

Analysis of erbium doped holey fiber using fundamental space filling mode

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Erbium-doped holey fiber with hexagonal lattice was modeled by using effective index method. In order to calculate the equivalent step index of the periodic structure of the cladding holey optical fiber, all-vectorial fundamental space filling mode approach was utilized. By using EH₁₁ mode, we have numerically solved the rate equations of a three-level pumping scheme for a fiber laser. The obtained results have shown a good agreement with the other experimental results, recently. The results have predicted amplifiers with gain efficiencies as high as 10 dB/mW.

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In erbium doped fiber amplifiers (EDFAs), to obtain high gain efficiency one needs to use fibers with high numerical aperture (NA) to achieve the maximum population inversion of the erbium ions. It is useful to confine the erbium ions to the central part of the core to enforce the pumping light propagating with the fundamental HE₁₁ mode^[1,2]. Germanium-doping, which is used in the core of high NA fiber, reduces the emission bandwidth of the Er³⁺ ions and limits its concentrations. This results in narrower gain bandwidths and increases the device lengths^[1,3].

Recently, holey optical fibers (HOFs) with high NA have become popular in making EDFAs with high gain efficiency^[4]. The stacking procedure used to fabricate HOFs makes it possible to locate the rare-earth ions accurately in the central region of the fiber, where has the maximum intensity of fundamental modes for pumping and signal fields^[1]. For amplification investigation in HOFs, the fundamental modes of core and cladding are to be known, which have been investigated by different analytical and numerical methods^[5–10].

The electromagnetic field profile of the propagating modes of an Er-doped holey fiber (EDHF) was carried out by a finite element method solver^[4]. The amplification properties of different HOFs have been studied by a full vector finite-element modal formulation combined with a population and propagation rate equation solver^[11]. The angular emission pattern was compared with the results of a multiple scattering simulation to study the emission of 532-nm light through the photonic crystal cladding of an optically pumped Er-doped silica-air HOF in Ref. [12]. In spite of these reports, the number of articles reporting the modeling of EDHF is rather small. Combining previous models of EDFAs^[13] and of HOFs^[10], in this paper, we presented a simple hybrid model which can predict the behavior of EDHFs. This model, unlike the FEM analysis, is easy to use for single-mode EDHFs and needs lesser time for computations.

In an infinitely self-similar hexagonal lattice, the exact

solutions of Maxwell's equations exhibit either an odd or an even parity with respect to planes marked either with S₁ or S₂ as in Fig. 1. The electromagnetic fields are unaffected whenever the symmetry planes S₁ or S₂ (dashed lines) are replaced either by a perfect electric or by a perfect magnetic conductor^[10].

The effective index of the cladding mode is strongly affected by the radius R of the hexagonal cell in HOFs, as shown in Fig. 2. In cylindrical coordinates (r, ϕ, z) , the electric and magnetic fields at point P in Fig. 2 on

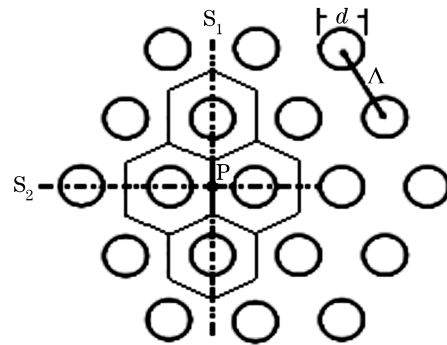


Fig. 1. Schematic of the infinitely self-similar hexagonal lattice. d : air-hole diameter; Λ : air-hole spacing.

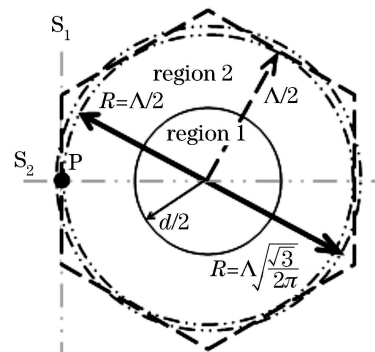


Fig. 2. Equivalent circular unit cell of a hexagonal cell with a radius of R .

the symmetry axes may satisfy boundary conditions, thus the hexagonal unit cell has often been approximated as a circular unit cell with a radius $R = \Lambda/2$ or with a preferable choice $R = \Lambda[\sqrt{3}/(2\pi)]^{0.5} (\approx 0.525\Lambda)$ ^[14].

