Effect of mutual inductance coupling on superconducting flux qubit decoherence

Yanyan Jiang (江燕燕)[1,3], Hualan Xu (徐华兰)[1], and Yinghua Ji (嵇英华)[1,2*]

[1]*College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang 330022*

[2]*Key Laboratory of Optoelectronic and Telecommunication of Jiangxi, Nanchang 330022*

[3]*Department of Physics, Anqing Teachers College, Anqing 246011*

[*]*E-mail: jyh2006@jxnu.edu.cn*

Received March 21, 2008

In the Born-Markov approximation and two-level approximation, and using the Bloch-Redfield equation, the decoherence property of superconducting quantum circuit with a flux qubit is investigated. The influence on decoherence of the mutual inductance coupling between the circuit components is complicated. The mutual inductance coupling between different loops will decrease the decoherence time. However, the mutual inductance coupling of the same loop, in a certain interval, will increase the decoherence time. Therefore, we can control the decoherence time by changing the mutual inductance parameters such as the strength and direction of coupling.

OCIS codes: 270.0270, 000.6800.

doi: 10.3788/COL20090701.0078.

In the quest for practical systems for carrying out quantum computations, solid-state systems that make use of the Josephson effect are available candidates[1,2]. Presently, three prototypes of superconducting qubits are studied experimentally[3−6]. The interaction between quantum system and environment will cause two demolishment processes, i.e., quantum dissipation and quantum decoherence[7,8]. The former will cause energy dissipation, and the latter will make the system degenerate from coherent state to classical state[9,10]. Compared with other qubit candidates (such as trapped ions, nuclear spins, and cavity quantum electrodynamics (QED)), decoherence presents a much more formidable challenge to superconducting qubits. For a true two-level qubit, decoherence occurs due to the coupling of the qubit to its environment. However, all of the proposed superconducting qubits have multiple energy levels which result in adverse effects on quantum gate operations. In fact, coupling between the computational bases, i.e., $|0\rangle$, $|1\rangle$ and states $|n \geq 2\rangle$ of the noncomputational subspace results in significant errors for one-qubit gate operations. Previous theoretical works on decoherence of superconducting qubits have typically relied on the widely used spin-boson model which postulates a purely two-level dynamics, therefore neglecting leakage effects[11]. As a basic model describing a quantum system, the spin-boson model provides a simple and effective way to study the quantum dissipation characteristics. Based on this model, the evolution of the quantum systems is described as a two-level dynamics process. Combining network graph theory with the Caldeira-Leggett model for dissipative elements, Burkard, Koch, and DiVincenzo (BKD) presented a multi-level quantum circuit theory of decoherence for a general circuit realization of a superconducting qubit[12,13]. The decoherence of "IBM qubit" was studied using the circuit theory. A number of decoherence mechanisms can be important, being both intrinsic characteristic to the Josephson junctions, and current and voltage from the external control circuits. The effect of the current or voltage fluctuations are related to the mutual inductance between external circuits and environment. In this letter, we study the effect of the mutual inductance between IBM qubit and environment for decoherence.

The IBM qubit is described by the electrical circuit shown in Fig. 1. We will investigate the decoherence property of superconducting flux qubits coupling with the environment. In the circuit, shunt resistors R, external impedances $Z(\omega)$, and bias current sources I_B form the environment.

The constraint relation between the current flowing through Josephson junction and voltage (flux) is

$$I_J = I_C \sin\varphi, \tag{1}$$

$$\frac{d\varphi}{dt} = \frac{\Phi_0}{2\pi} V_J(t), \tag{2}$$

where, I_J is the super-current of Josephson junction, I_C is the critical current of the junction, φ is the pulse difference across the junction, V_J is the voltage across

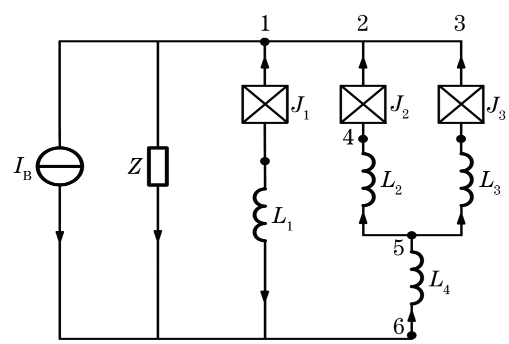

Fig. 1. Quantum circuit with flux qubit. Impedance Z reflects the dissipative effect of environmental electromagnetic fluctuation on qubit. The coefficient of the mutual inductance between L_1 and L_2, L_3 and L_4 is M_1, while that between L_3 and L_2, L_1 and L_4 is M_2. J_i $(i = 1, 2, 3)$: Josephson junctions.

the junction, $\Phi_0 \equiv h/2e$ is the flux quantum.

