Analysis of Doppler-effect on satellite constellations with wavelength division multiplexing architectures

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With the development of optical space communications, a global space-based optical backbone network is currently proposed by using broadband laser inter-satellite links (ISLs) which enable routing traffic through the space. Satellite optical networking techniques based on wavelength division multiplexing (WDM) ISLs can transit significantly high data rates signals. In this letter, a new function of wavelength excursion due to Doppler-effect is developed for the ISLs, considering the conception of pointing ahead mechanism. The characteristic of wavelength excursion induced by Doppler-effect is examined in one of low earth orbit (LEO) satellite constellation networks named the next-generation LEO system (NeLS) with WDM ISLs assumed, and the influence on its communications caused by wavelength excursion is analyzed.

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Corresponding to microwave communications, optical space communications have the major advantages of high rates and better real-time performance. They have drawn much attention in telecommunication community[1−3]. Broad-band applications in satellite communications impulsed by massive data traffic demand caused by exploring outer space and other requires such as Internet are arisen, which has become a key driver for satellite constellations. A few low earth orbit (LEO) satellite constellation providing broadband communications to users are currently proposed, such as Iridium[4], Teledesic[5], Celestri[6] and the next-generation LEO system (NeLS)[7]. Every satellite in the constellations, as a high-speed carrier, causes inevitably the Doppler-effect to signals. That means the wavelength of a channel that routes the signal to destination satellite goes beyond the receiving band of the corresponding channel for the wavelength division multiplexing (WDM) systems. In this letter, on the assumption that WDM architectures were available as the inter-satellite lines (ISLs) with wavelength routing, NeLS constellation is analyzed on wavelength excursion induced by Doppler-effect. This issue is related to the routing and wavelength allocation (RWA) problems of a global wireless optical WDM network. Furthermore, in the case of pointing, acquisition and tracking (PAT) for the optical satellite communications, a notion of pointing ahead mechanism is considered.

According to Doppler-effect concerned to relativistic effects[8], the frequency ω_d of the detector in the destination, on condition that the light source and the detector were all locomotory, is defined by

$$\omega_\mathrm{d} = \omega_\mathrm{s}\sqrt{1-u_\mathrm{s}^2/c^2}\,[1-(u_\mathrm{d}/c)\cos\theta_\mathrm{d}] \Big/ \left\{\sqrt{1-u_\mathrm{d}^2/c^2}\,[1-(u_\mathrm{s}/c)\cos\theta_\mathrm{s}]\right\}, \quad (1)$$

where ω_s is the frequency of light source, u_s and u_d are the velocities of the source and the detector, respectively. θ_d is the angle between directions of the wave propagation and the detector movement, θ_s is the angle between directions of the wave propagation and the source motion.

As for inter-satellite laser communications, the source satellite and the destination satellite are in the three-dimension space for which it is difficult to acquire angles between directions of the wave propagation and the object movement. Therefore, parameters of angles need to be converted. With the hypothesis that a uniform plane wave was transmitted between the source satellite and the destination satellite, the angles become those between the vectors of the relative position and the objects velocities.

Relative position vector $\vec{r}\,(t)$ of the source satellite and the destination satellite can be obtained via

$$\vec{r}\,(t) = \vec{r}_\mathrm{d}\,(t) - \vec{r}_\mathrm{s}\,(t)\,, \quad (2)$$

where $\vec{r}_\mathrm{s}\,(t)$ and $\vec{r}_\mathrm{d}\,(t)$ are position vectors of the source satellite and the destination satellite in J2000 earth core inertial (ECI) coordinates, respectively. Equation (1) can be reformed as follows:

$$\omega_\mathrm{d} = \omega_\mathrm{s}A\,(t)\,[c-|\vec{u}_\mathrm{d}\,(t)|\cos\theta_\mathrm{d}\,(t)] \,/[c-|\vec{u}_\mathrm{s}\,(t)|\cos\theta_\mathrm{s}\,(t)]\,, \quad (3)$$