The solutions of the two-dimensional (2D) Helmholtz equation, which satisfy the boundary conditions and finite field at the center of the fiber, result in responses in regions 1 (air) and 2 (silica), respectively. The diminishing fields at radius R , because of full-conductivity together with continuous conditions, are given by^[10,14]

$$P_1(r) = J_1(ur/a)Y_1(uR/a) - Y_1(ur/a)J_1(uR/a), \quad (1)$$

where J , and Y are Bessel functions of first kind and Neumann function, respectively. The mode EH_{11} for a full conductor boundary condition is the fundamental mode given by^[10]

$$\frac{I_2(w)}{I_1(w)} = -\frac{1}{w} - \frac{w}{2} \left(1 + \frac{\varepsilon_{r2}}{\varepsilon_{r1}}\right) p'_1 - w \sqrt{\frac{1}{4} \left(1 - \frac{\varepsilon_{r2}}{\varepsilon_{r1}}\right)^2 p_1'^2 + \left(\frac{1}{w^2} + \frac{1}{u^2}\right) \left(\frac{1}{w^2} + \frac{\varepsilon_{r2}}{\varepsilon_{r1}} \frac{1}{u^2}\right)}, \quad (2)$$

where

$$p'_1(u) = \frac{a}{u^2} \frac{\partial P_1(r)/\partial r}{P_1(r)} \Big|_{r=a}, \quad \varepsilon_{1r} = \sqrt{n_1}, \quad \varepsilon_{2r} = \sqrt{n_2},$$

$$w = \frac{2\pi}{\lambda} a \sqrt{n_{\text{FSM}}^2 - n_1^2}, \quad u = \frac{2\pi}{\lambda} a \sqrt{n_2^2 - n_{\text{FSM}}^2},$$

I is the modified Bessel function, and λ is the free space wavelength of light, $a = d/2$ is the radius of air hole, and $n_{\text{FSM}} = \beta/k_0$ is refractive index of fundamental space filling mode, $k_0 = 2\pi/\lambda$ is the wave number in free space.

We define a SIF with the core radius $\rho = 0.64\Lambda$ and refractive indices of n_{core} for $r \leq \rho$ and n_{FSM} for $r > \rho$. The electric field component E_x of EH_{11} mode is obtained as^[14]

$$E_x = \begin{cases} \frac{E_0 J_0(Ur/\rho)}{J_0(U)} & r \leq \rho \\ \frac{E_0 K_0(Wr/\rho)}{K_0(W)} & r > \rho \end{cases}, \quad (3)$$

where

$$\frac{J_0(U)}{U J_1(U)} = \frac{K_0(W)}{W K_1(W)}, \quad (4)$$

$$\begin{cases} U = \rho \sqrt{n_{\text{core}}^2 k_0^2 - \beta^2} \\ W = \rho \sqrt{\beta^2 - n_{\text{FSM}}^2 k_0^2} \\ V_{\text{eff}} = k_0 a_{\text{eff}} \sqrt{n_{\text{core}}^2 - n_{\text{FSM}}^2} \quad \text{or} \\ V = k_0 \rho \sqrt{n_{\text{core}}^2 - n_{\text{FSM}}^2} \end{cases}, \quad (5)$$

and J_0 and J_1 are the zero and first order Bessel functions of the first kind, and K_0 and K_1 are the zero and first order modified Bessel functions of the second kind, respectively. E_0 is the constant power carried by the mode.

By solving Eq. (4), the value of n_{eff} is found, and by substituting $E_y = 0$, the electric field component E_x of EH_{11} mode will be obtained. The intensity pattern

$i(r, \phi)$, proportional to $|E_x|^2$, is approximated as a Gaussian envelop function as

$$i(r, \phi) = \frac{e^{-r^2/\Omega^2}}{\pi\Omega^2}, \quad (6)$$

where the effective mode field diameter Ω is defined as^[15]

$$\Omega = \rho J_0(U) \frac{V K_1(W)}{U K_0(W)}. \quad (7)$$

The Gaussian envelop in Eq. (6) is used in the following rate equations of a three-level pumping scheme of an EDHF.