The Hamiltonian of the total system is

$$H(t) = H_{\rm S} + H_{\rm B} + H_{\rm SB}, \tag{3}$$

where $H_{\rm S}$ is the Hamiltonian of the system, $H_{\rm B}$ is the Hamiltonian of the reservoir, $H_{\rm SB}$ is the interaction of the quantum system and reservoir.

$$H_{\rm S} = \frac{1}{2}Q_C^T C^{-1} Q_C + \left(\frac{\Phi_0}{2\pi}\right)^2 U(\boldsymbol{\varphi}), \tag{4}$$

C is the capacitance of the junction and $U(\boldsymbol{\varphi})$ is the potential energy of the system. From $H_{\rm S}$, the canonical coordinate and canonical momentums of the system are respectively

$$X = \left(\frac{\Phi_0}{2\pi}\right)\varphi,$$

$$Q_C = \frac{\Phi_0}{2\pi}C\dot{\varphi}.$$

$H_{\rm B}$ is given by

$$H_{\rm B} = \frac{1}{2}\sum_\alpha \left(\frac{1}{m_\alpha}p_\alpha^2 + m_\alpha \omega_\alpha^2 x_\alpha^2\right). \tag{5}$$

$H_{\rm B}$ is the Hamiltonian describing a bath of harmonic oscillators position and momentum operators x_α and p_α, mass m_α, and oscillator frequency ω_α. Adopting coordinate-coordinate mode, $H_{\rm SB}$ inherits the coupling with a dissipative environment,

$$H_{\rm SB} = (\mathbf{g}\cdot\boldsymbol{\varphi})\sum_\alpha c_\alpha x_\alpha + (\mathbf{g}\cdot\boldsymbol{\varphi})^2 \sum_\alpha \frac{c_\alpha^2}{2m_\alpha\omega_\alpha^2}. \tag{6}$$

c_α is the coupling coefficient, $\mathbf{g}$ is the normalization vector:

$$\mathbf{g} = \frac{1}{\sqrt{E}}\begin{pmatrix} L_2L_3 + L_2L_4 + L_3L_4 + M_1(2L_2 + L_3 - 3M_2) + M_2(L_2 + L_3 - 2L_4 - 3M_2) \\ L_1L_3 + 2M_1^2 + (L_1 + L_3 + L_4 + 2M_2)M_1 - M_2^2 - (L_1 - L_3)M_2 \\ L_1L_2 - 2M_1^2 - (L_1 + L_4 + 3M_2)M_1 - M_2^2 - (L_1 - L_2)M_2 \end{pmatrix}, \tag{7}$$

where

$$\begin{aligned} E(M) = &[L_1(L_3 + M_1 - M_2) + L_3(M_1 + M_2) + L_4M_2 + 2M_1^2 - M_2^2 + 2M_1M_2]^2 \\ &+[L_1(M_1 + M_2 - L_2) + L_4M_1 - L_2M_2 + 2M_1^2 + M_2^2 + 3M_1M_2]^2 \\ &+[L_3(L_2 + L_4 + M_1 + M_2) + L_2(L_4 + 2M_1 + M_2) + M_2(2L_4 + 3M_1 + M_2)]^2. \end{aligned}$$

From $H(t)$, we first inspect the Hamilton equations for the bath and the system coordinates. The Fourier transform representations of the dynamical equations of the system are obtained as

$$-\omega^2 C\boldsymbol{\varphi}(\omega) = -\frac{\partial U}{\partial \boldsymbol{\varphi}}(\omega) - \left(\frac{2\pi}{\Phi_0}\right)^2 \mathbf{g}(\mathbf{g}\cdot\boldsymbol{\varphi})\sum_\alpha \frac{c_\alpha^2}{m_\alpha(\omega^2 - \omega_\alpha^2)}. \tag{8}$$

