Tolerance on tilt error for coherent combining of fiber lasers

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Limited by the precision of optical machining and assembling, the optical axes of lasers in an array cannot be strictly parallel to each other, which will result in the beam quality degradation of the combined beam. The tolerance on tilt error for coherent combining of fiber lasers is studied in detail. The complex amplitude distribution in the far field for the Gaussian beam with tilt angle is obtained by a novel coordinate transform method. Effect of tilt error on coherent combining is modelled analytically. Beam propagation factor is used to evaluate the effect of coherent combining. Numerical results show that for ring-distributed fiber laser array with central wavelength λ and geometry size D, if the root-mean-square (RMS) value of the tilt error is smaller than $0.72\lambda/D$, the energy encircled in the diffraction-limited bucket can be ensured to be more than 50% of the value when there is no tilt error. The results are helpful to the designing and manufacturing of fiber array for coherent combining.

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Recent advances show that fiber lasers have the capability to generate kilowatt output power with good beam quality[1,2]. But the scalability of diffraction-limited output power is restricted in terms of thermal load, fiber damage, nonlinearity, and the pump power and brightness of pump laser diodes (LDs). Coherent beam combining of fiber lasers can solve the power limitation[3,4]. In coherent combining, several fibers are packaged together into an array. All the array elements operate with the same wavelength and the relative phases of the elements are controlled. Fiber lasers are particularly well-suited to beam combining because of their inherent compact size.

Theoretical analysis on coherent combining of fiber lasers has been discussed extensively[5−8]. The effects of phase error, asymmetric intensity distribution, and fill factor on the beam quality of coherently combined beam have been studied. However, the optical axes of lasers emitted from each array element were assumed to be parallel to each other in these researches. In fact, limited by the precision of optical machining and assembling, the optical axes of lasers in an array cannot be strictly parallel to each other[9], which is defined as tilt error. Tilt error will inevitably result in beam quality degradation of the combined beam. However, to the best of our knowledge, the effect of tilt error has not yet been discussed. In this letter, the effect of tilt error on coherent combining is modelled analytically by a novel coordinate transforming method, and the tolerance on tilt error for coherent combining is studied numerically.

We consider a ring distributed fiber laser array shown in Fig. 1 for its high fill factor value[7], which is a great advantage for coherent combining. This array can contain more elements in a given area than other array shapes. The ring distributed array consists of a central element and several surrounding rings of elements. An array with N rings will contain L lasers, where $L = 1 + 3N\,(N + 1)$. It should be noted that the core diameters of double-clad fibers used for generating high power lasers are about 20 μm while the outer clad diameter is about 400 μm, and there is a large distance between the center of beamlets, which puts disadvantage to the effect of coherent combining, meanwhile the beam waist of 10-μm level corresponds to a relatively large diffraction angle. So this laser array consisting of a bundled double-clad fiber is not practical for long-range use. As shown in Fig. 1(b), the laser beams can be expanded and collimated by using microlens array, and in this way the distance between the adjacent elements will become smaller compared with the beam waist. Suppose that each fiber laser beam has a Gaussian single mode field distribution. The beam waist of each laser is ω_0. Fiber lasers are arranged in the array with the nearest neighbor separated by a distance of d.