Steady state condition for a laser model, describing the erbium transitions in communication window, is $dN_1/dt = 0$, where N_1 is the ground state population of energy level. To investigate the EDHF under steady state condition, we consider a radial distribution of homogenous doping in the core with radius ρ as

$$N_t(r) = \begin{cases} N_0 & 0 \leq r \leq \rho \\ 0 & r > \rho \end{cases}, \quad (8)$$

The rate equations of erbium transitions of ordinary Er-doped fiber are given by^[13,15]

$$\frac{dP_P(z)}{dz} = -2\pi\sigma_{\text{pa}}N_0 \times \int_0^b \frac{1 + \frac{\eta_s P_S(z) i_S(r)}{I_{\text{so}}}}{1 + \frac{P_P(z) i_P(r)}{I_{\text{po}}} + \frac{P_S(z) i_S(r)}{I_{\text{so}}}} P_P(z) i_P(r) r dr, \quad (9)$$

$$\frac{dP_S(z)}{dz} = 2\pi\sigma_{\text{sa}}N_0 \times \int_0^b \frac{\eta_s \frac{P_P(z)}{I_{\text{po}}} i_P(r) - 1}{1 + \frac{P_P(z) i_P(r)}{I_{\text{po}}} + \frac{P_S(z) i_S(r)}{I_{\text{so}}}} P_S(z) i_S(r) r dr, \quad (10)$$

where P_P and P_S are the z -dependent mode powers, σ_{pa} , σ_{sa} , and σ_{se} are pumping, signal absorption, and signal emission cross-sections, respectively, τ_{sp} is the transition time between two upper levels of erbium, $\eta_s = \sigma_{\text{se}}/\sigma_{\text{sa}}$, $i_S(r)$ and $i_P(r)$ are the qualitative patterns of the signal and pumping propagations, considered as Gaussian envelopes with respect to fundamental mode propagation in an HOF with a hexagonal lattice, given by

$$I_P(r, z) = P_P(z) i_P(r), \quad I_S(r, z) = P_S(z) i_S(r). \quad (11)$$

The signal I_{so} and pumping I_{po} saturation intensities are defined as

$$I_{\text{so}} = \frac{h\nu_s}{\sigma_{\text{sa}}\tau_{\text{sp}}(1 + \eta_s)}, \quad I_{\text{po}} = \frac{h\nu_p}{\sigma_{\text{pa}}\tau_{\text{sp}}}. \quad (12)$$

The rate equations for signal and pumping powers are solved, numerically, by an initial value problem, such as Runge-Kutta-Fehlberg method^[16]. In all calculations, we have considered the input signals equal to -30 dBm ($1 \mu\text{W}$).

It is shown that EDFA's conversion efficiency is the lowest near the amplifier gain peak at 1530 nm, because of the high absorption and emission cross sections at this signal wavelength^[13]. We have drawn in Fig. 3 the

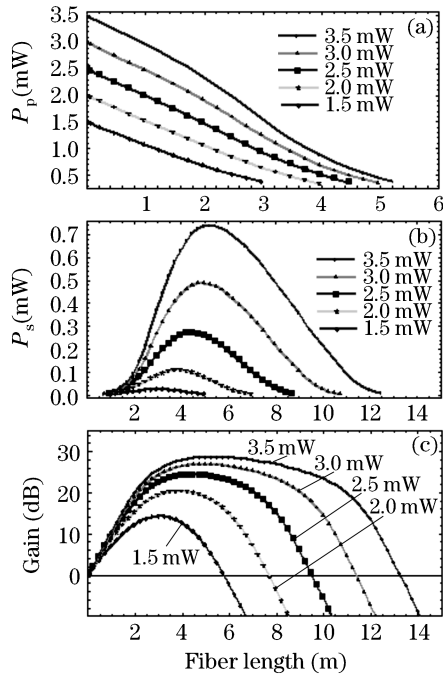


Fig. 3. Amplifying diagrams of an EDHF at $\lambda_s = 1531$ nm, $\lambda_p = 980$ nm, $\rho = 0.64\Lambda$, $N_0 = 150$ ppm. (a) Pumping power, (b) gain, and (c) signal power gain versus amplifier length.

amplification diagrams of an EDHF with typical parameter values of $d/\Lambda = 0.54$ and $\Lambda = 2 \mu\text{m}$ at the signal wavelength $\lambda_s = 1531$ nm. A uniformly doped core $\rho = 0.64\Lambda$ with erbium density of about $N_0 = 150$ ppm at 980-nm pumping wavelength is considered. The pumping and signal absorptions, and signal emission cross sections are respectively taken as $\sigma_{pa} = 2.58 \times 10^{-25} \text{ m}^2$, $\sigma_{sa} = 5.94 \times 10^{-25} \text{ m}^2$, and $\sigma_{se} = 7.04 \times 10^{-25} \text{ m}^2$.

