

# Influence of turbulent atmosphere on the far-field coherent combined beam quality

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Propagation of coherent combined laser beams in turbulent atmosphere is numerically studied based on the extended Huygens-Fresnel principle. By choosing beam propagation factor (BPF) and beam quality factor (BQ) to characterize the far-field irradiance distribution properties, the influence of turbulence on far-field coherent combined beam quality is studied in detail. The investigation reveals that with the coherence length decreasing, the irradiance distribution pattern evolves from typical non-Gaussian shape with multiple side-lobes into Gaussian shape which is seen in the incoherent combining case. In weak turbulent atmosphere, the far-field beam quality suffers less when the laser array gets more compact and operates at a longer wavelength. In strong turbulent atmosphere, the far-field beam quality degrades into the incoherent combining case without any relationship with the fill factor and laser wavelength.

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In the past decade, coherent combining of laser beams have been widely investigated theoretically and experimentally due to their wide applications such as long-range energy delivering<sup>[1]</sup>. Fiber lasers are particularly well-suited to beam combining because of their inherent compact size. Robust coherent combining of a small number of fiber lasers has been demonstrated<sup>[2-4]</sup>. Propagation of coherent combined laser beams in free space has been studied in detail, and the effects of phase error, asymmetric intensity distribution, and fill factor on the far-field coherent combined beam quality have been discussed theoretically<sup>[5-8]</sup>. Laser beams have to propagate in the atmosphere for long-range use. Propagation of coherent combined laser beams in the atmosphere has not yet been studied until very recently<sup>[9-11]</sup>. The analytical expression for far-field irradiance distribution of coherent combined laser beams in a turbulent atmosphere has been derived and calculated numerically<sup>[9,10]</sup>. However, further calculation is needed to give a quantitative study on the influence of atmosphere turbulent.

In this letter, we study the influence of atmosphere turbulence on the far-field coherent combined beam quality. Propagation properties and far-field irradiance distribution are obtained by using extended Huygens-Fresnel principle, and beam propagation factor (BPF)<sup>[12]</sup> and beam quality factor (BQ)<sup>[13]</sup> are chosen as characteristic parameters of beam quality in the far field. Our motivation is to learn the influence of atmosphere turbulence quantitatively and obtain some references for design optimization of practical engineering systems.

We assume that the fiber laser array is located at the source plane ( $z = 0$ ) and suppose that each fiber laser beam has a Gaussian single mode field distribution. The laser beams propagate along the  $z$  axis in the Cartesian coordinate system. The field distribution of the laser array at the source plane is written as

$$E(x, y, 0) = \sum_i^M E_i(x, y, 0), \quad (1)$$

where  $E_i(x, y, 0)$  denotes the  $i$ th Gaussian beam with the initial beam width  $\omega_0$  and located at the position  $(a_i, b_i, 0)$ .  $E_i(x, y, 0)$  can be expressed as

$$E_i(x, y, 0) = \exp \left[ -\frac{(x - a_i)^2 + (y - b_i)^2}{\omega_0^2} \right]. \quad (2)$$

By using extended Huygens-Fresnel principle, the far-field average irradiance distribution (at  $z = L$  plane,  $L$  denotes the propagation distance) of coherent combined laser beam in a turbulent atmosphere can be expressed as<sup>[9-11,14-16]</sup>

$$\begin{aligned} & \langle I(p, q, L) \rangle \\ &= \frac{k^2}{(2\pi L)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, 0) E^*(\xi, \eta, 0) \\ & \times \exp \left( \frac{ik}{2L} \left[ (p-x)^2 + (q-y)^2 + (p-\xi)^2 + (q-\eta)^2 \right] \right) \\ & \times \langle \exp[\psi(x, y, p, q) + \psi^*(\xi, \eta, p, q)] \rangle dx dy d\xi d\eta, \quad (3) \end{aligned}$$

where  $k = 2\pi/\lambda$  is the wave number,  $\lambda$  is the wavelength,  $(p, q)$  denotes the transverse coordinates at the far-field plane, and angle brackets indicate the ensemble average over the medium statistics covering the log-amplitude and phase fluctuations due to the turbulent atmosphere.

The ensemble average term in Eq. (3) can be expressed as

$$\begin{aligned} & \langle \exp[\psi(x, y, p, q) + \psi^*(\xi, \eta, p, q)] \rangle \\ &= \exp[-0.5D_\psi(x - \xi, y - \eta)] \\ &= \exp \left\{ -\frac{1}{r_0^2} \left[ (x - \xi)^2 - (y - \eta)^2 \right] \right\}, \quad (4) \end{aligned}$$

where  $D_\psi$  is the wave structure function,  $r_0 = (0.545C_n^2 k^2 L)^{-3/5}$  is the coherence length of spherical

wave propagating in the turbulent medium with  $C_n^2$  being the structure constant of the refractive index. It has to be noted that Eq. (4) is valid only under the Rytov approximation. Equation (3) provides a convenient and effective way for studying the propagating properties of coherent combined beams in a turbulent atmosphere.

