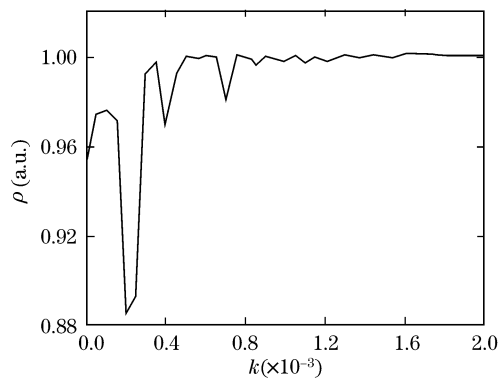

Fig. 4. Photon density correlation coefficient of two responding lasers ρ versus modulation intensity k.

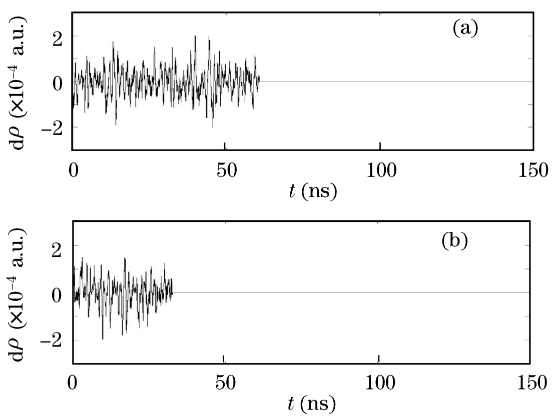

Fig. 5. Photon density difference of two responding lasers dρ versus time t at different modulation intensities. (a) k=0.0016; (b) k=0.0020.

intensity. It is observed that the photon density correlation coefficient is less than 1.00 if the mudulation intensity is very small and it is equal to 1.00 if the modulation intensity is in the range of $k > 1.40 \times 10^{-3}$. So the two modulated laser systems achieve precise synchronization and the range of parameter in which two responding lasers achieve precise synchronization is $k > 1.40\times 10^{-3}$.

In order to analyze the effect of synchronization more clearly, we calculate how long it will take to achieve synchronization from unsynchronization at different modulation intensities, namely, how long the transient process will take. It is clear that the shorter the transient process is, the faster the synchronization is, and the better the efficiency of synchronization is. We give the photon density difference of two modulated lasers versus time at mudulation intensities of 0.0016 and 0.0020 in Fig. 5. It is observed from Fig. 5(a) that the photon density difference of two responding lasers becomes 0 in the range of $t>$ 60.85 ns and from Fig. 5(b) that the photon density difference becomes 0 in the range of $t>$ 32.59 ns. It indicates that the systems can achieve precise synchronization in a shorter time as the modulation intensity increases. We make a conclusion that the greater the mudulation intensity is, the easier it is for the two modulated systems to achieve precise synchronization.

External chaotic signal parameter modulation can also be applied to systems with a number of modulated lasers. With the same chaotic signal from one laser, a great many modulated lasers can reach broad synchronization with the modulating laser, and the modulated lasers achieve precise synchronization with each other.

In conclusion, we get the synchronization of the two chaotic VCSELs by external chaotic signal parameter modulation. The numerical calculation of modulated lasers' correlation coefficient shows that the two lasers can achieve chaotic synchronization well. We also find that the greater the modulation intensity is, the easier it is for modulated lasers to achieve synchronization. The modulated systems can be extended to many lasers. By this means, modulated lasers can reach broad synchronization with the modulating laser and achieve precise synchronization with each other. It is of great significance for multiple-terminal secure communication with synchronization of chaotic VCSELs.

References

1. L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64,** 821 (1990).
2. Y.-J. Liu, S.-H. Zhang, and X.-Z. Qian, Chin. Phys. **16,** 463 (2007).
3. M. Shao, S.-H. Zhang, and X.-Z. Qian, Laser Journal (in Chinese) **28,** (3) 7 (2007).
4. V. Bindu and V. M. Nandakumaran, J. Opt. A **4,** 115 (2002).
5. S. Yan, Chin. Opt. Lett. **3,** 283 (2005).
6. L. Han, C. Jia, L. Liu, L. Liu, C. Zhang, and H. Zhao, Photon Technol. (in Chinese) (4) 181 (2006).
7. J. Song, Y. Fu, Y. Liu, J. Yin, B. Kang, and G. Du, J. Optoelectron. Laser (in Chinese) **11,** 345 (2000).
8. W. Liu, J. Zhong, and C. Yan, Chin. J. Lumin. (in Chinese) **27,** 519 (2006).
9. H. Guan, G. Huang, X. Huang, B. Guo, H. Shu, and K. Gao, Chinese J. Lasers (in Chinese) **34,** 33 (2007).
10. W. Zhang, W. Pan, B. Luo, X. Li, X. Zou, and M. Wang, Chinese J. Lasers (in Chinese) **34,** 55 (2007).
11. Y. Hong, M. W. Lee, P. S. Spencer, and K. A. Shore, Opt. Lett. **29,** 1215 (2004).
12. M. W. Lee, Y. Hong, and K. A. Shore, IEEE Photon. Technol. Lett. **16,** 2392 (2004).
13. Y. Hong, J. Paul, P. S. Spencer, and K. A. Shore, J. Opt. Soc. Am. B **23,** 2285 (2006).
14. Y. Hong, M. W. Lee, J. Paul, P. S. Spencer, and K. A. Shore, Opt. Lett. **33,** 587 (2008).
15. W. W. Chow, K. D. Choquette, M. H. Crawford, K. L. Lear, and G. R. Hadley, IEEE J. Quantum Electron. **33,** 1810 (1997).
16. P. V. Mena, J. J. Morikuni, S.-M. Kang, A. V. Harton, and K. W. Wyatt, J. Lightwave Technol. **17,** 865 (1999).
17. S. F. Yu, W. N. Wong, P. Shum, and E. H. Li, IEEE J. Quantum Electron. **32,** 2139 (1996).
18. W. Pan, X. Zhang, B. Luo, H. Lu, and J. Chen, Semicond. Photon. Technol. **6,** 40 (2000).
19. H. D. I. Abarbanel, N. F. Rulkov, and M. M. Sushchlk, Phys. Rev. E **53,** 4528 (1996).
20. S.-H. Zhang and K. Shen, Chin. Phys. **11,** 894 (2002).
21. R. Wang and K. Shen, Phys. Rev. E **65,** 016207 (2002).
22. S. Tang and J. M. Liu, Opt. Lett. **26,** 596 (2001).