where $A\,(t) = \sqrt{\left(c^2-|\vec{u}_\mathrm{s}\,(t)|^2\right)/\left(c^2-|\vec{u}_\mathrm{d}\,(t)|^2\right)}$, $\vec{u}_\mathrm{s}\,(t)$ and $\vec{u}_\mathrm{d}\,(t)$ are velocity vectors of the source satellite and the destination satellite in J2000 ECI coordinates, respectively. We import the relative position vector $\vec{r}\,(t)$ to Eq. (3), the function is given by

$$\omega_\mathrm{d} = \omega_\mathrm{s}A\,(t)\,[c-\vec{u}_\mathrm{d}\,(t)\cdot(\vec{r}\,(t)\,/\,|\vec{r}\,(t)|)] \,/[c-\vec{u}_\mathrm{s}\,(t)\cdot(\vec{r}\,(t)\,/\,|\vec{r}\,(t)|)]\,. \quad (4)$$

Converting frequency into wavelength, wavelength excursion can be given by

$$\Delta\lambda = \lambda_{\rm s} \times \left[\frac{1}{A(t)} \left(1 + \frac{\left(\vec{u}_{\rm d}(t) - \vec{u}_{\rm s}(t)\right) \cdot \left(\vec{r}(t)/|\vec{r}(t)|\right)}{c - \vec{u}_{\rm d}(t) \cdot \left(\vec{r}(t)/|\vec{r}(t)|\right)} \right) - 1 \right]. \quad (5)$$

With a view to the mutual movement of two satellites, beam from the source satellite must be pointed ahead along the track of the destination satellite for an additional distance in time delays. The time delays can be defined by

$$\Delta t = \frac{|\vec{r}(t)|}{c} + \frac{|\vec{r}(t+\Delta t)|}{c} + t_{\rm d}, \quad (6)$$

where $t_{\rm d}$ is responding time of the source satellite. Because of a little change in the relative position, Δt can be described by

$$\Delta t = 2\frac{|\vec{r}(t)|}{c} + t_{\rm d}. \quad (7)$$

Hence, wavelength excursion induced by Doppler-effect in inter-satellite lasercom is

$$\Delta\lambda = \lambda_{\rm s} \times \left[\frac{1}{A(t)} \left(1 + \frac{\left(\vec{u}_{\rm d}(t+\Delta t) - \vec{u}_{\rm s}(t)\right) \cdot \left(\vec{r}'(t)/|\vec{r}'(t)|\right)}{c - \vec{u}_{\rm d}(t+\Delta t) \cdot \left(\vec{r}'(t)/|\vec{r}'(t)|\right)} \right) - 1 \right], \quad (8)$$

where $\vec{r}'(t) = \vec{r}_{\rm d}(t+\Delta t) - \vec{r}_{\rm s}(t)$.

NeLS employed with 2π constellation is designed for a global mobile communications. Constellation parameters can be expressed as a walker notation of 120/10/1. Table 1 lists its primary parameters for the orbits. Figure 1 is a schematic of NeLS.

Based on the assumption of available WDM architectures and a full field of view for satellite terminals, NeLS was simulated and wavelength excursion during 6600 s was investigated. Because of the periodic characteristic of the constellation, statuses of satellites in the same planes are coequal. In other words, they have the same

Table 1. Primary Parameters for NeLS Constellation

Parameters	Value
Orbital Inclination	55°
Orbital Altitude	1200 km
Eccentricity	0
Orbital Period	6565 s
No. of Orbital Planes	10
No. of Sats per Orbital Plane	12
True Anomaly Phasing	3°
RAAN Increment	36°
Intraorbit ISL Distance	3922 km
Interorbit ISL Distance	< 4909 km

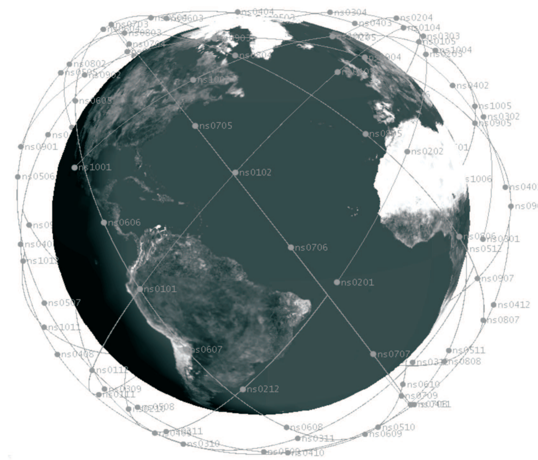

Fig. 1. Schematic of the NeLS.