In Fig. 3(a), pumping power decreases when propagates through waveguide. Figure 3(b) shows that signal power reaches to a maximum level in an optimum length, and then decreases while propagating along the holey fiber. By comparing Figs. 3(a) and (b), it is observed that after an optimum length, where the pumping power diminishes, the signal power would decrease, accordingly. Figure 3(c) illustrates gain versus fiber length for different pumping powers. The optimum lengths of various pumping powers, the saturation power of the pumping and signal wavelengths, and the threshold pumping power are listed in Table 1.

The gain variations versus pumping powers are shown in Fig. 4, for different lengths of EDHF. The maximum gain observed in pumping powers is higher than 12 mW

Table 1. Amplifying Characteristics of the EDHF in Fig. 3

Initial Pumping Power (mW)	Optimum Length (m)
1.5	2.962
2.0	3.963
2.5	4.462
3.0	4.962
3.5	5.212

Saturation pumping power = 0.23 mW, saturation signal power = 0.034 mW, threshold pumping power = 0.194 mW.

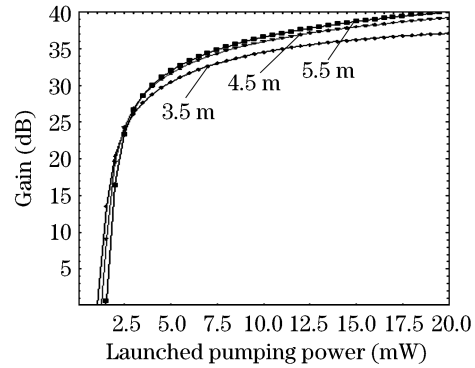


Fig. 4. Gain versus launched pumping power for various EDHF lengths with $d/\Lambda = 0.54$, $\lambda = 2 \mu\text{m}$, $\rho = 0.64\Lambda$, $N_0 = 150$ ppm, $\lambda_s = 1531$ nm, and $\lambda_p = 980$ nm.

and in length longer than 5.5 m. The gain curves for various amplifier lengths are converged at the high pumping powers, which predict a high amplifying limit up to gains of 40 – 45 dB. Such an amplifier with gain-coefficients of 6.5 – 9.0 dB/mW has high gain efficiency. The gain coefficients decrease in higher pumping powers or longer lengths of EDHF, as shown in Fig. 4 for 3 – 5-mW pumping power range.

In the experiment reported in Ref. [1], an EDHF was fabricated with eight rings of air holes using stack- and draw-process, where a ~ 1000 -ppm Er-doped aluminosilicate rod by weight was used to make the core within the preform. Its NA, air-hole diameter d , and the pitch Λ were reported to be 0.14, 1, and $2 \mu\text{m}$, respectively. The value of silica refractive index n_{silica} was not reported, so in our calculation, based on other references, we consider its value as 1.46. By using NA = 0.14 of the perform^[1], the refractive index of the doped core will be $n_{\text{core}} = (\text{NA}^2 + n_{\text{silica}}^2)^{0.5} \approx 1.4667$. We chose the radius of the doped core less than the empirical radius $\rho = 0.64\Lambda$, so that the HOF remains single-moded. On the basis of various investigations, we chose the maximum doped-core radius to be $\rho_{\text{er}} = 0.56\Lambda$ ^[17]. The characteristics of such a theoretical SIF for the signal and pumping powers of $\lambda_s = 1531$ nm and $\lambda_p = 980$ nm are shown in Table 2.

