## Adaptive interference hyperspectral image compression with spectrum distortion control

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As one of the next generation imaging spectrometers, interferential spectrometer has been paid much attention. With traditional spectrum compression methods, the hyperspectral images generated by interferential spectrometer can only be protected with better visual quality in spatial domain, but its optical applications in Fourier domain are often ignored. So the relation between the distortion in Fourier domain and the compression in spatial domain is analyzed in this letter. Based on this analysis, a novel coding scheme is proposed, which can compress data in spatial domain while reducing the distortion in Fourier domain. The bitstream of set partitioning in hierarchical trees (SPIHT) is truncated by adaptively lifting the rate-distortion optimization theory. Experimental results show that the proposed scheme can achieve better performance in Fourier domain while maintaining the image quality in spatial domain.

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Interferential spectrometer is carried on the satellite for the purpose of space exploration, military reconnaissance, and weather  $forecast^{[1]}$ . Because of its advantages of high throughput, multi-channel, and great resolution, interferential spectrometer has been used on the Chinese Chang'e Moon Exploration Satellite for the substance classification, resource investigation, and remotesensing. Its unique character is that its optical applications are mainly in Fourier domain, which is different from the dispersive spectrometer data, such as airborne visible/infrared imaging spectrometer (AVIRIS). Interference hyperspectral images should be compressed immediately when formed on the charge coupled device (CCD) of interferential spectrometer on the satellite, which is in spatial domain. Furthermore, a complex and non-real-time post-processing is adopted to recover the spectrum on the ground in Fourier domain<sup>[2]</sup>. Great amount of data will be generated during the imaging on interferential spectrometer. Thus, challenges are brought and spectrum should be well protected during the highratio data compression.

Several lossless compression methods<sup>[3,4]</sup> have been proposed for the interferential spectrometer data. However, lossy compression is more efficient for transmission if spectrum distortion can be controlled in an acceptable level for optical application. Although there are some lossy compression methods<sup>[5]</sup> for dispersive spectrometer, they do not fit the requirements of interferential spectrometer due to different optical imaging principles. Hence, some methods [6-8] are proposed for interferential hyperspectral image but they only focus on the visual information rather than the spectrum information. Due to the importance in the application for spectrum analyzer, spectrum distortion in Fourier domain should be paid more attention. Therefore, lossy compression method with spectrum distortion control for interference hyperspectral image is crucial for the interferential spectrometer's further applications on satellites.

In this letter, the relation between the compression in spatial domain and the distortion control in Fourier domain is analyzed. Based on the achieved conclusion, we propose the weighted rate-distortion optimization for set partitioning in hierarchical trees (WRDO-SPIHT) which imposes different priorities on various optical path difference (OPD) pixels with rate-distortion optimization truncation for bitstream allocation of SPIHT according to the characteristics of interference hyperspectral images in Fourier domain. This method can not only improve the compression performance of SPIHT in spatial domain but also achieve the balance of bitstream allocation between spatial and spectral distortion.

The optical principle of interferential spectrometer is that the light forms two slim targets on the fore-focal planes and is separated into a pair of coherent lights in the interferometer. The interferogram shown in Fig. 1(a) is collected from CCD detector by the optic collecting system. The data I(x) in the same row of interferogram generated by interference of coherent lights is called the interference curve as shown in Fig. 1(b) and can be expressed as

$$I(x) = \int_{\nu_{\min}}^{\nu_{\max}} B(\nu) \mathrm{e}^{2\pi\nu x} \mathrm{d}\nu, \qquad (1)$$

where  $B(\nu)$  is the spectral distribution of source,  $v_{\min}$  and  $v_{\max}$  are extremums of wave number, x represents OPD in interference curve.

According to the theory of irradiance, radiation consists of a series of waves with finite lengths. It means that there is one waveform during the coherence time  $\tau_{\rm c}$ . When B(v) = 1, I(x) can be expressed as

$$I(x) = \sqrt{\frac{2}{\pi}} \frac{\sin((x - x_0)\tau_c/2)}{x - x_0} = \tau_c \sqrt{\frac{1}{2\pi}} Sa\left(\frac{(x - x_0)\tau_c}{2}\right).$$
(2)

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Equation (2) shows that the interference curve is a conversion of Sa function whose mainlobe width is  $4\pi/\tau_c$  and central frequency is  $x_0$ . Although the incident rays amplitudes change, most interference curves in the interferogram produced by the same interferential spectrometer follow the model of  $Sa((x - x_0)\tau_c/2)$ , where  $x_0$  and  $\tau_c$  are the parameters of interferential spectrometer.

