Repeat-until-success measurement-based scheme for controlled phase gates in a cavity

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We propose a new repeat-until-success (RUS) measurement-based scheme to implement quantum controlled phase gates according to the effect of dipole-induced-transparency (DIT) in a cavity and single-photon interference at a 50:50 beam-splitter. In our scheme, the DIT effect can appropriately attach a photon to the state of the dipoles according to their initial state, and in this way, a suitably encoded dipole-photon state is thus prepared. The measurement of the photon after it passing through a 50:50 beam-splitter can project the encoded matter-photon state to either a desired phase gate operation for the matter qubits or to their initial states. The recurrence of the initial state permits us to implement the desired entangling gate in a RUS way.

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As well known, a quantum computer made up of quantum logic gates, two kinds of noncommutable single-qubit gates, and one kind of two-qubit entangling logic gate can construct a universal quantum computation[1,2]. To implement two-qubit entangling gates, many proposals have been presented by coherently controlling the qubit-qubit interactions and some proof-of-principle experiments have been performed, such as trapped ions[3,4], cavity quantum electrodynamics (QED)[5−12], and quantum dots[13], etc. These proposals successfully demonstrated the principles and the possibilities of the quantum computation.

On the other hand, quantum computation can be implemented by the measurement of the prepared quantum states. Knill *et al.* proposed a measurement-based linear-optics quantum computation scheme[14], in which quantum gates can be probabilistically constructed when certain measurement results are obtained. To achieve deterministic quantum logic gates, two different methods have been developed, one is the one-way measurement-based quantum computation[15,16] where certain multiqubit cluster states[17] are initially prepared and the subsequent one-qubit measurements on the selected particles could sufficiently lead to any desired quantum gate operations deterministically. The cluster states can be generated probabilistically by photon interference effects and photon measurements. Proposals[18,19] have been presented for the improvements on the linear optics quantum computation in Ref. [14]. Another method to implement deterministic entangling logic gates is the so-called repeat-until-success (RUS) scheme[20,21] in which the two-qubit entangling gates can be probabilistically implemented by measuring the photons in a mutually unbiased basis. If, however, one fails to implement the desired entangling logic gates by measurement, the quantum information stored in matter qubits is not destroyed and can be recovered to its original form for further manipulating. Thus one can repeat the procedure again and again until success. Very recently, a new RUS measurement-based quantum computation scheme has been proposed[22] by using the effect of dipole-induced-transparency (DIT)[23] and the single-photon interference at a 50:50 beam-splitter. In the scheme[22], the cavities are assumed to contain a single dipoles each. In this letter, we present a scheme for the implementation of the controlled-phase (CP) gates when multiple dipoles are placed in a single cavity. In comparison with Ref. [22], the scheme proposed here has at least two advantages: 1) it simplifies the requirements for quantum computation; 2) it reduces the decoherence due to photon losses, because in this scheme the photon is transmitting only via a single cavity, thus the fidelity of the resulting gates can be increased.

Let us firstly consider the DIT effect with multiple dipoles in a cavity. In a cavity-waveguide coupling system, an optical field would normally be transmitted from one waveguide to another through the resonant coupling to the cavity, in other words, the waveguide is normally opaque at the cavity resonance. Reference [23] showed that the situation will be greatly changed when a dipole (atom, quantum dot, etc.) is placed in the drop-filter cavity. In fact, the resonant coupling between the dipole and the cavity makes the waveguide highly transparent even in the bad cavity regime, this is the so-called DIT effect. The DIT effect has been proposed to generate and detect Bell states of two dipoles[23] and Greenberger-Horne-Zeilinger (GHZ) states of many dipoles[24]. Similar to the DIT effect with a single dipoles in a cavity, when multiple identical dipoles are placed in a cavity and each resonantly couples to the cavity, the resonant coupling between the the cavity and dipoles can obviously make the waveguide highly transparent.

Next, let us take into account the situation where the multiple identical dipoles are of the three-level Λ-type configuration with an excited state $|e\rangle$, a ground state $|0\rangle$, and a long lived metastable state $|1\rangle$ as shown in Fig. 1. In our scheme, the states $|0\rangle$ and $|1\rangle$ of the dipoles construct the computational basis. We assume that the

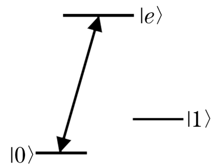

Fig. 1. Three-level structure of the dipoles.

