Effects of external magnetic trap on two dark solitons of a two-component Bose-Einstein condensate

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Two dark solitons are considered in a two-component Bose-Einstein condensate with an external magnetic trap, and effects of the trap potential on their dynamics are investigated by the numerical simulation. The results show that the dark solitons attract, collide and repel periodically in two components as time changes, the time period depends strictly on the initial condition and the potential, and there are obvious self-trapping effects on the two dark solitons.

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It is well known that the Gross-Pitaevskii equation (mean-field model) supports solitonic solutions. Exact analytical solutions to one and two-dimensional (2D) mean-field models describing dynamics of Bose-Einstein condensates (BECs) are investigated, and the interaction in different components is studied for a miscible case[1−5]. Bright solitons have been detected only very recently using an optical red-detuned laser beam along the axial direction of the sample to impose a transverse confinement, while dark solitons of Bose condensed atoms were experimentally observed few years ago. A dark soliton in a BEC is a macroscopic excitation of the condensate with a corresponding positive scattering length, and is characterized by a local density minimum and a phase gradient of the wave function at the position of the minimum[6]. The stability of dark solitons, which are defined as stationary objects, in a nonlinear system is a critical property. Behavior of a BEC strongly depends on parameters of trap potentials which can be controlled by either the intensity or geometry of laser beams. For example, the matter wave dynamics can be effectively manipulated by external fields. The harmonic trap induces a dynamical instability of the soliton, culminating in sound emission. The interaction of dark solitons may be strongly repulsive, and the cross-phase modulation has an important influence on both the formation and interaction of the dark solitons[7−9].

A multi-component BEC presents novel and fundamentally different scenarios. In particular, it is observed that the BEC can reach an equilibrium state characterized by the separation of the species in different domains, and the creation of dark solitons in a two-component BEC is investigated. The magnetic trap provides the principle increase of phase space density to the BEC transition, and the atoms are cooled in the magnetic trap using energetically selective spin transitions[10]. A relevant interesting issue is to learn how to control the motion of the condensates and different types of dark solitons in the multi-component BEC, then the question arises how one could affect or even guide the motion of the dark solitons. In this letter, the effects of external trap on two dark solitons of a two-component BEC are investigated by the numerical simulation, and some novel results are obtained.

The two vector components are evolving under the Gross-Pitaevskii equation, which are the macroscopic wave functions of Bose-condensed atoms in two different internal states. By rescaling process, the general equations may easily be put into the dimensionless form, and the procedure results in the following coupled Schrödinger equations[11,12]

$$j\frac{\partial u_1}{\partial t}+\frac{1}{2}\frac{\partial^2 u_1}{\partial z^2}-[|u_1|^2+g_1|u_2|^2+V(z)+\mu_1]u_1=0, \tag{1a}$$

$$j\frac{\partial u_2}{\partial t}+\frac{1}{2}\frac{\partial^2 u_2}{\partial z^2}-[|u_2|^2+g_2|u_1|^2+V(z)+\mu_2]u_2=0, \tag{1b}$$

where u_i $(i=1,2)$ are the condensate wave-functions, μ_i $(i=1,2)$ are the chemical potentials which are connected to the number of atoms of the condensate, g_i $(i=1,2)$ are the interaction strengths describing the inter-atomic interaction between two components of the BEC, $V(z)$ is the normalized confining potential of the components in the longitudinal direction (z direction). In the present work, we assume a cylindrical highly anisotropic trapped potential, and under this circumstance, one can approximate the field as Eqs. (1a) and (1b)[13]. Namely, the transverse potential is much stronger than the longitudinal one, it is natural to reduce the full three-dimensional (3D) Schrödinger equation to an effective one-dimensional (1D) Schrödinger equation. Different approaches were proposed to achieve this purpose under different conditions[14].

Additionally, we assume that the interaction strengths satisfy $g_1=g_2=g$, and the assumption matches well with the experimental conditions. The μ_i $(i=1,2)$ term can be eliminated by using the transformation $u_i\rightarrow u_i\exp(-j\mu_i t)$ $(i=1,2)$, and the coupled 1D equations can be given by

$$j\frac{\partial u_1}{\partial t}+\frac{1}{2}\frac{\partial^2 u_1}{\partial z^2}-[|u_1|^2+g\,|u_2|^2+V(z)]u_1=0, \quad \text{(2a)}$$

$$j\frac{\partial u_2}{\partial t}+\frac{1}{2}\frac{\partial^2 u_2}{\partial z^2}-[|u_2|^2+g\,|u_1|^2+V(z)]u_2=0. \quad \text{(2b)}$$

The external magnetic trap is cylindrically symmetric with the harmonic frequency in the radial direction, and the potential of the external trap is

$$V(z)=\frac{1}{2}\Omega^2 z^2, \quad (3)$$

where Ω is the harmonic frequency in the radial direction (z direction).