Table 2. Durations of Accessing Time

Tag of Sats	Duration (s)	Tag of Sats	Duration (s)
ns0102	6600	ns0606	1685
ns0112	6600	ns0607	1685
ns0201	6552	ns0608	1233
ns0211	2193	ns0703	1245
ns0212	6600	ns0704	1746
ns0310	1257	ns0705	1747
ns0311	3003	ns0706	1151
ns0312	2602	ns0802	1811
ns0409	1523	ns0803	2125
ns0410	2125	ns0804	1523
ns0411	1811	ns0901	2602
ns0507	1151	ns0902	3003
ns0508	1747	ns0903	1257
ns0509	1746	ns1001	6600
ns0510	1245	ns1002	2193
ns0605	1233	ns1012	6556

distributions of wavelength excursion. The rest satellites may be deduced by analogy. So we choose a satellite tagged by ns0101 as the source satellite for the investigation. Its true anomaly phase is 0°, and right ascension of ascending node (RAAN) is 0°.

As shown in Table 1, link distance of the intraorbits is a constant quantity of 3922 km, and that of the interorbits is less than 4909 km. When satellite ns0101 was operating for 6600 s, about one orbital period, it has been accessed by 32 satellites, including two intra-orbital plane satellites and 30 inter-orbital plane satellites. Table 2 shows the durations of 32 satellites' accessing. There are only four full-time ISLs in the orbital period, including two intraorbit ISLs and two adjacent interorbit ISLs. Angles between vectors of the relative position $\vec{r}'(t)$ and velocities of two satellites in the intraorbit are constant and equivalent quantities, and time delay Δt is also unchanged. Consequently, no wavelength excursion can be detected. By Eq. (8) where obviously $A(t) = 1$, we can be clearly aware of the statuses of wavelength excursions of two interorbit ISLs for ns0101. As shown in Fig. 2, it

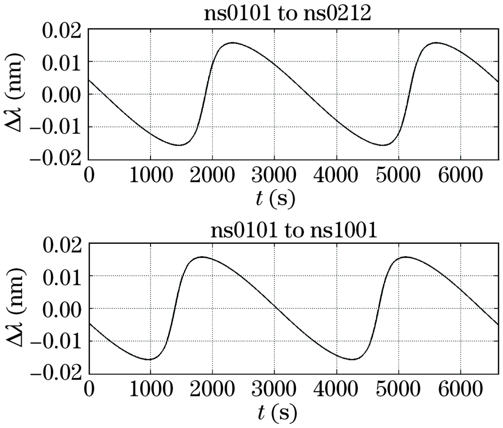

Fig. 2. Variation of wavelength excursions for two inter-orbital plane ISLs ($\lambda_s = 1550$ nm).

Table 3. Maximum Values of Wavelength Excursion

ISL of Sats	Value (nm)	ISL of Sats	Value (nm)
ns0101 to ns0201	0.0091	ns0101 to ns0607	0.0560
ns0101 to ns0211	0.0064	ns0101 to ns0608	0.0432
ns0101 to ns0310	0.0152	ns0101 to ns0703	0.0394
ns0101 to ns0311	0.0314	ns0101 to ns0704	0.0533
ns0101 to ns0312	0.0260	ns0101 to ns0705	0.0518
ns0101 to ns0409	0.0338	ns0101 to ns0706	0.0357
ns0101 to ns0410	0.0475	ns0101 to ns0802	0.0387
ns0101 to ns0411	0.0387	ns0101 to ns0803	0.0475
ns0101 to ns0507	0.0357	ns0101 to ns0804	0.0338
ns0101 to ns0508	0.0518	ns0101 to ns0901	0.0260
ns0101 to ns0509	0.0533	ns0101 to ns0902	0.0314
ns0101 to ns0510	0.0394	ns0101 to ns0903	0.0152
ns0101 to ns0605	0.0432	ns0101 to ns1002	0.0064
ns0101 to ns0606	0.0560	ns0101 to ns1012	0.0091

varies periodically, with a time period of 3282.5 s. Maximum value is 0.0156 nm with a variation of 0.0312 nm.