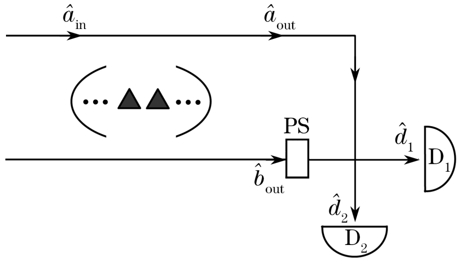

Fig. 2. Schematic setup of the RUS scheme for implementing the CP gate based on the DIT effect with multiple dipoles in a cavity. The dipoles are supposed to have identical level structures. PS: phase-shifters; BS: 50:50 beam-splitter; D_1, D_2: photodetectors.

transition $|0\rangle \leftrightarrow |e\rangle$ is resonant to the cavity, while the transition $|1\rangle \leftrightarrow |e\rangle$ is largely off-resonant to the cavity. For the input light fields, the waveguide will be opaque if and only if the dipoles are all in the state $|1\rangle$, otherwise it will be highly transparent if one or more dipoles are in the state $|0\rangle$. In an ideal case, when a photon is sent from the upper waveguide as plotted in Fig. 2, it will be transmitted through the lower or upper waveguides depending on whether or not the dipoles are all in the state $|1\rangle$. We now suppose that there are N dipoles in the cavity and the general initial state of these N dipoles can be simply expressed as two terms, one is $c_{1\cdots 1}|1\rangle \otimes |1\rangle \otimes |1\rangle \otimes \cdots$ where $c_{1\cdots 1}$ is a coefficient, indicating all the dipoles are in the state $|1\rangle$, which is denoted by $|\tilde{1}\rangle$, and the other term denoted by $|\tilde{0}\rangle$, consisting of all the contributions that at least one dipole is in the state $|0\rangle$. The initial state of the N dipoles is thus of the form

$$|\Psi\rangle_{\text{init}} = |\tilde{0}\rangle + |\tilde{1}\rangle. \quad (1)$$

Now we show how to encode the state (1) with a single photon by means of the DIT effect (see Fig. 2). Supposing the input optical field is in a single photon state $\hat{a}_{\text{in}}^{\dagger}|\text{vac}\rangle$, where $|\text{vac}\rangle$ denotes the vacuum state of the light fields, the initial state of the input optical field and the N dipoles at time $t = 0$ is

$$|\Phi(0)\rangle = \hat{a}_{\text{in}}^{\dagger}|\text{vac}\rangle \otimes \left(|\tilde{0}\rangle + |\tilde{1}\rangle\right). \quad (2)$$

According to the principle of the DIT effect with multiple dipoles in a cavity discussed above, the output photon will pass through the cavity and be transmitted from the lower waveguide if and only if all the dipoles are in state $|1\rangle$, otherwise the photon will be transmitted along the upper waveguide. We assume that the phase shift in the lower waveguide provides an extra phase in contrast to the upper waveguide, and then when the optical field mode in the upper waveguide arrives at the beam-splitter at time $t = t_1$, the state (2) will evolve to

$$|\Phi(t_1)\rangle = \hat{a}_{\text{out}}^{\dagger}|\text{vac}\rangle \otimes |\tilde{0}\rangle + \mathrm{e}^{\mathrm{i}\phi}\hat{b}_{\text{out}}^{\dagger}|\text{vac}\rangle \otimes |\tilde{1}\rangle. \quad (3)$$

The state (3) is exactly the encoded state for the implementation of the multiqubit CP gates.

To implement the multiqubit CP gates, the optical modes from the lower and upper waveguides are incident upon a 50:50 beam-splitter as shown in Fig. 2 and they interfere there. The output modes of the beam-splitter $\hat{d}_1$ and $\hat{d}_2$ are finally detected by photodetectors D_1 and D_2 respectively. For the 50:50 beam-splitter, the relations between the input and the output modes are

$$\hat{d}_1^{\dagger} = \frac{1}{\sqrt{2}}\left(\hat{b}_{\text{out}}^{\dagger} + \mathrm{i}\hat{a}_{\text{out}}^{\dagger}\right),$$

$$\hat{d}_2^{\dagger} = \frac{1}{\sqrt{2}}\left(\mathrm{i}\hat{b}_{\text{out}}^{\dagger} + \hat{a}_{\text{out}}^{\dagger}\right). \quad (4)$$

Therefore after the evolution through the beam-splitter at time $t = t_2$, the state (3) becomes

$$|\Phi(t_2)\rangle = \frac{1}{\sqrt{2}}\Big\{-\mathrm{i}\hat{d}_1^{\dagger}|\text{vac}\rangle \otimes \left(|\tilde{0}\rangle + \exp\left[\mathrm{i}\left(\phi + \pi/2\right)\right]|\tilde{1}\rangle\right) + \hat{d}_2^{\dagger}|\text{vac}\rangle \otimes \left(|\tilde{0}\rangle + \exp\left[\mathrm{i}\left(\phi - \pi/2\right)\right]|\tilde{1}\rangle\right)\Big\}. \quad (5)$$

If the photodetector D_1 measures the photon, the state (5) will collapse to $|\tilde{0}\rangle + \exp[\mathrm{i}(\phi+\pi/2)]|\tilde{1}\rangle$ with 50% probability. Suppose the phase factor ϕ is chosen as $2k\pi+\pi/2$, with k a positive integer, then the collapsed state is $|\tilde{0}\rangle - |\tilde{1}\rangle$. This state is actually our target state corresponding to executing a multiqubit CP gate operation on the initial state (1). If, however, the photodetector D_2 measures the photon also with 50% probability, the state (5) will collapse into $|\tilde{0}\rangle + \exp\left[\mathrm{i}\left(\phi - \pi/2\right)\right]|\tilde{1}\rangle$. Considering $\phi = 2k\pi + \pi/2$, this state is exactly the initial state (1). In the latter case, although one cannot realize a multiqubit CP gate operation, the initial state (1) is not destroyed and can be used to repeat above procedure again, in this way one can finally realize a multiqubit CP gate until the photodetector D_1 measures the photon.