We can perform a series of direct numerical simulations for the coupled 1D Schrödinger equations (Eqs. (2a) and (2b)), and solutions of the two dark solitons are

$$u_1(z,t)=\phi(z)v_1(z,t), \quad \text{(4a)}$$

$$u_2(z,t)=\phi(z)v_2(z,t), \quad \text{(4b)}$$

where $\phi(z)$ describes the background wave function, v_i $(i=1,2)$ describe the dark solitons, and[11,12]

$$v_1(z,t)=j\cos\theta-\sin\theta\tanh[\sin\theta(z-\Delta/2)], \quad \text{(5a)}$$

$$v_2(z,t)=j\cos\theta+\sin\theta\tanh[\sin\theta(z+\Delta/2)], \quad \text{(5b)}$$

where Eqs. (5a) and (5b) represent a kink-antikink situation, i.e., when the phase fronts of the solitons are facing each other, θ is the distributing angle, Δ is the relative distance between the two dark solitons, $\cos\theta$ is the dark soliton speed relating to the speed of sound, $\sin\theta$ is depth of the dark soliton, $\phi(z)$ is the stationary function, which can be approximated in the Thomas-Fermi (TF) framework as

$$\phi(z)=\sqrt{1-V(z)}=\sqrt{1-\frac{1}{2}\Omega^2 z^2}, \quad (6)$$

in the region where $V(z)<1$ and $\phi(z)=0$ outside.

In Fig. 1, we show the evolution of two dark solitons in a condensate confined in the external magnetic trap of the different harmonic frequency, the interaction strength $g=0.20$, the initial relative distance $\Delta=4$ and the initial distributing angle $\theta=\pi/2$. We can see two dark solitons attract, collide and repel reciprocally in the periodical evolution as time changes, and the colliding time period (time interval) depends strictly on the harmonic frequency. The time period becomes large when harmonic frequency goes large. We have systematically compared the evolution of the two dark solitons, and found that there are obvious self-trapping effects on the two dark solitons. The features show that two dark solitons are independent of their evolution because of their robust features.

The evolution of two dark solitons in a two-component condensate confined in the external magnetic trap $V(z)$ with $\Omega=0.2$ is shown in Fig. 2, the initial relative distance $\Delta=4$ and the interaction strength $g=0.20$ for different initial distributing angles. It is known that smaller values of the depth for dark soliton are physically less interesting because the very shallow dark soliton is hardly distinguishable from sound. The colliding time period depends strictly on the initial distributing angles. For example, the time period becomes small when depth of the dark soliton becomes shallow, and the shape of the dark soliton may go distorting.

In Fig. 3, we pertain to the same harmonic frequency $\Omega=0.2$, the initial relative distance $\Delta=4$, and the initial distributing angles $\theta=\pi/2$ for different interaction strengths and find that the colliding time period is