The rest of interorbit ISLs are not full-time functional, namely, accesses of ns0101 to satellite cluster vary with the time passing by. Number of accessible satellite cluster remains about 12 at different time. Values of wavelength excursion for different ISLs are not equivalent at the same time. Moreover, breakdown of interorbit ISLs suspends accessing which results in different and intermittent curves for wavelength excursions. However, they are still periodic curves, and have the same period mentioned above. Table 3 lists the maximum values of wavelength excursion for every single interorbit ISL. It is shown that Doppler-effect induces maximum wavelength excursion up to 0.056 nm. That is to say, the wavelength ($\lambda_s = 1550$ nm) fluctuation spread of interorbit ISLs can reach 0.112 nm. See Fig. 3 for maximum wavelength fluctuation spread of interorbit ISLs.

Following the International Telecommunication Union (ITU) optical frequency allocation plan, wavelength spacing should be set at least to 1.6 nm (200 GHz), allowing for about 16 channels in the common erbium-doped fiber amplifier (EDFA) band. In the WDM systems, signal power of single channel received by the detector can be expressed by

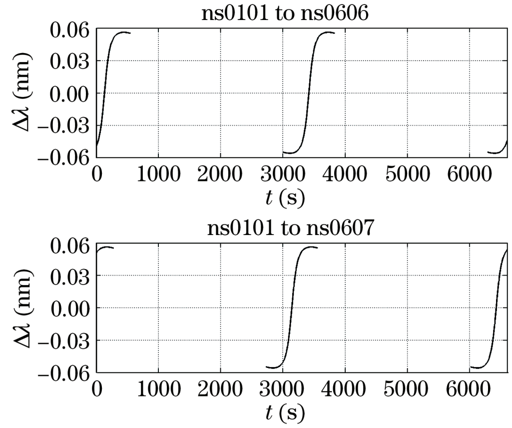

Fig. 3. Maximum wavelength fluctuation spread of inter-orbital plane ISLs ($\lambda_s = 1550$ nm).

$$P_s = \int_{-\infty}^{\infty} S(f)\,|H(f)|^2\,\mathrm{d}f. \tag{9}$$

And the one for crosstalk power is

$$P_{ct} = \int_{-\infty}^{\infty} S(f + f_d)\,|H(f)|^2\,\mathrm{d}f, \tag{10}$$

where f_d is the frequency spacing for WDM channels, $S(f)$ and $H(f)$are the power spectral density of signal and filter spectrum, respectively. They can be described as a Lorentz equation by

$$S(f) = \frac{P_r}{2\pi}\frac{B_s}{(B_s/2)^2 + [f - (\Delta f + f_{Lc})]^2},$$

$$H(f) = \frac{(B_f/2)^2}{(B_f/2)^2 + (f - f_{Dex})^2}, \tag{11}$$

where f_{Lc} and f_{Dex} represent central frequencies of the laser and filter, B_s is the 3-dB bandwidth of the laser, and B_f is the 3-dB bandwidth of the filter, P_r is the mean receiving power of the signal, Δf denotes Doppler frequency shift.

We have taken $B_f = 2B_s = 70$ GHz and $f_{Lc} = f_{Dex} =$ 193.55 THz (1550 nm). It is seen in Fig. 4 that Doppler frequency shift results in descending of signal power, on the other hand, it makes parts of signal spectrum of adjacent channels invade into filter band which leads to

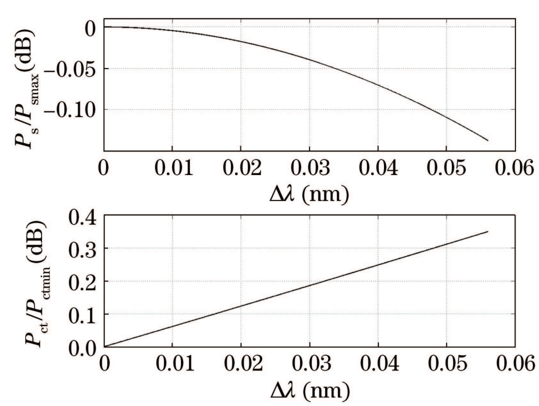

Fig. 4. Variation of normalized signal and crosstalk power with wavelength excursion.