According to Fourier transform spectroscopy, the spectrum curve can be obtained by Fourier transform of interference curve. Perform Fourier transform on Eq. (1), the spectrum curve is

$$B(f) = \int_0^{\delta_{\rm m}} I(x) \mathrm{e}^{-2\pi f x} \mathrm{d}x, \qquad (3)$$

where  $\delta_{\rm m}$  is the maximum OPD.

In the substance classification and recognition, the shape of spectrum curve can be used to identify existence sand contents of materials<sup>[9,10]</sup>. A practical example of substance recognition can be found in Ref. [10], which shows the spectrum curves  $(0.4-2.5 \ \mu\text{m})$  of a camou-flaged tank and the located grassplot. Spectrum analyzer can distinguish the camouflaged tank from grassplot by the shape of spectrum curve at 1.7  $\mu$ m. Therefore, in the compression the shape and contour information in the spectrum curve should not be lost.

Suppose I(x) and  $I^*(x)$  are the original and reconstructed interference curve in spatial domain, and B(f) and  $B^*(f)$  are their respective spectrum curve. Consider the 2nd-order Taylor formula B(f) = $B(f_0)+B'(f_0)(f-f_0)+\dots+o[(f-f_0)^2]$ . We know that the first-order derivative  $B'(f_0)$  is the most important parameter which determines the shape of spectrum curve. Let dB(f)/df and  $dB^*(f)/df$  denote the first-order derivatives of B(f) and  $B^*(f)$ , respectively. The mean square error (MSE) of the first-order derivative (FD-MSE) of spectrum curve in Fourier domain is

$$\mathrm{MSE}_{\mathrm{FD}}(f) = \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \left| \frac{\mathrm{d}B(f)}{\mathrm{d}f} - \frac{\mathrm{d}B^*(f)}{\mathrm{d}f} \right|^2 \mathrm{d}f.$$
(4)

The relation between the FD-MSE in Fourier domain and the distortion of reconstructed images in spatial domain is

$$\underbrace{\operatorname{MSE}_{\operatorname{FD}}(f)}_{\operatorname{Fourier Domain}} = \underbrace{8\pi \int_{0}^{\delta_{\operatorname{m}}} x^{2} \left(I(x) - I^{*}(x)\right)^{2} \mathrm{d}x}_{\operatorname{Spatial Domain}}.$$
 (5)

Let  $\hat{B}(f) = \int_0^{\delta_m} I(x) e^{-j2\pi x f} dx$ , we have

$$B(f) = B(f) + B(-f).$$

According to the property of Fourier Transform, one obtains

$$I(x) + I(-x) \xrightarrow{\text{FT}} \hat{B}(f) + \hat{B}(-f)$$

and then we have the following relation

$$-jx(I(x) + I(-x)) \xleftarrow{FT} \frac{dB(f)}{df}$$

Take the recovered spectrum curve into consideration, then

$$\begin{aligned} x\left((I(x)+I(-x))-(I^*(x)+I^*(-x))\right) \\ \xleftarrow{\mathrm{FT}} j\left(\frac{\mathrm{d}B(f)}{\mathrm{d}f}-\frac{\mathrm{d}B^*(f)}{\mathrm{d}f}\right). \end{aligned}$$

According to Parseval's theorem, there will be

$$\int_{-\infty}^{+\infty} \left( x \left( (I(x) + I(-x)) - (I^*(x) + I^*(-x)) \right) \right)^2 \mathrm{d}x$$
$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| \frac{\mathrm{d}B(f)}{\mathrm{d}f} - \frac{\mathrm{d}B^*(f)}{\mathrm{d}f} \right|^2 \mathrm{d}f.$$

From Eq. (1), I(x) is an even function, i.e., I(x) = I(-x). Considering  $f \in [f_{\min}, f_{\max}]$ , then one gets

$$MSE_{FD}(f) = \int_{f_{min}}^{f_{max}} \left| \frac{\mathrm{d}B(f)}{\mathrm{d}f} - \frac{\mathrm{d}B^*(f)}{\mathrm{d}f} \right|^2 \mathrm{d}f$$
$$= 8\pi \int_{-\infty}^{+\infty} x^2 (I(x) - I^*(x))^2 \mathrm{d}x$$

Due to  $x \in [0, \delta_m]$ 

$$MSE_{FD}(f) = 8\pi \int_0^{\delta_m} x^2 \left( I(x) - I^*(x) \right)^2 dx$$

The relation of distortions between Fourier domain and spatial domain is established by this theorem. Based on Eq. (5), it is known that if  $\text{MSE}_{\text{FD}}(f)$  in Fourier domain is restricted by the application requirement, in spatial domain, the larger |x| is, the greater the difference between I(x) and  $I^*(x)$  contributes to the error of the first-order derivative of spectrum. Therefore in the small OPD area of spatial domain, the error of compression could be large, while in the large OPD area, the error should be small. Consequently, due to the firstorder derivative, in the valid OPD area  $(x < \delta_m)$ , as OPD increasing, the pixels in interferogram are so important that they should be protected efficiently when compressed in spatial domain.