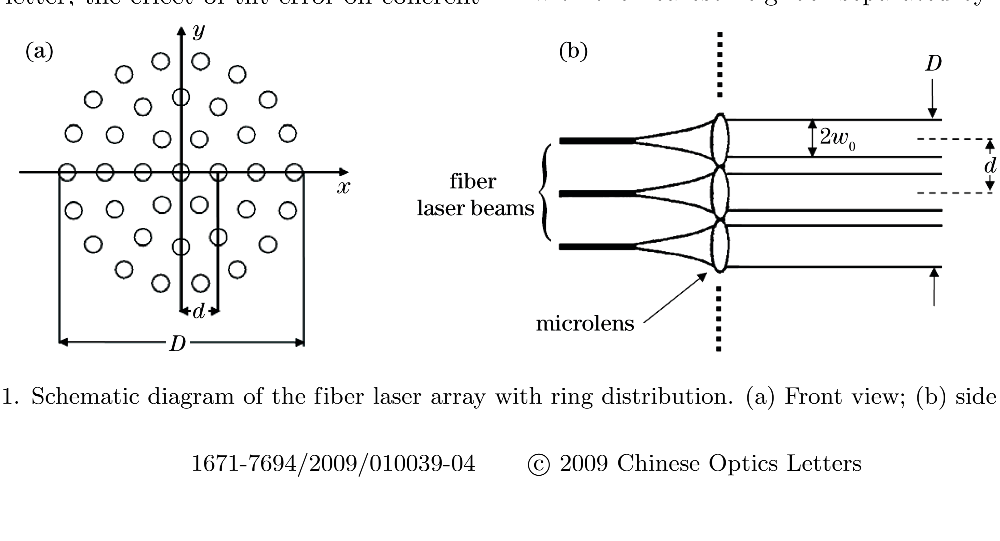

Fig. 1. Schematic diagram of the fiber laser array with ring distribution. (a) Front view; (b) side view.

We define the fill factor t as $(d - 2\omega_0)/\omega_0$ to describe the compactness of the array. A smaller t corresponds to a more compact array. The diameter of the whole fiber laser array D is $2Nd$.

If there is no tilt error, the optical axis of each fiber laser is parallel to each other, the complex field of the nth laser beam at the z plane can be simply written as

$$E_n(x,y,z) = \exp\left(-\frac{((x-x_n)^2+(y-y_n)^2)}{w^2(z)}\right) \times \exp\left(-\mathrm{i}\left\{k\left[\frac{((x-x_n)^2+(y-y_n)^2)}{2R(z)}+z\right]-\psi+\psi_{n0}\right\}\right), \tag{1}$$

where (x_n, y_n) denotes the central position of the nth element, ψ_{n0} is the original phase of the nth element. If all the laser array elements are phase locked and coherently combined, $\psi_{n0} = 0$. Also in Eq. (1), $k = \frac{2\pi}{\lambda}$, $Z_0 = \frac{\pi w_0^2}{\lambda}$, $w(z) = w_0\sqrt{1+\left(\frac{z}{Z_0}\right)^2}$, $R(z) = Z_0\left(\frac{z}{Z_0}+\frac{Z_0}{z}\right)$, $\psi = \arctan\left(\frac{z}{Z_0}\right)$, λ is the wavelength of the laser beam.

The far-field intensity distribution of the coherently combined beam is

$$I(x,y,z) = \left(\sum_n E_n(x,y,z)\right)\left(\sum_n E_n(x,y,z)\right)^*. \tag{2}$$

Equation (2) is often used for analyzing the advantage of coherent combining and effect of phase error on coherent combining.

For an array with tilt error, the complex field of the nth laser beam at the z plane is not easy to be obtained. In Ref. [10], a fast numerical method for free-space beam propagation between arbitrary oriented planes is developed. The complex field tilted in one direction (for instance, zenith angle) can be calculated. However, as shown in Fig. 2, for an element in the fiber laser array, the tilt angle can be in coursing (θ_x) and pitching (θ_y) directions at the same time. Suppose that the coordinate system (x, y, z) denotes the laser beam propagating along the z axis with no tilt error, and the coordinate system (α, β, γ) denotes the laser beam propagating along the γ axis with tilt angle (θ_x, θ_y). Thus a coordinate transform can be used for calculating the tilted far field.

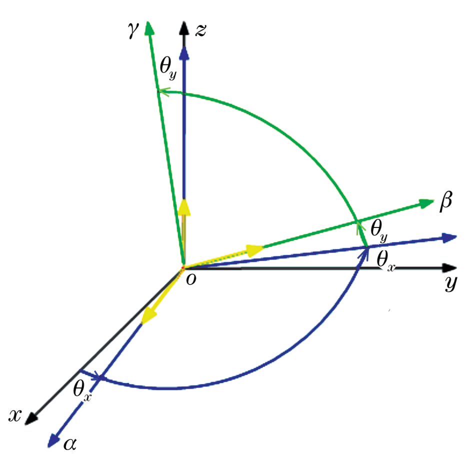

Fig. 2. Coordinate systems for the laser beam propagation with tilt error in both coursing and pitching directions.