Then the noise spectrum coupling to the qubit is

$$J_{\rm eff}^{(T)}(M,\omega) = J_{\rm eff}(M,\omega)\left(\frac{\Phi_0}{2\pi}\right)^2 \coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right), \tag{9}$$

where $k_{\rm B}$ is the Boltzmann constant, T is the temperature of the thermal bath, and

$$J_{\rm eff}(M,\omega) = \frac{\omega E(M){\rm Re}Z(\omega)}{[D(M){\rm Im}Z(\omega) + \omega D_Z(M)]^2 + [D(M){\rm Re}Z(\omega)]^2} \tag{10}$$

is the effective spectral density. In order to describe the decoherence in the weak damping limit, we use the Bloch-Redfield formalism. According to the quantum statistics principle, the time evolution of the total density operator $\rho_T(t)$ under unitary evolution is

$$i\hbar\frac{{\rm d}\rho_T(t)}{{\rm d}t} = [H(t), \rho_T(t)].$$

To study the dynamic process of the system, we take the partial trace over bath modes and obtain the reduced density operator $\rho(t) = {\rm Tr}_{\rm B}\rho_T(t)$.

We suppose that the initial state of the whole system is divided into a system part $\rho(0)$ and an equilibrium bath part $\rho_{\rm B} = Z_{\rm B}^{-1}\exp(-\beta H_{\rm B})$, with the partition function $Z_{\rm B} = {\rm Tr}\exp(-\beta H_{\rm B})$, $\beta = 1/(k_{\rm B}T)$ is the reciprocal of the temperature. Using the Hamiltonian $H(t)$, we can deduce the evolution master equation of the system density matrix. In Born-Markov approximation, the eigenbasis vector should be the eigenstate $|n\rangle$ of Hamiltonian $H_{\rm S}$, with the reduced density matrix element $\rho_{nm} = \langle n|\rho|m\rangle$, $H_{\rm S}|n\rangle = \omega_n|n\rangle$, which obeys Redfield equation[14]

$$\dot{\rho}_{nm}(t) = -i\omega_{nm}\rho_{nm}(t) - \sum_{kl} R_{nmkl}\rho_{kl}(t), \tag{11}$$

$\omega_{nm} = \omega_n - \omega_m$. The Redfield relaxation tensors R_{nmkl} comprise the dissipative effects of the coupling of the system with the environment. The elements of the Redfield relaxation tensor read

$$R_{nmkl} = \delta_{lm} \sum_r \Gamma^{(+)}_{nrrk} + \delta_{nk} \sum_r \Gamma^{(-)}_{lmnk} - \Gamma^{(+)}_{lmnk} - \Gamma^{(-)}_{lmnk}. \quad (12)$$

In the above deduction, we assume that the interaction between the system and the environment bath is linear. For our system-bath interaction Hamiltonian (5), after tracing out the bath degrees of freedom, we obtain

$$\mathrm{Re}\Gamma^{(+)}_{lmnk} = (\mathbf{g}\cdot\boldsymbol{\varphi})_{lm}(\mathbf{g}\cdot\boldsymbol{\varphi})_{nk} \frac{J(|\omega_{nk}|)}{\hbar}\frac{\exp(-\beta\hbar\omega_{nk}/2)}{\sinh(\beta\hbar|\omega_{nk}|/2)}, \quad (13)$$

$$\mathrm{Im}\Gamma^{(+)}_{lmnk} = (\mathbf{g}\cdot\boldsymbol{\varphi})_{lm}(\mathbf{g}\cdot\boldsymbol{\varphi})_{nk} \frac{2}{\pi\hbar} P \times \int_0^\infty \mathrm{d}\omega \frac{J(\omega)}{\omega^2 - \omega^2_{nk}}\left[\omega_{nk}\coth\left(\frac{\beta\hbar\omega}{2}\right) - 2\right], \quad (14)$$

where $(\mathbf{g}\cdot\boldsymbol{\varphi})_{lm} = \langle l|\,\mathbf{g}\cdot\boldsymbol{\varphi}\,|m\rangle$.