Fig. 1. Interference hyperspectral image. (a) Interferogram, (b) interference curve.



Fig. 2. Coding scheme.

Bit Rate	WRDO-SPIHT	$\operatorname{RDO-SPIHT}$	$\rm JPEG2000$	$\operatorname{SPHIT}$
(bpp)	(dB)	(dB)	(dB)	(dB)
0.2	47.37	47.35	47.01	46.33
0.4	48.12	48.08	47.90	47.67
0.6	48.68	48.65	48.59	48.35
0.8	49.38	49.36	49.34	48.96
1.0	50.40	50.33	50.12	49.85

 Table 1. PSNR Performance Comparison

SPIHT<sup>[11]</sup> is an efficient wavelet-based progressive image compression technique, designed to minimize the MSE between the original and reconstructed images. However, low-amplitude wavelet coefficients which may be important for spectrum classification are given low priority by the conventional SPIHT. The above theorem indicates that the significance of coefficients in the interferogram gradually changes with OPD. Therefore we propose WRDO-SPIHT for interference hyperspectral image compression.

The coding scheme of WRDO-SPIHT is shown in Fig. 2. A zerotree  $T_i$  is constituted by all offsprings of Z(i, j) whose root is (i, j) in the LL subband after wavelet decomposition. A zerotree containing a set of coefficients from different bands represents the same spatial region in the image. The root of each tree can indicate OPD position. Then each tree performs SPIHT coding independently for every bitplane to produce bitstream  $B_i$ , which is weighted according to the OPD of its root (i, j) following the above theorem. Bitstream is truncated into L layers by rate-distortion, such as  $B_i^1, B_i^2, \dots, B_i^L$ , which successively and monotonically improve the image quality. Finally, the  $B_i^l, 1 \leq l \leq L$ , of each  $T_i$  will be used to generate the bitstream.

In the following, we will analyze the bitstream truncated process which includes weighting bit-stream according to OPD and rate-distortion optimized truncation in Fig. 2. Given the rate  $R_{\text{max}}$ , suppose that the truncated rate of the embedded bitstream produced in tree  $T_i$  is  $R_i^{n_i}$ , where  $n_i$  is truncated point. The reconstructed distortion of the coefficients in the tree  $T_i$  is  $D_i^{n_i}$ . According to the method of Lagrange multipliers, the problem is equivalent to minimize

$$\sum_{i} \left( R_i^{n_i} + \lambda D_i^{n_i} \right), \tag{6}$$

where  $\lambda$  should be adjusted until the rate produced by the truncation points which minimize Eq. (6) satisfying  $R = R_{\text{max}}$ . The problem of minimizing Eq. (6) is a separate optimization problem for each individual tree. Specifically, for each tree  $T_i$ , the truncation point  $n_i$ which minimizes  $(R_i^{n_i} + \lambda D_i^{n_i})$  needs to be found.

After minimizing Eq. (6), the slope  $S_i^{n_i} = \Delta D_i^{n_i} / \Delta R_i^{n_i}$  is calculated to determine the candidate truncated points. It is known that the greater  $S_i^{n_i}$  is, the earlier  $n_i$  will be chosen. So in the interferogram, important pixels' slopes should be lifted in order to be truncated earlier. Our proposed algorithm selects weighted mean-squared error distortion as the distortion criterion according to the OPD of every pixel on the spectrum in Fourier domain. According to Eq. (2), with

Sa function being seen as a sine wave squeezed between the two envelopes  $y = \pm 1/x$ , the digression trend of I(x)is  $y = |2/(x - x_0)\tau_c|$ . Let

$$\omega(x) = \frac{2}{(2\delta_{\rm m} + x - x_0)\tau_{\rm c}}\tag{7}$$

be the weight of distortion which is the symmetry of  $y = |2/(x - x_0)\tau_c|$  to  $x = \delta_m$ , where  $\delta_m$  is the maximum of OPD. Therefore the distortion of tree  $T_i$  is

$$D = \sum_{m,n} \omega(x) D_i^n[m,n] = \sum_{m,n} \omega(x) \left( c_i[m,n] - c'_i[m,n] \right)^2,$$

where  $c_i[m, n]$  is the wavelet coefficient of original image and  $c'_i[m, n]$  is the wavelet coefficient of reconstructed image.