ascending of crosstalk power. The normalized signal power decreases up to 0.14 dB, while the normalized crosstalk power increases about 0.35 dB. We set ε as a ratio of signal power to crosstalk power, with which the ability of interference-proof is described as follows:

$$\varepsilon = \frac{P_{\rm s}}{P_{\rm ct}}. \qquad (12)$$

Thus, it is clear that the ratio drops as Doppler frequency shift increases about 0.5 dB, as shown in Fig. 5.

If we ignore the background light power and suppose that the receiving detector is thermal noise limit, the bit error rate (BER) of optical communications with an on-off keying (OOK) modulation format is depicted by[9]

$$\mathrm{BER} = \frac{1}{2}\left[\frac{1}{2}Q\left(\frac{m_{11}-\upsilon_{\rm T}}{\sigma_n}\right) + \frac{1}{2}Q\left(\frac{m_{10}-\upsilon_{\rm T}}{\sigma_n}\right) + \frac{1}{2}Q\left(\frac{\upsilon_{\rm T}-m_{01}}{\sigma_n}\right) + \frac{1}{2}Q\left(\frac{\upsilon_{\rm T}-m_{00}}{\sigma_n}\right)\right], \qquad (13)$$

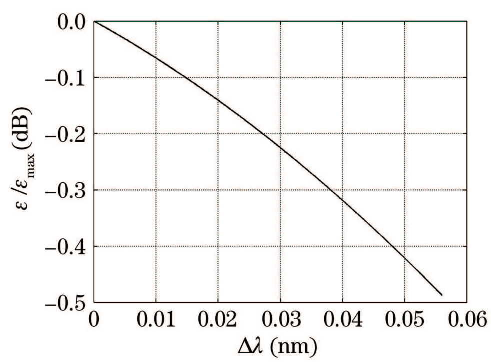

Fig. 5. Variation of normalized ε with wavelength excursion.

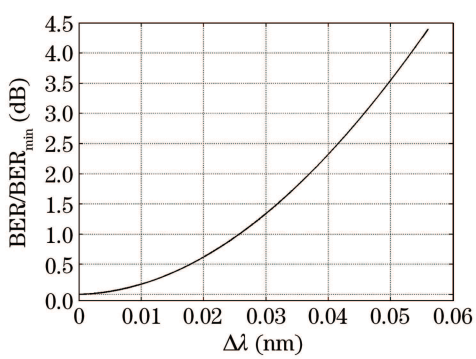

Fig. 6. Variation of normalized BER with wavelength excursion.

where $\upsilon_{\rm T}$ and σ_n^2 are the decoding threshold and variance of thermal noise. We set $\upsilon_{\rm T} = e\bar{g}K_{\rm s}/2$ and $K_{\rm s} = \eta P_{\rm s}T_{\rm b}/hf$ for the decoding threshold. For convenience, we have taken $\eta = 0.8$, $T_{\rm b} = 0.4$ ns and $\bar{g} = 10$. Figure 6 shows the variation tendency of BER as Doppler frequency shift increases. It is seen that wavelength excursion makes BER of WDM systems increase, with a maximum increase of 4.4 dB. And also, the more wavelength excursion Doppler-effect induced, the more rapidly BER curve rises.

In conclusion, Doppler frequency shift leads to ascending of crosstalk power and descending of signal power which directly makes BER increase. It might be not a direct factor to bring serious impacts on the ISLs, but under some circumstances, such as central frequency deviation caused by temperature variation of outer space, radiation, or fluctuation of the wavelength, and it can get the quality of the ISLs worse, even breakdown. In WDM systems, the most important issue is the stability of communicating wavelengths for each channel. Therefore, Doppler wavelength shift must be considered in the ISLs as WDM architectures when wavelength routing is available.

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