We consider a ring distributed fiber laser array shown in Fig. 1 for its high fill factor value<sup>[7]</sup> which is a great advantage for coherent combining. An array with  $N$  rings contains  $M$  lasers,  $M = 1 + 3N(N + 1)$ . The ring distributed array consists of a central element and several surrounding rings of elements. The core diameters of double-clad fibers used for generating high power lasers are about  $20 \mu\text{m}$  and the laser beam waist is thus rather small, which corresponds to a relatively large diffraction angle and not practical for long-range use. As shown in Fig. 1(b), the laser beams can be expanded and collimated by microlens array. The beam waist of each laser beam after expanding is  $\omega_0$ . Fiber lasers are arranged in the array with the nearest neighbor separated by a distance of  $d$ . Fill factor is defined as  $t = (d - 2\omega_0)/\omega_0$  to describe the compactness of the array. A smaller  $t$  corresponds to a more compact array. The diameter of the whole fiber laser array is  $D = 2Nd$ .

In order to investigate the effect of turbulence on coherent combined beam, we firstly consider a fiber laser array with 4 rings (61 lasers). The parameters are taken as follows:  $\lambda = 1 \mu\text{m}$ ,  $\omega_0 = 1 \text{ cm}$ ,  $t = 0.5$ ,  $d = 2.5 \text{ cm}$ ,  $D = 20 \text{ cm}$ , and  $L = 10 \text{ km}$ . Figures 2(a)–(d) show the contour plots of far-field irradiance distributions for the coherent combined beam propagation in turbulent atmosphere with different coherence lengths. Figure 2(e) shows the contour plot for the coherent combined beam propagation in free space, and Fig. 2(f) shows the contour plot for the incoherent combined beam propagation in the atmosphere with the same coherence length as in Fig. 2(d).

One can see from Fig. 2 that in atmosphere with weak turbulence, the beamlets propagate to the far-field plane, overlap and interfere with others. The on-axis irradiance becomes the maximum. However, as the turbulence

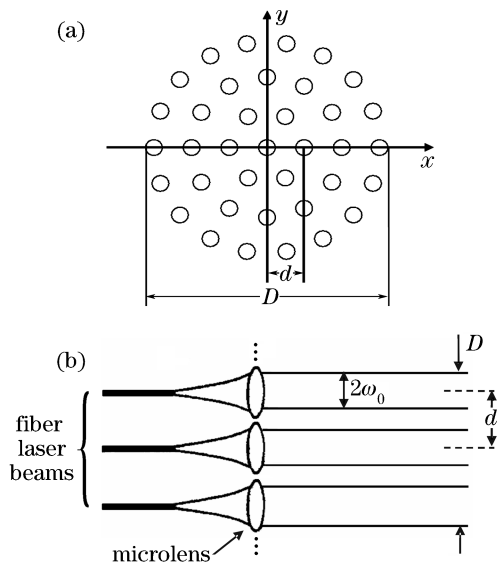


Fig. 1. Fiber laser array with ring distribution. (a) Front view; (b) side view.

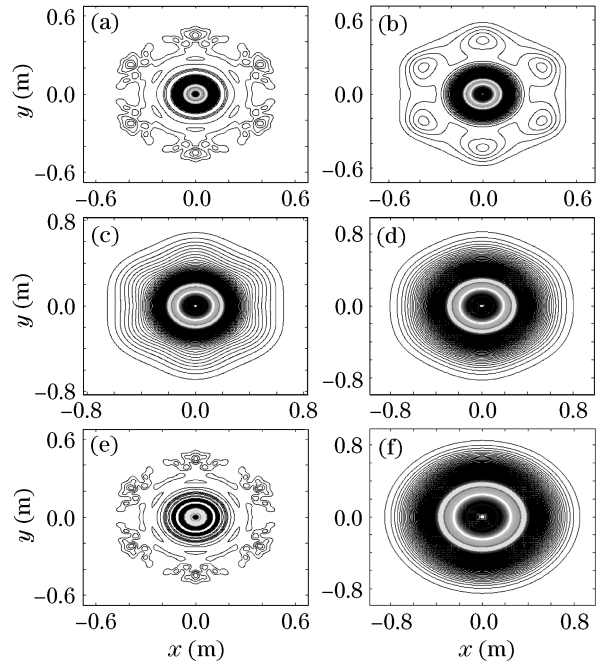


Fig. 2. Contour plots of the far-field irradiance distributions for coherent combined beam propagation with different coherence length  $r_0$ . (a)  $D/r_0 = 1$ ; (b)  $D/r_0 = 5$ ; (c)  $D/r_0 = 10$ ; (d)  $D/r_0 = 15$ ; (e)  $D/r_0 = 0$  (free space); (f)  $D/r_0 = 15$  (incoherent combining).