Suppose that the weighted lifting function on the OPD direction is  $\gamma(x)$ , and the lifted slope of the candidate truncated points  $S_i^n(\gamma(x))$  is

$$S_i^n(\gamma(x)) = \gamma(x) \cdot S_i^n = \gamma(x) \cdot \Delta D_i^n / \Delta R_i^n$$

We have known that as OPD increasing, the candidates will be more important in Fourier domain. So we can use OPD x as the weight to adjust the priority of pixel's R-D slopes in order to truncate the important candidates preferentially when compress in spatial domain. Let

$$\gamma(x) = px, \ p > 0.$$

The lifted slope of candidate truncated points according to OPD will be

$$S_i^n(x) = px \cdot \Delta D_i^n / \Delta R_i^n$$
.

Provided that the maximum slope is  $S_{\max}$  and the minimum is  $S_{\min}$ , the choice of p must ensure  $S_i^n(x)$  in the range of  $[S_{\min}, S_{\max}]$ . The value of p is a constant determined by the properties of the image. In our evaluations, p is chosen properly to obtain the best compression quality. For the tested interference hyperspectral spectrometer data, p is about 1.47. As a result, the candidates' slopes are adjusted with OPD changing. As OPD increasing, the candidates will be truncated according to their priorities.

In order to test the efficiency of WRDO-SPIHT both in spatial and Fourier domain, its performance is compared to original SPIHT<sup>[11]</sup>, JPEG2000 image compression standard, and rate-distortion optimization for SPIHT (RDO-SPIHT)<sup>[12]</sup>.

Table 1 lists the peak signal to noise ratio (PSNR) of the four methods to show the performance in spatial domain. From this table, we learn that in spatial domain compression, the PSNR of WRDO-SPIHT achieves 0.3–1-dB improvement compared with SPIHT and 0.04–0.36-dB improvement compared with JPEG2000. WRDO-SPIHT has almost the same performance as RDO-SPIHT and also the best performance in these popular image compression methods.



Fig. 3. Fourier domain results comparison. (a) REQ, (b) FD-MSE.

It is known that there are close relation between the spatial domain and Fourier domain. Fourier transform, the bridge between spatial domain and Fourier domain, is a linear calculation. So if the distortion in Fourier domain is reduced, the distortion in spatial domain would be also reduced. Obviously, our attempt to minimize FD-MSE in Fourier domain has positive effect on the reduction of distortion in spatial domain. As a result, WRDO-SPIHT can have improvement in PSNR, although the optimization is in Fourier domain.

For interferential spectrometer data, Fourier domain is more important for the pattern/target recognition and classification. In the following evaluation, we will discuss the performance in Fourier domain. As a common criterion, spectral relative quadratic error  $(RQE)^{[2]}$  reflects the spectrum error introduced in compression. Its expression is

$$\mathrm{RQE} = \frac{\sqrt{\int_{f_{\min}}^{f_{\max}} |B^*(f) - B(f)|^2 \,\mathrm{d}f}}{\int_{f_{\min}}^{f_{\max}} B(f) \,\mathrm{d}f},$$

where B(f) is the original spectrum and  $B^*(f)$  is the reconstructed spectrum.  $f_{\text{max}}$  and  $f_{\text{min}}$  are extremums of frequency. Figure 3(a) shows RQE of spectrum curves from the 1st row to the 150th row in Fig. 1. The RQE of WRDO-SPIHT improves almost 3 times compared with SPIHT, about 1.8 times compared with RDO-SPIHT, and about 1.5 times compared with JPEG2000. Take the FD-MSE defined in Eq. (4) as the criterion to evaluate the distortion of contour and absorption peaks of spectrum curve. The FD-MSE produced by the four algorithms are shown in Fig. 3(b). It is clear that WRDO-SPIHT improves FD-MSE obviously. The average FD-MSE of WRDO-SPIHT is 6.03, with that of JPEG2000 being 9.54, RDO-SPIHT being 12.99, and SPIHT being 28.24.

In conclusion, based on the relation between the compression in spatial domain and the distortion in Fourier domain, we propose a new algorithm, weighted ratedistortion optimization for SPIHT for the interferometer hyperspectral image compression. In order to protect the shape of spectrum curve which is the evaluation criteria for substance classification and recognition in spectrum, the slope of rate-distortion curve is lifted according to their contribution of pixels in various optical path differences based on the rate-distortion optimization theory. The proposed method can make an optimization of bits allocation between spatial domain and Fourier domain. Compared with RDO-SPIHT, JPEG2000, and SPIHT, the proposed method can not only achieve a better performance in spatial domain but also reduce the distortion in Fourier domain. The shapes of spectrum curves are better protected.

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