The Bloch-Redfield equation (11) can describe multi-leveled dynamic process of the quantum system and can be applied to an arbitrary circuit with superconducting ring. The circuit can describe not only a single qubit, but also several qubits. If we only consider two-level approximation, Bloch-Redfield equation becomes Bloch equation. So, the decoherence time T_2 is given by

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}. \quad (15)$$

The potential energy $U(\varphi)$ will form an asymmetry double-well potential under a certain condition, with the left and right wells having a minimum value, respectively. Under semiclassical approximation, we only consider two-level question. When the potential barrier is very high, there exist bind states $|L\rangle$ and $|R\rangle$, respectively. However, when the potential barrier is limited, there will exist a potential barrier tunneling effect, with the two states being combined.

Under semiclassical approximation, T_1, T_2, and the dephasing time of the system are given by

$$\frac{1}{T_1} = \frac{1}{\hbar}\left(\frac{\Phi_0}{2\pi}\right)^2\left(\frac{\Delta\delta\boldsymbol{\varphi}}{\omega_{01}}\right)^2 \times J_{\mathrm{eff}}(M,\omega)\coth\left(\frac{\hbar\omega_{01}}{2k_{\mathrm{B}}T}\right)\Big|_{\omega=\omega_{01}}, \quad (16)$$

$$\frac{1}{T_\phi} = \frac{1}{\hbar}\left(\frac{\Phi_0}{2\pi}\right)^2\left(\frac{\varepsilon\delta\boldsymbol{\varphi}}{\omega_{01}}\right)^2 \times \frac{J_{\mathrm{eff}}(M,\omega)}{\omega}\coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\Big|_{\omega\to 0}, \quad (17)$$

where $\omega_{01} = \sqrt{\Delta^2+\varepsilon^2}$, ε is the classical energy difference and $\Delta = \langle L|\,H_{\mathrm{S}}\,|R\rangle$ is the tunneling amplitude between the two wells, $\delta\boldsymbol{\varphi} = \varphi_{\mathrm{L}} - \varphi_{\mathrm{R}}$.

Equations (16) and (17) show that, in a certain condition, T_2 and T_1 are usually expressed in terms of the spectral density of the heat bath fluctuations. The effective spectral density expression of Eq. (10) is closely related to the impedances $Z(\omega)$, showing that, for flux qubit, the magnetic field fluctuation of the environment is an important factor of decoherence. The spectral density expression is also closely related to the mutual inductance coupling of the circuit parameters and inductors. These electrical circuit elements can necessarily control and read out the state of the superconducting qubit. They also have a large influence over the spectral density. The research result shows that for a certain circuit, the parameters such as ε and Δ are mainly affected by the bias sources and external magnetic fluxes. Compared with that of the bias sources and external magnetic fluxes, the influence of the circuit element parameters over the parameters such as ε and Δ is very small. Now we mainly discuss the effect of the mutual inductance coupling on the flux qubit. We regard the parameters ε and Δ as constants.

Under ohmic environment, Fig. 2 describes the mutual dependent relationship between spectral density $J(\omega)$ and environment impedance. From Eq. (10) and Fig. 2 we can clearly see that, under the ohmic impedances environment, with other conditions constant, the spectral density $J(\omega)$ is in reverse proportion to the environment dissipation, showing that energy relaxation time and decoherence time are proportional to the dissipation, which is consistent with the results of Refs. [15 − 18].

Under ohmic impedance environment, we get the evolution curve of the spectral density $J(\omega)$ and mutual inductance parameters. Figure 3 describes the influence of the mutual inductance effects among inductance coils over the spectral density. It is shown that the influence of the mutual inductance coupling with the spectral density is complicated. Figure 3(a) shows that, with the increase of the mutual inductance coupling effects between L_1 and L_2, the spectral density increases. Supposing that the mutual inductance coupling has no influence over the