In the above analysis, we have concentrated on the ideal case. In practice, the cavity loss, the atomic decay, and the limited efficiency of the DIT effect will reduce the fidelity of the resultant entangling gates. In the following, let us investigate the fidelity of a two-qubit CP gate operation when there are only two dipoles in the cavity. The general initial state of two dipoles can be written as $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$, where the coefficients α, β, γ, and δ meet the normalized relation. In Ref. [23] a formula about the input-output relation was given under the conditions of cavity decay, dipole decay, and some potential leaky modes. In the case that the cavity resonantly couples to the dipoles (at least one dipole is in the state $|0\rangle$) and to the waveguide with one input field as shown in Fig. 2, the input-output relations are

$$\hat{a}_{\text{out}} = \frac{\left(\frac{1}{2}\kappa + 2\tau g^2\right)\hat{a}_{\text{in}} - \sqrt{\kappa\lambda}\hat{e}_{\text{in}}}{\lambda + \frac{1}{2}\kappa + 2\tau g^2},$$

$$\hat{b}_{\text{out}} = \frac{-\lambda\hat{a}_{\text{in}} - \sqrt{\kappa\lambda}\hat{e}_{\text{in}}}{\lambda + \frac{1}{2}\kappa + 2\tau g^2}, \quad (6)$$

where λ is the coupling constant between the cavity and the waveguide, g is the vacuum Rabi frequency of the

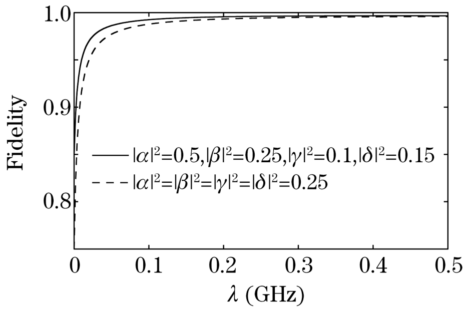

Fig. 3. Fidelity for one round operation as a function of λ. The fidelity of the CP gate operation is the same as that of the recurrent initial state.

dipole ($g = 0$ when all the dipoles are in $|1\rangle$), $1/2\tau$ is the decay rate of the dipole operator, κ is the cavity decay rate, $\hat{e}_{\rm in}$ is the operator of the potential leaky modes. With the input-output relation (6), the fidelity F, defined as $F = |\langle\psi|\bar{\psi}\rangle|^2$ with $|\bar{\psi}\rangle$ ($|\psi\rangle$) being the outcome state after the photon measurement in nonideal (ideal) situation, can be worked out. Figure 3 plots F for one round implementation as a function of λ. According to Ref. [12], the parameters are set as $\kappa = 0.01$ THz, $g = 0.33$ THz, and $\tau^{-1} = 1$ GHz. The results show that both the gate operation and the recurrent initial state have high fidelities when λ is ten times larger than κ. In contrast to Ref. [22], under the same circumstances the scheme proposed in this paper has a higher fidelity. Furthermore, the fidelity of the CP gate operation is the same as that of the recurrent initial state. This is different from the situation in the two cavity scheme[22] where the fidelities of the CP gate operation and the recurrent initial state are different.

Finally, we discuss a situation in which any photon is not detected by either photon detector in our DIT-based scheme. This situation occurs due to the photon loss during its transmission in the DIT device or to the limited efficiency of the photon detectors. In such situations, the final state is actually an incoherent mixture of the desired gate-operated state of the matter qubits and their initial state, and our RUS scheme thus fails. We have to abandon the results and restart the quantum computation from the beginning. In order to reduce such unfavorable effects, high quality waveguides and cavity as well as highly sensitive photon detectors should be used, so that the photon loss can be minimized. In our scheme, the photon detectors are not required to resolve the photon numbers, thus it is sufficient to use bucket or vacuum detectors which discriminate no photon from many photons. From the current technique, the bucket detectors can be made more sensitive than number-solving ones[25].

In conclusion, we have presented a new RUS measurement-based scheme to implement multiqubit CP gates by using single-photon interference and DIT effect with multiple dipoles in a cavity. In comparison with Ref. [22], our new scheme utilizes less sources such as cavities and waveguides and reduces the decoherence from the transmission of the optical light fields.

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