Note that we have not exactly used the same values of pumping and signal wavelengths as reported in Ref. [1], since their exact erbium cross sections are not known to us. Therefore, for comparison, the values of pumping and signal wavelengths utilized in our model are chosen very close to their experimental results. Such an EDHF has

Table 2. Characteristics of a Theoretical SIF with $d/\Lambda = 0.5$, $\Lambda = 2 \mu\text{m}$, $\rho_{\text{er}} = 0.56 \Lambda$, $N_0 = 1000$ ppm, $n_{\text{silica}} = 1.46$

Parameters	Pumping	Signal
λ (μm)	0.98	1.531
n_{FSM}	1.42931	1.40495
n_{eff}	1.4489	1.42965
Ω (μm)	0.8897	0.9959
NA	0.3290	0.4211
Waveguide Parameter	2.3627	1.9355
Confinement Ratio	0.8205	0.722

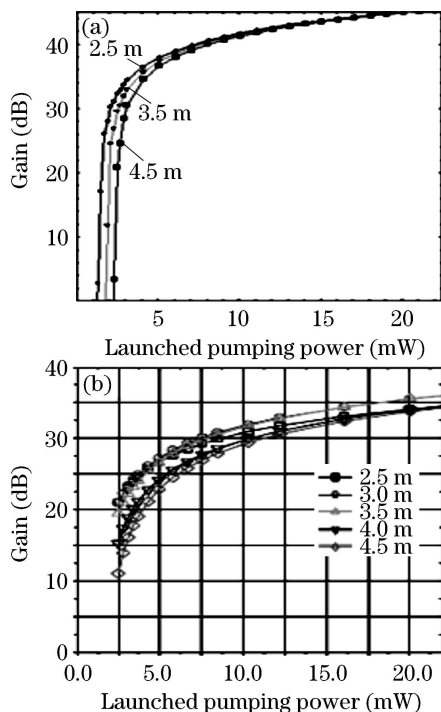


Fig. 5. Gain versus launched pumping powers for different fiber lengths; $d/\Lambda = 0.5$, $\Lambda = 2 \mu\text{m}$, and $N_0 = 1000 \text{ ppm}$. (a) Theoretical values: $\rho_{\text{er}} = 0.56\Lambda$, $\lambda_s = 1.531 \mu\text{m}$, $\lambda_p = 0.980 \mu\text{m}$, and conversion efficiency = 0.4. (b) Experimental values: $\lambda_s = 1.533 \mu\text{m}$, $\lambda_p = 0.976 \mu\text{m}$, and conversion efficiency ≥ 0.4 .

a high NA and a good confinement ratio more than 80% for pumping wavelength, which provides a good overlap between erbium ions and pumping propagation. Conventional Er-doped fibers drawn from a preform of Er-doped aluminosilicate were found to show good optical power conversion efficiency more than 40%, despite the relatively high dopant concentration^[1]. So, on the basis of this result, we considered 40% conversion efficiency for solving the rate equations.

Figure 5 shows gain versus pumping powers for different lengths of EDHF. Gain coefficient is smaller at longer lengths. One can compare the gain coefficients of 4.5- and 2.5-m EDHF at pumping of 3 mW, and they are about 9.3 and 10 dB/mW, respectively. In higher pumping powers, the more the length, the more will be the gain coefficient. Our theoretical results have a good agreement with the experimental data of Ref. [1].

We have predicted a high gain coefficient of about 6–10 dB/mW in 2–5 mW of pumping range for lengths of 2.5, 3.5, and 4.5 m, while in Ref. [1], the corresponding gain coefficients are reported as 8.61, 7.95, and 5.25 dB/mW in the same pumping interval, at signal and pumping wavelengths of $\lambda_s = 1533 \text{ nm}$ and $\lambda_p = 976$

nm, respectively.

In conclusion, by using effective index method, erbium-doped holey fiber with hexagonal lattice is modeled. To obtain the equivalent step index of the periodic structure of the cladding holey optical fiber, all-vectorial fundamental space filling mode approach is utilized. We have used EH_{11} mode to solve the rate equations of a three-level pumping scheme of a fiber laser. Our model, which is very similar to the reported experimental results, has a high numerical aperture and a good confinement ratio more than 80% for pumping wavelength, which provides a good overlap between erbium ions and pumping propagation. The theoretical results have predicted amplifiers with gain efficiencies as high as 10 dB/mW at pumping power of 5 mW for lengths of 2.5, 3.5, and 4.5 m.

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