From the basic knowledge of linear algebra, the relationship between the two coordinate systems can be expressed as

$$\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = \begin{pmatrix}\cos\theta_x & 0 & \sin\theta_x\\ \sin\theta_y\sin\theta_x & \cos\theta_y & -\sin\theta_y\cos\theta_x\\ -\sin\theta_x\cos\theta_y & \sin\theta_y & \cos\theta_x\cos\theta_y\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} \tag{3}$$

and

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}\cos\theta_x & \sin\theta_y\sin\theta_x & -\sin\theta_x\cos\theta_y\\ 0 & \cos\theta_y & \sin\theta_y\\ \sin\theta_x & -\sin\theta_y\cos\theta_x & \cos\theta_x\cos\theta_y\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix}. \tag{4}$$

Then the complex field of the nth laser beam with tilt angle at the z plane is

$$E_{n\_\mathrm{tilt}}(x,y,z) = \exp\left[-\frac{(\eta_n^2+\xi_n^2)}{w^2(\chi_n)}\right] \times \exp\left\{-\mathrm{i}\times k\left[\frac{(\eta_n^2+\xi_n^2)}{2\Delta R_n}+\chi_n\right]-\Delta\psi_n\right\} \tag{5}$$

with

$$\begin{cases}\eta_n = \cos\theta_{x,n}x + \sin\theta_{y,n}\sin\theta_{x,n}y \\ \qquad -\sin\theta_{x,n}\cos\theta_{y,n}z - x_n\\ \xi_n = \cos\theta_{y,n}y + \sin\theta_{y,n}z - y_n\\ \chi_n = -\sin\theta_{x,n}x - \sin\theta_{y,n}\cos\theta_{x,n}y \\ \qquad +\cos\theta_{x,n}\cos\theta_{y,n}z\\ \Delta R_n = Z_0(\chi_n/Z_0 + Z_0/\chi_n)\\ \Delta\psi_n = \arctan(\chi_n/Z_0)\\ w(\chi_n) = w_0\sqrt{1+(\chi_n/Z_0)^2}\end{cases}. \tag{6}$$

Compared with Ref. [10], this method is more universal, and the tilted complex fields in both coursing and pitching directions can be calculated.

The irradiance distribution for the coherently combined laser beams with tilt errors can be written as

$$I_{\mathrm{tilt}}(x,y,z) = \left(\sum_n E_{n\_\mathrm{tilt}}(x,y,z)\right)\left(\sum_n E_{n\_\mathrm{tilt}}(x,y,z)\right)^*. \tag{7}$$

In order to investigate the effect of tilt error on coherently combined beam, we firstly consider a fiber laser array with five rings (91 lasers). The parameters are taken as follows: $\lambda = 1$ μm, $\omega_0 = 5$ mm, $d = 12.5$ mm, z is taken to be 1×10^5 m, a number large enough to ensure the far-field condition, and thus the Fresnel number $N_F \ll 1$. The irradiance distribution for the coherently combined laser beams with tilt errors is calculated and plotted in Fig. 3. It is shown that the tilt error will cause the decrease of energy contained in the main-lobe and beam quality degradation.