effect gets stronger, the coherence between laser array elements is destructed, thus the far-field beam quality degrades. The far-field irradiance distribution pattern gradually evolves into Gaussian shape, which is often the distribution characteristic of incoherent combining. This phenomenon is caused by the isotropic influence of the atmosphere turbulence<sup>[10]</sup>. It has to be noted that the irradiance distribution is statistically averaged for multiple short-exposure irradiance patterns. It cannot be used to embody the practical irradiance distribution of coherent combined beams propagating in turbulent atmosphere at specified time due to the varying characteristics of turbulence.

$M^2$  factor and  $\beta$  parameters are often used to evaluate single laser beam<sup>[17]</sup>. In order to study the influence of atmosphere turbulence on the far-field coherent combined beam quality, BPF and BQ are chosen as characteristic parameters of coherent combined beam quality in the far field. BPF is defined as the laser output power in a specified far-field bucket  $P$  divided by the total output power from the effective near-field exit aperture of the combined laser beam  $P_{DL}$ ,

$$\text{BPF} = (P/P_{DL}), \quad (5)$$

and BQ is defined as<sup>[13]</sup>

$$\text{BQ} = \sqrt{P_{DL}/P}. \quad (6)$$

According to Ref. [12], the far-field bucket is defined as  $A_{DL} = (\pi/4)(\theta_{DL}z)^2$ , which is the diffraction-limited bucket and  $\theta_{DL} = 2.44\lambda/D$ , where  $D$  is the effective exit aperture of the combined laser beam, namely, the diameter of the whole array. There is a simple mathematical relationship between BPF and BQ. In general, BPF is

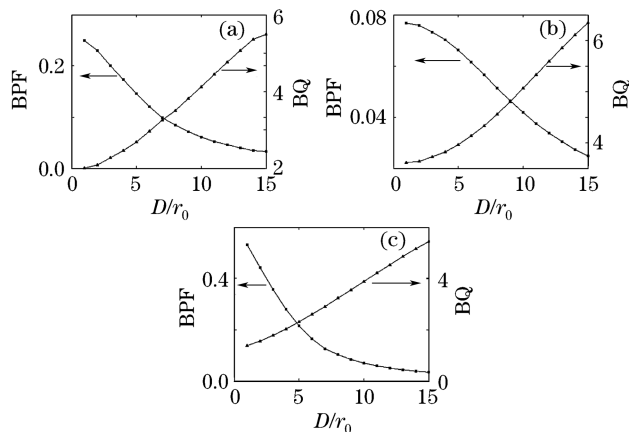


Fig. 3. Dependence of BPF and BQ on the atmosphere coherence length. (a)  $\lambda = 1 \mu\text{m}$ ,  $\omega_0 = 1 \text{ cm}$ ,  $t = 0.5$ ,  $d = 2.5 \text{ cm}$ ,  $D = 20 \text{ cm}$ ; (b)  $\lambda = 1 \mu\text{m}$ ,  $\omega_0 = 1 \text{ cm}$ ,  $t = 1$ ,  $d = 3 \text{ cm}$ ,  $D = 24 \text{ cm}$ ; (c)  $\lambda = 2 \mu\text{m}$ ,  $\omega_0 = 1 \text{ cm}$ ,  $t = 0.5$ ,  $d = 2.5 \text{ cm}$ ,  $D = 20 \text{ cm}$ .

smaller than 1 and BQ is larger than 1. The closer to 1 these two parameters are, the better the beam quality is.

Figure 3(a) shows the dependence of BPF and BQ on the atmosphere coherence length. The array parameters of the laser array are the same as those used in Fig. 2. When the fill factor  $t = 1$ , the distance between array elements become larger than that in Fig. 3(a) and the dependence of BPF and BQ on the atmosphere coherence length will be different, as shown in Fig. 3(b). Figure 3(c) shows the dependence of BPF and BQ on the atmosphere coherence length when the laser wavelength  $\lambda = 2 \mu\text{m}$ , while the other array parameters conserves as those used in Fig. 3(a).

