Method for rapid measuring retardation of a quarter-wave plate based on simultaneous phase shifting technique

Kun Yang (杨坤)[1,3], Aijun Zeng (曾爱军)[2], Xiangzhao Wang (王向朝)[1],
Feng Tang (唐锋)[1], and Hua Wang (王华)[1,3]

[1]*Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800*
[2]*Shanghai Hengyi Optics and Fine Mechanics Co., Ltd, Shanghai 201800*
[3]*Graduate University of Chinese Academy of Sciences, Beijing 100049*

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A method for rapid measuring retardation of a quarter-wave plate based on simultaneous phase shifting technique is presented. The simultaneous phase shifting function is realized by an orthogonal grating, a diaphragm, an analyzer array, and a 4-quadrant detector. The intensities of the light beams from the four analyzers with different azimuths are measured simultaneously. The retardation of the quarter-wave plate is obtained through the four light intensity values. In this method, the major axis position of the quarter-wave plate need not be determined in advance. In addition, the measured result is free of the intensity fluctuation of light source. The feasibility of the method is verified by the experiments.

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Quarter-wave plate is one of the most important optical elements. It can change the light polarization state and is widely used in the field of polarization measurement[1−4]. Retardation is a main parameter of the quarter-wave plate. In the measurements, the retardation of the quarter-wave plate directly affects the measured results, so determination of the quarter-wave plate retardation is important. The reported methods for measuring retardation of a quarter-wave plate include polarizer rotating method[5], wave plate rotating method[6], heterodyne interferometric method[7] , and phase modulation method[8−10], etc. The measured retardation of the quarter-wave plate is affected by the intensity fluctuation of the light source in the polarizer rotating method, the wave plate rotating method and the heterodyne interferometric method. In the phase modulation method, the measured retardation of the quarter-wave plate is independent of the intensity fluctuation of the light source, but the major axis position of the quarter-wave plate is needed to determine in advance. Simultaneous phase shifting technique based on a grating has been applied in interferometry[11−14]. In this letter, a method for rapid measuring retardation of a quarter-wave plate based on simultaneous phase shifting technique is proposed. In this method, the major axis position of the quarter-wave plate is not needed to determine in advance. The simultaneous phase shifting function is realized by an orthogonal grating, a diaphragm, an analyzer array, and a 4-quadrant detector. The intensities of the light beams from the four analyzers with different azimuths are measured simultaneously. The retardation of the quarter-wave plate is obtained through the four light intensity values. So the rapid measurement on retardation of quarter-wave plate is achieved.

The optical arrangement for measuring the retardation of the quarter-wave plate is illustrated in Fig. 1. The optical measuring system is composed of a laser, a polarizer, the quarter-wave plate to be measured, an orthogonal grating, a diaphragm, an analyzer array, and a 4-quadrant detector. The azimuth of the polarizer is 0° ("0°" position is defined by the horizontal direction. The signs of the angles are positive for counter-clockwise rotation and negative for clockwise rotation.). The diffractive beams of 0th order and ($\pm m$, $\pm n$) orders are generated through the orthogonal grating, where m and n are positive integers. The diaphragm is put after the grating, which is composed of four pinholes and the pinholes are arranged in square. With the diaphragm, four diffractive beams of $(\pm 1, \pm 1)$ orders with the same optical intensity are passed, and other order beams are prevented. Each of the four diffractive beams is incident on one of the analyzer in the analyzer array, respectively. The analyzer array is composed of four analyzers (2×2), where the polarization directions differ in turn by 45°. The azimuths of the analyzers are 0°, 45°, 90°, and 135°, respectively. The intensity signals of the four beams are detected by a 4-quadrant detector simultaneously.

The Stokes vector of the polarizer with the azimuth 0° is

$$\mathbf{S} = I_0 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \tag{1}$$

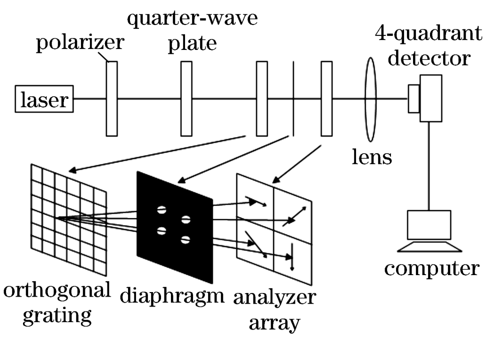

Fig. 1. Optical arrangement for measuring the retardation of quarter-wave plate.

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where I_0 is the intensity of the light from the polarizer. The Mueller matrix of the quarter-wave plate with a retardation of δ and an angle θ of the fast axis is

$$\mathbf{M}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta+\sin^2 2\theta\cos\delta & (1-\cos\delta)\sin 2\theta\cos 2\theta & -\sin 2\theta\sin\delta \\ 0 & (1-\cos\delta)\sin 2\theta\cos 2\theta & \sin^2 2\theta+\cos^2 2\theta\cos\delta & \cos 2\theta\sin\delta \\ 0 & \sin 2\theta\sin\delta & -\cos 2\theta\sin\delta & \cos\delta \end{pmatrix}. \tag{2}$$