Figure 3 shows a qualitative effect of tilt error. Beam quality factor should be introduced in order to get a detailed result. As shown in Fig. 3, the far-field intensity profile of the coherently combined beam is typically

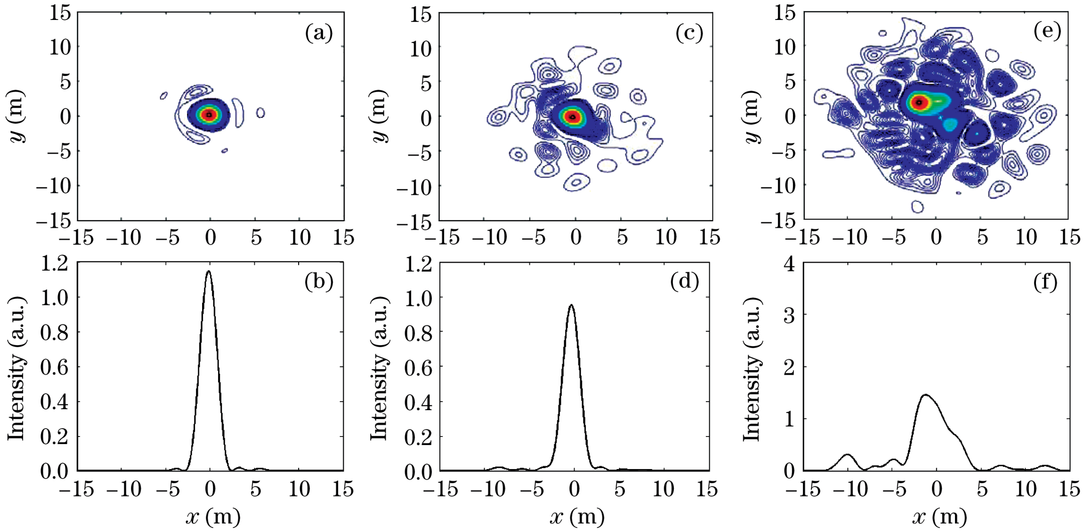

Fig. 3. Far-field irradiance distribution with tilt errors. (a),(c),(e): Contour plots for far-field irradiance distribution with tilt errors, RMS values 5, 15, and 30 μrad, respectively; (b),(d),(f): far-field irradiance profiles along the x axis with tilt errors, RMS values 5, 15, and 30 μrad, respectively.

non-Gaussian distributed. Besides the main-lobe that contains most of the far-field energy, there are also some side-lobes. M^2 factor is not proper to evaluate this kind of beam profile[11]. What we are most interested in is the energy contained in the main-lobe. In fact, the beam quality of the coherently combined beam can be characterized by a beam propagation factor (BPF)[12]:

$$\mathrm{BPF} = (P/P_{\mathrm{DL}}), \tag{8}$$

where P is the laser output power in a specified far-field bucket, and P_{DL} is the total output power from the effective near-field exit aperture of the laser beam array. According to Ref. [12], the far-field bucket is defined as $A_{\mathrm{DL}} = (\pi/4)(\theta_{\mathrm{DL}} z)^2$, which is the diffraction-limited bucket, and $\theta_{\mathrm{DL}} = 2.44\lambda/D$, where D is the effective exit aperture of the combined laser beam, namely, the diameter of the whole array. For a certain fiber laser array, the ideal value of BPF without tilt error can be numerically calculated through Eq. (2). We define a tolerance on the RMS value of the tilt error when BPF becomes 50% of the value when there is no tilt error. Before numerical calculation, we should note that the divergent angle for an ideal laser beam is proportional to λ/D. Most of the laser energy is encircled in the bucket with a radius of $\lambda z/D$. So laser array with a larger D will have a relatively more strict tolerance on the tilt error. Laser array with a fixed number of lasers varies in D owing to different fill factors. For this reason, we normalized the RMS values (denoted by r) of the tilt error by dividing them with λ/D. For an array with 91 lasers (5 rings), the dependence of the BPF on the normalized RMS value of the tilt error with different fill factors is given in Fig. 4 (the parameters are taken to be the same as those used in Fig. 3). Totally 30 steps are used in the numerical calculation. It is shown that BPF will decrease as the RMS value of the tilt error increasing.

Effect of normalized tilt error can be studied more accurately by plotting the dependence of normalized BPF on normalized tilt error (for a laser array with determinate fill factor, the normalized BPF is defined as the BPF value divided by the ideal value when there is no tilt error). As shown in Fig. 5, there is almost no difference among the curves of normalized BPF for different fill factors.