Figures 3(a) and (b) demonstrate that the fill factor plays an important role on the far-field coherent combined beam quality in weak turbulent atmosphere. A more compact laser array corresponds to a better far-field beam quality in weak turbulence. However, the beam quality degrades faster for a more compact laser array as the turbulence effect gets stronger. Figures 3(a) and (c) demonstrate that for laser arrays with the same geometric size but difference laser wavelengths, longer laser wavelength leads to better far-field beam quality in weak turbulent atmosphere. Again, the beam quality degrades faster for the array with longer laser wavelength as the turbulence effect gets stronger. The BPF and BQ values for the three laser arrays are all about 0.03 and 6 respectively when  $D/r_0 = 15$ . Comparing the BPF curves and BQ curves in Fig. 3, one can tell that in strong turbulence, the beam quality does not depend on the fill factor and laser wavelength anymore. Figure 3 also shows that for different laser arrays, the BQ value has an approximate linear relationship with  $D/r_0$ , which can be used for quantitative analysis of far-field beam quality in turbulent atmosphere for any given laser array.

In conclusion, the propagation of coherent combined laser beams in turbulent atmosphere is numerically studied based on the extended Huygens-Fresnel principle, and the influence of turbulence on far-field coherent combined beam quality is studied in detail by choosing BPF

and BQ to describe the far-field irradiance distribution properties. We find that the irradiance distribution pattern evolves from the typical non-Gaussian one with multiple side-lobes into Gaussian shape with the coherence length decreasing. The investigation also reveals that more compact laser array and longer laser wavelength correspond to better beam quality in weak turbulence. However, the beam quality degrades into the incoherent combining case in strong turbulence, which does not depend on the fill factor and laser wavelength.

It should pointed out that for coherent combined beams propagating in the atmosphere, only the influence of turbulence on the far-field coherent combined beam quality is studied in this paper. The influence of extinction, scattering, and thermal blooming is to be studied in the future to give a comprehensive understanding of the multiple laser system performance.

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## References

1. T. Y. Fan, *IEEE J. Sel. Top. Quantum Electron.* **11**, 567 (2005).
2. J. Anderegg, S. Brosnan, E. Cheung, P. Epp, D. Hammons, H. Komine, M. Weber, and M. Wickham, *Proc. SPIE* **6102**, 61020U (2006).
3. B. He, Q. Lou, J. Zhou, J. Dong, Y. Wei, and Z. Wang, *Acta Opt. Sin.* (in Chinese) **26**, 1279 (2006).
4. J. Hou, R. Xiao, Z. Jiang, B. Shu, J. Chen, and Z. Liu, *High Power Laser and Particle Beams* (in Chinese) **18**, 1585 (2006).
5. C. D. Nabors, *Appl. Opt.* **33**, 2284 (1994).
6. B. Lü and H. Ma, *Appl. Opt.* **39**, 1279 (2000).
7. Y. Li, L. Qian, D. Lu, D. Fan, and S. Wen, *Opt. Laser Technol.* **39**, 957 (2007).
8. Z. Hu, S. Le, and D. Yang, *Proc. SPIE* **1635**, 118 (1992).
9. X. Chu, Z. Liu, and Y. Wu, *J. Opt. Soc. Am. A* **25**, 74 (2008).
10. Y. Cai, Y. Chen, H. T. Eyyuboğlu, and Y. Baykal, *Appl. Phys. B* **88**, 467 (2007).
11. H. T. Eyyuboğlu, Y. Baykal, and Y. Cai, *Appl. Phys. B* **91**, 265 (2008).
12. C. M. Stickley, "Architecture for diode high energy laser systems" <http://www.darpa.mil/mto/programs/adhels/index.htm> (July 20, 2007).
13. G. D. Goodno, C. P. Asman, J. Anderegg, S. Brosnan, E. C. Cheung, D. Hammons, H. Injeyan, H. Komine, W. H. Long, Jr., M. McClellan, S. J. McNaught, S. Redmond, R. Simpson, J. Sollee, M. Weber, S. B. Weiss, and M. Wickham, *IEEE J. Sel. Top. Quantum Electron.* **13**, 460 (2007).
14. Y. Zhang and G. Wang, *Chin. Opt. Lett.* **4**, 559 (2006).
15. Y. Zhang, M. Tang, and C. Tao, *Chin. Opt. Lett.* **3**, 559 (2005).
16. Y. Zhang and T. Zhu, *Chin. Opt. Lett.* **6**, 79 (2008).
17. Y. Su and M. Wan, *High Energy Laser System* (in Chinese) (National Defence Industry Press, Beijing, 2006) p.39.