For the diffractive beam of the first order, the Mueller matrix of the orthogonal grating is

$$\mathbf{G}=\beta\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

where β is the diffraction efficiency of the first order beams. The Mueller matrix of the analyzer is

$$\mathbf{A}=\frac{1}{2}\begin{pmatrix} 1 & \cos 2\alpha & \sin 2\alpha & 0 \\ \cos 2\alpha & \cos^2 2\alpha & \sin 2\alpha\cos 2\alpha & 0 \\ \sin 2\alpha & \sin 2\alpha\cos 2\alpha & \sin^2 2\alpha & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{4}$$

where α is the azimuth of the analyzer. The Stokes vector of the emerging beam from the analyzer is

$$\mathbf{S}'=\mathbf{AGMS}=\frac{\beta I_0}{2}\begin{pmatrix} 1+\cos 2\alpha(\cos^2 2\theta+\cos\delta\sin^2 2\theta)+0.5\sin 2\alpha(1-\cos\delta)\sin 4\theta \\ \cos 2\alpha+\cos^2 2\alpha(\cos^2 2\theta+\cos\delta\sin^2 2\theta)+0.25\sin 4\alpha(1-\cos\delta)\sin 4\theta \\ \sin 2\alpha+0.5\sin 4\alpha(\cos^2 2\theta+\cos\delta\sin^2 2\theta)+0.5\sin^2 2\alpha(1-\cos\delta)\sin 4\theta \\ 0 \end{pmatrix}. \tag{5}$$

Thus, the light intensity signal reaching the detector is

$$I=\frac{\beta I_0}{2}(1+\cos 2\alpha(\cos^2 2\theta+\cos\delta\sin^2 2\theta)+\sin 2\alpha(1-\cos\delta)\sin 2\theta\cos 2\theta). \tag{6}$$

Four intensity values measured at $\alpha = 0°$, $45°$, $90°$, and $135°$ can be proved as

$$I(0°)=\frac{\beta I_0}{4}[4-(1-\cos 4\theta)(1-\cos\delta)], \tag{7}$$

$$I(45°)=\frac{\beta I_0}{4}[2+\sin 4\theta(1-\cos\delta)], \tag{8}$$

$$I(90°)=\frac{\beta I_0}{4}[(1-\cos 4\theta)(1-\cos\delta)], \tag{9}$$

$$I(135°)=\frac{\beta I_0}{4}[2-\sin 4\theta(1-\cos\delta)], \tag{10}$$

respectively. According to Eqs. (7)—(10), we can obtain

$$k_1=\frac{I(0°)-I(90°)}{I(0°)+I(90°)}=\frac{1}{2}[\cos 4\theta(1-\cos\delta)+(1+\cos\delta)], \tag{11}$$

$$k_2=\frac{I(45°)-I(135°)}{I(45°)+I(135°)}=\frac{1}{2}\sin 4\theta(1-\cos\delta). \tag{12}$$

Using $\sin^2 2\theta+\cos^2 2\theta=1$, the retardation of the quarter-wave plate can be expressed as

$$\delta=a\cos\left(\frac{k_1^2+k_2^2-k_1}{k_1-1}\right). \tag{13}$$

In our experiment, the light source was a He-Ne laser, and the laser wavelength was 632.8 nm. The polarizer was a Glan-Tayor prism with an extinction ratio greater than 10^5:1. The analyzers in the analyzer array were manufactured using the same polaroid. The groove density of the grating was 200 lines/mm. The 4-quadrant detector was Si Pin detector manufactured by Chongqing Optoelectronic Research Institute, China. The data acquisition card was 4-channel NuDAQ-9812, and the analog input resolution was 12-bit.

In this letter, "0°" position for the angle of the reference axis of the quarter-wave plate was defined by the horizontal plane. The retardations were measured as the quarter-wave plate was rotated at 15° intervals from 0° to 180°. The measured results are shown in Fig. 2. It can be seen from Fig. 2 that the difference between the

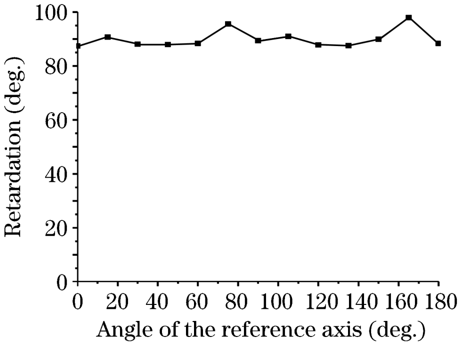

Fig. 2. Measured retardation as the quarter-wave plate rotating from 0° to 180° (interval: 15°).

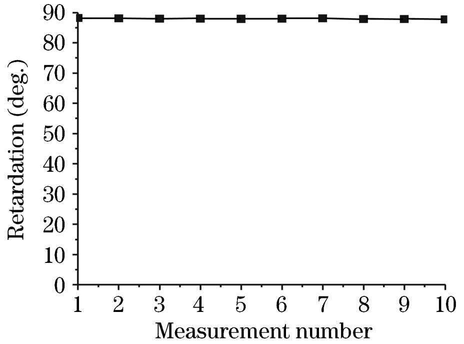

Fig. 3. Measured results of the retardation of quarter-wave plate for ten consecutive measurements.

measured retardation values, when the position of reference axis is 75° or 165° and other positions, is large. In this experiment, $I(90°)$ closed to zero when angle of the reference axis of the quarter-wave plate was 75° or 165°. According to Eq. (13), the background noise affects the measured result seriously. So we should avoid to measure the quarter-wave plate retardation on the position of $I(90°) \approx 0$. Without above-mentioned two positions, the average of the measured retardations is 88.778° and the standard deviation is 1.26°.

The results of ten consecutive measurements at the same position of the quarter-wave plate are shown in Fig. 3. The average and standard deviation for the ten measurements are 87.980° and 0.158°, respectively. So the repeated accuracy of the method is high.

Compared to the consecutive measurements, the standard deviation in rotating measurement is relatively large. The possible causes include: the rotating axis of the quarter-wave plate is not parallel to the optical axis exactly, or the rotating center of the quarter-wave plate is not in the optical axis.

In summary, a method for rapid measuring the retardation of a quarter-wave plate based on simultaneous phase shifting technique is presented. In this method, the major axis position of the quarter-wave plate need not be determined in advance. In addition, the measured result is free of the intensity fluctuation of light source.

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