The averaged dependence of normalized BPF on the normalized tilt error can be exactly fitted to be a 6th polynomial, given as

$$\mathrm{BPF}_{\mathrm{tilt\_error}}(r) = 0.25563r^6 - 1.4233r^5 + 2.5507r^4 - 1.0614r^3 - 0.99629r^2 - 0.04239r + 0.9998. \tag{9}$$

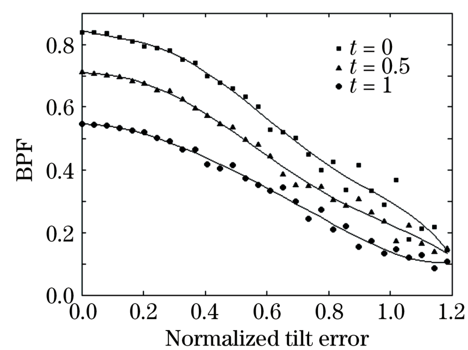

Fig. 4. Dependence of BPF on the normalized tilt error for different fill factors t.

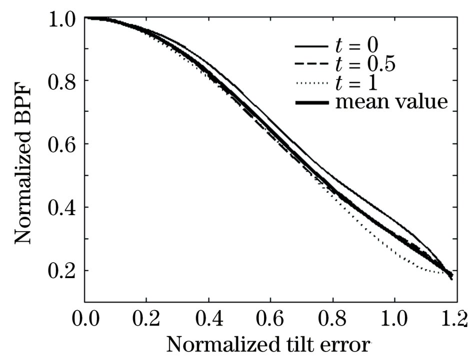

Fig. 5. Dependence of normalized BPF on the normalized tilt error for different fill factors.

From Eq. (9), it can be concluded that for the ring distributed 91-laser array, if the RMS value of the tilt error is smaller than $0.72\lambda/D$, the energy encircled in the diffraction-limited bucket can be ensured to be more than 50% of the value when there is no tilt error.

We also calculated the averaged dependence of normalized BPF on the normalized tilt error for several other laser arrays with different numbers of lasers, i.e., ring distributed fiber laser arrays with ring number N of 1, 2, 5, 6 (corresponding to laser arrays with 7, 19, 91, 127 lasers). The dependence of normalized BPF on the normalized tilt error is plotted in Fig. 6. Generally, there is no difference among the curves for different laser arrays, except for the case when the normalized tilt error is relatively large (i.e., larger than 1). Laser array that contains less lasers suffer less from large tilt errors. However, the criterion of $0.72\lambda/D$ to ensure energy encircled in the diffraction-limited bucket more than 50% of the ideal value still holds true for different laser arrays. The array size D is often designed to be at several-ten-centimeter level in practice for long-range energy delivering use, and then the RMS tilt error should be controlled at 10-μrad level, which is a great challenge for optical machining and assembling.

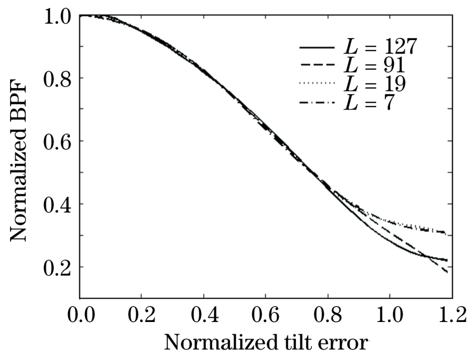

Fig. 6. Dependence of normalized BPF on the normalized tilt error for laser arrays with different laser numbers L.

In conclusion, the tolerance on tilt error for coherent combining of fiber lasers is studied numerically. Results show that the tolerance value depends on laser wavelength λ and array size D. For laser arrays with different laser numbers, the energy encircled in the diffraction-limited bucket can be ensured more than 50% of the ideal case if the RMS tilt error is smaller than $0.72\lambda/D$. This conclusion offers a guide for designing and manufacturing the fiber array used for coherent combining.

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