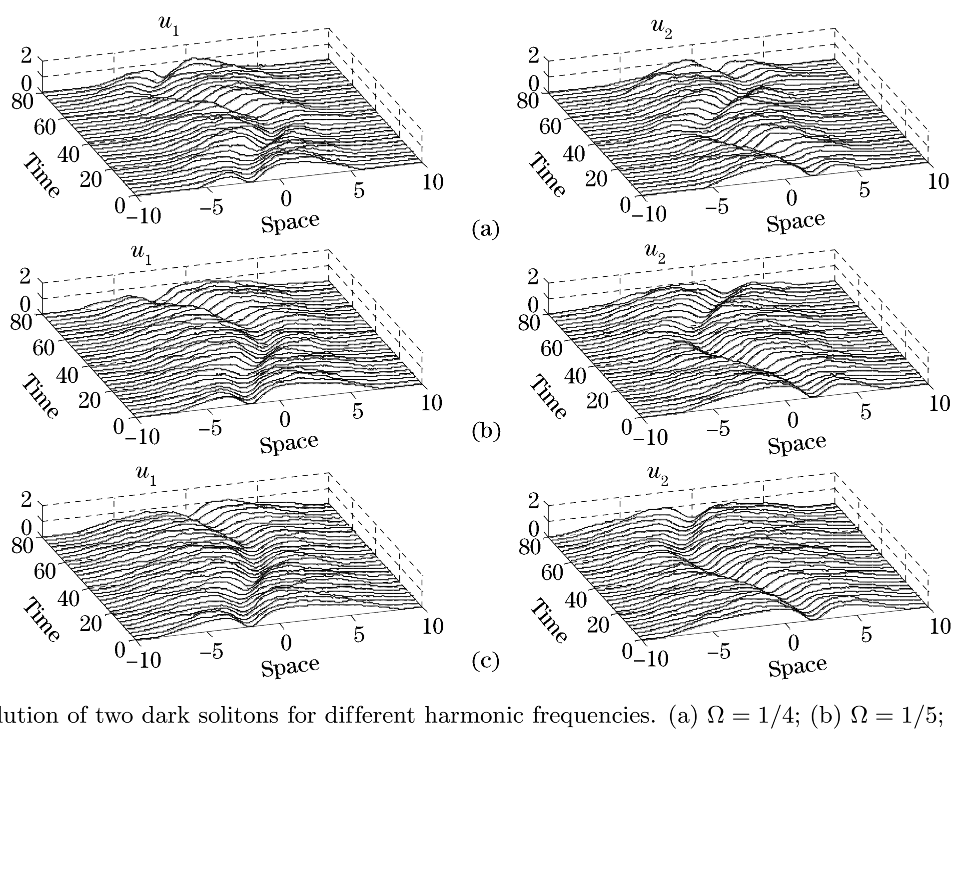

Fig. 1. Evolution of two dark solitons for different harmonic frequencies. (a) $\Omega=1/4$; (b) $\Omega=1/5$; (c) $\Omega=1/6$.

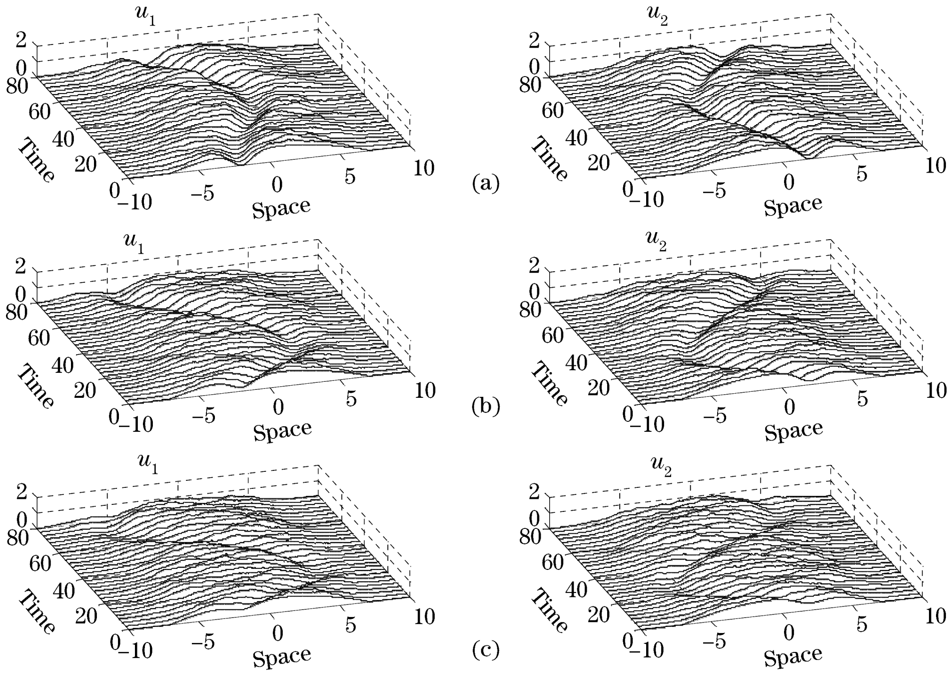

Fig. 2. Evolution of two dark solitons for different initial distributing angles. (a) $\theta = \pi/2$; (b) $\theta = \pi/3$; (c) $\theta = \pi/4$.