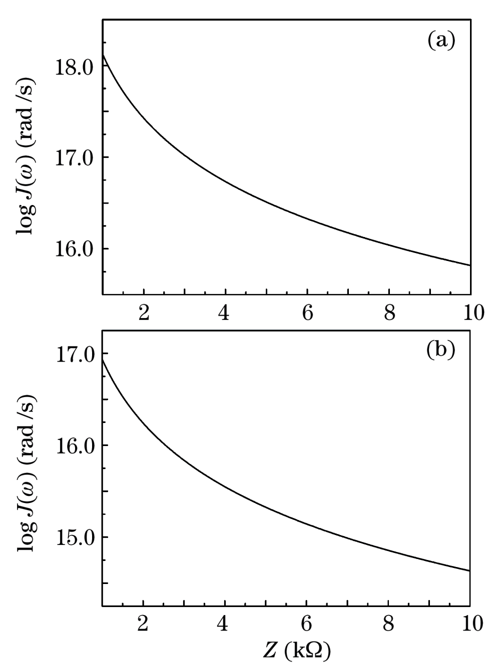

Fig. 2. Relation between spectral density $J(\omega)$ and impedance $Z(\omega)$. (a) With mutual inductance coupling between inductance $M_1 = M_2 = 1$ pH; (b) without mutual inductance coupling between inductance $L_1 = L_4 = 100$ pH, $L_2 = L_3 = 4$ pH, $\omega = 60$ GHz.

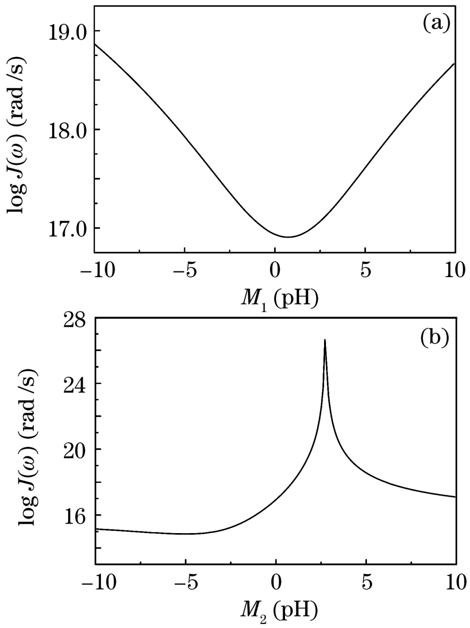

Fig. 3. Influence of mutual inductance effect over spectral density $J(\omega)$. (a) The influence of M_1, (b) the influence of M_2. $L_1 = L_4 = 100$ pH, $L_2 = L_3 = 4$ pH, Re$Z(\omega_{01}) = 1$ kΩ, $\omega = 60$ GHz.

tunnel, then the above result actually shows that, with the increase of the magnetic coupling of control circuit and main circuit, the energy relaxation time and decoherence time are decreasing. This is reasonable. For the main circuit, the control circuit is the external environment. The stronger coupling effect of the system and environment, the more clearly the decoherence.

M_2 reflects the coupling effects among the inductance in the main circuit. From Fig. 3(b), we can see that when M_2 coupling is strong, the spectral density will reduce with the increase of the coupling. When other conditions are constant, the decoherence time is increasing. Actually, the above result means that, for an actual quantum system, the decoherence effect results from not only the unavoidable entanglement effect between the whole quantum system and the external environment, but also the internal freedom degree in the quantum system. The mutual environment among each internal subsystem in the quantum system may also has a great influence over the decoherence. It is also seen that the influence of the mutual inductance effect in the same loop over the spectral density is strongly dependent on the coupling intensity and relative direction.

To sum up, the influence of the mutual inductance coupling among the elements is complicated. Our research results indicate that the relative orientation of mutual inductance coupling does not affect the trend of spectral density changing with the impedance in ohmic environment. Further analysis shows that the behavior of the dephasing time T_ϕ changing with the mutual inductance parameters and the ohmic environmental impedances is the same as that of spectral density.

It is a useful work to study the influence of mutual inductance factors between loops over the qubit decoherence. In this letter, in the Born-Markov approximation, according to Bloch-Redfield equation, we study the decoherence effect of the flux qubit. In ohmic environment, the dephasing time T_ϕ rises in proportional to dissipation; the energy relaxation time T_1 from the high energy level $|j\rangle$ to the low energy level $|i\rangle$ rises directly with dissipation; then the decoherence time T_2 rises directly with dissipation. The influence of mutual inductance coupling between elements over the spectral density is relatively complicated. Under certain conditions, we can reduce decoherence in the system through optimizing the circuit design and making use of the coupling effect between induction coils. Generally speaking, weakening the magnetic coupling between the control loop and the main loop helps to increase the decoherence time.

This work was supported by the National Natural Science Foundation of China under Grant No. 10864002.

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