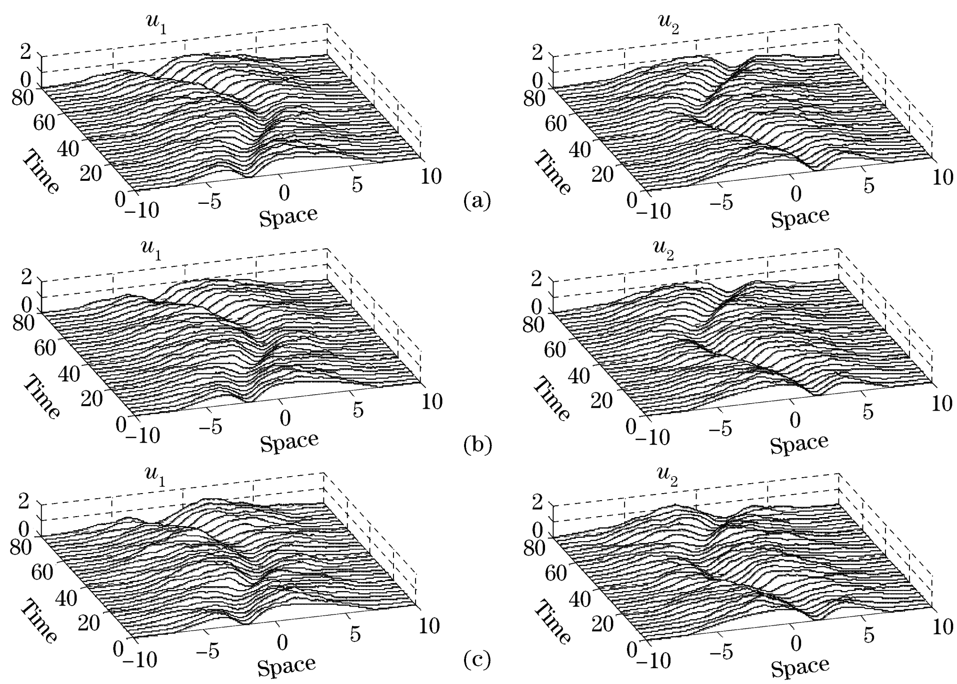

Fig. 3. Evolution of two dark solitons for different interaction strengths. (a) $g = 1/20$; (b) $g = 1/5$; (c) $g = 1/2$.

independent of the interaction strength, while the time period almost does not change when the interaction strength becomes large or small. Furthermore, we pertain to the fixed harmonic frequency $\Omega = 0.2$, the interaction strength $g = 0.20$ and the initial distributing angle $\theta = \pi/2$ for different initial relative distances in Fig. 4 and find that the colliding time period depends strictly on the initial relative distance. For example, the time period becomes large as the increase of initial relative distance. There are also obvious self-trapping effects on the two dark solitons, and they are different from those of two bright solitons in a two-component BEC[15], in which there are obvious switching effects.

In summary, the effects of the trap potential on dynamics of two dark solitons in a two-component BEC are investigated by the numerical simulation. The two dark solitons attract, collide and repel periodically in two components as time changes, and time period of the periodical collision depends strictly on the initial distributing angle, the initial relative distance and the harmonic frequency of the external potential, but is independent of the interaction strength. The dark soliton has robust

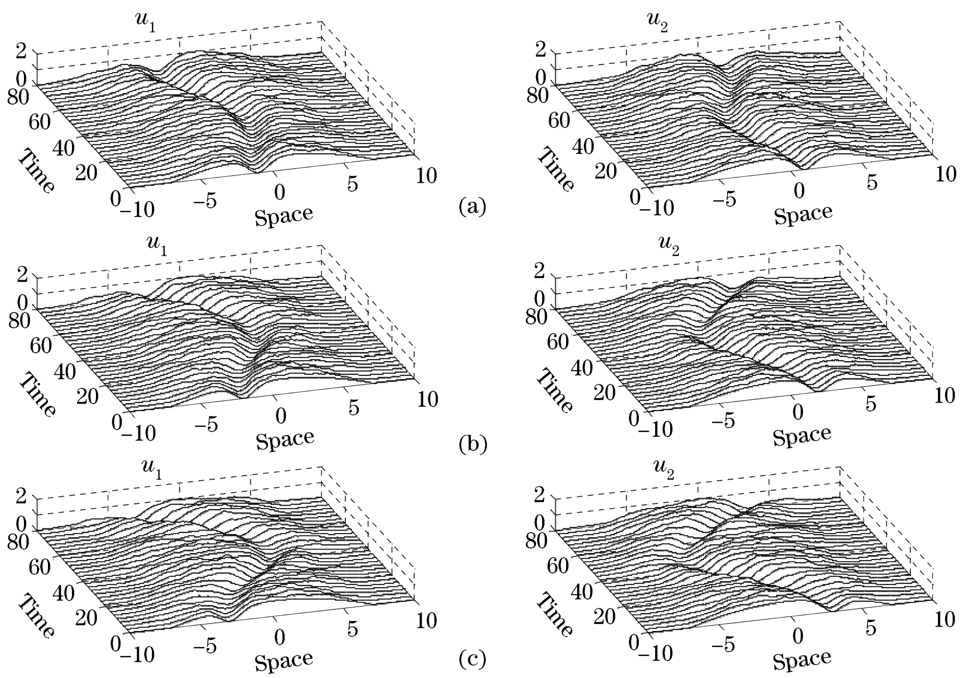

Fig. 4. Evolution of two dark solitons for different initial relative distances. (a) $\Delta = 2.0$; (b) $\Delta = 4.0$; (c) $\Delta = 6.0$.

features, and there are obvious self-trapping effects on two dark solitons in a two-component BEC. The results show that the two dark solitons are independent between two components in their evolution under the external magnetic trap because of their robust features, and the features may be used in a two-component atom laser which is a device generating an intense coherent beam (the dark soliton train) of atoms in two components through a stimulated process.

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