A method of measuring the refractive index of extraordinary ray in uniaxial crystal with optic axis at an arbitrary orientation

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A new equation to measure the refractive index of extraordinary ray in uniaxial crystal with the optic axis at an arbitrary orientation has been given in this letter, and the term in this equation makes the measurements to be relatively easy. The theoretical study shows that the accuracy achieved in the experiments attains to the order of magnitude in 10^{-3}.

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Nonlinear optical techniques have been playing an important role in modern optics, especially in extending laser frequency bands. In recent years, nonlinear optical materials are rapidly developed. However, people have long been puzzled by how to practically define refractive index of the extraordinary (e-) ray of crystals with optic axis at an arbitrary orientation. Knowing the index is the key in making crystals be nonlinear optical devices. For instance, phase matching techniques used in nonlinear optical devices are just to make the index of the e-ray equal the ordinary (o-) ray for arranging the optical axis at certain orientation[1].

Theoretically, the indices of the e-ray, as the axis at arbitrary orientation in crystals, can be calculated by the index ellipsoid[2]:

$$\frac{1}{n_{\mathrm{e}}^2(\beta)} = \frac{\cos^2\beta}{n_{\mathrm{o}}^2} + \frac{\sin^2\beta}{n_{\mathrm{e}}^2}, \tag{1}$$

where β indicates the angle of the e-ray subtended to the optical axis, n_{o} and n_{e} are the principal indices for the o-ray and the e-ray, respectively. In fact, β has only theoretical meaning but is not measurable, although we have created a group of equations to help the measurement[3−5]. For the time being, as we know, the indices of the e-ray have not been measured yet except the special case of the axis parallel to the surface of the crystals.

Presuming a beam is normally incident on the crystal with optic axis at arbitrary orientation, the birefringence occurs inside the crystal. Generally the e-ray passes a different way and deviates a small angle from the o-ray, except the special cases of the axis parallel or vertical to the surface. The optical path and the phase of the e-ray are different from the o-ray while they arrive at the emerging surface of the crystal. In negative crystals, such as YVO_4, the e-ray will have a higher speed (same frequency) and a longer wavelength. We can write the equation regarding the phase difference

$$\delta = \frac{2\pi d}{\lambda}\left[n_{\mathrm{o}} - \frac{n_{\mathrm{e}}(\beta)}{\cos\alpha}\right], \tag{2}$$

where d is the thickness of the crystal, α indicates the walk-off angle of the e-ray from the o-ray, λ denotes the incident wavelength. If $\beta = \pi/2$, the optical path of the e-ray is the same with the o-ray, the index of the e-ray $n_{\mathrm{e}}(\beta)$ is equal to the principal index n_{e}, and then Eq. (2) simplifies to the well known formula $\delta = \frac{2\pi d}{\lambda}(n_{\mathrm{o}} - n_{\mathrm{e}})$.

From Eq. (2), consequently, we have the index of the e-ray at normal incidence:

$$n_{\mathrm{e}}(\beta) = \left[n_{\mathrm{o}} - \frac{\delta\lambda}{2\pi d}\right]\cos\alpha. \tag{3}$$

Evidently, the terms in the right hand of Eq. (3) are experimentally measurable. n_{o} and λ are easily known. d can be measured in the order of magnitude of micrometer. α can be given by measuring the separation of the birefringence on the emerging surface. So as to the key factor δ, how to define its absolute value is tedious and hard work. To get the absolute value, we suggest to perturb the paths of the birefringence to create a new phase difference δ'. If the perturbation is so suitable that the error of $n_{\mathrm{e}}(\beta)$ and α, caused by the perturbation, are negligible, then we can get a group of new data pertaining to δ'. With the help of the new data we use the difference of $\delta - \delta'$, instead of the absolute values of δ to solve $n_{\mathrm{e}}(\beta)$. To realize this idea, the perturbation of the paths has been done by tilting the crystal just a very little to the normal incidence.

If slightly tilting the crystal at a tiny angle i, which equals the incidence, the phase difference becomes

$$\delta' = \frac{2\pi d}{\lambda}\left[\frac{n_{\mathrm{o}}}{\cos r} - \frac{n_{\mathrm{e}}(\beta)}{\cos(\alpha \pm r)}\right], \tag{4}$$

where r indicates the refractive angle of the o-ray and is given by $\sin r = \frac{\sin i}{n_{\mathrm{o}}}$. Which of the sign "±" before r in Eq. (4) should be taken depends on orientation and inclination of the crystal.

Finally, subtracting Eq. (2) from Eq. (4), we have

$$n_{\mathrm{e}}(\beta) = \frac{n_{\mathrm{o}}\left[\frac{1}{\cos r} - 1\right] + \frac{\Delta\delta\lambda}{2\pi d}}{\frac{1}{\cos(\alpha \pm r)} - \frac{1}{\cos\alpha}}, \tag{5}$$

where $\Delta\delta = \delta - \delta'$. Obviously, according to Eq. (5) we no longer need to know the absolute δ. What we need to do is to monitor the relative variation of $\Delta\delta$ which is a function of the tilting angle. This makes the measurements be relatively easy.

Here we will measure the index. In the apparatus shown in Fig. 1, a He-Ne laser ($\lambda = 632.8$ nm) was incident on the crystal. The telephoto system TP consists of two positive lenses with focal lengths of $f_1 = 4$ mm and $f_2 = 120$ mm respectively, and it expands the double refractions to compress the divergence to be less than 0.1 mrad, as well as makes the double refractions overlap each other to form polarized interferential fringes. Actually, the interference is duality due to path difference and polarization. There are two pieces of YVO_4 whose sizes are $10 \times 10 \times 15$ (mm) and $10 \times 10 \times 10$ (mm) respectively, and the YVO_4 crystals with optic axes oriented at $\pi/4$ and $\pi/3$ to the incident surface were tested respectively. The rectangular crystal is convenient for determining the azimuth of the optic axis. The measured YVO_4 lay on an adjustable deck which is precisely adjustable with micrometers. The deck was inserted into a pair of polarizors, P and A, whose polarized axes are orthogonal and at $\pi/4$ to the rims of the section of the crystal. P is for polarizing the incidence and A acts as the analyzer. For initial collimation the whole system was aimed at normal incidence as strictly as possible to get the "zero order" fringe. The fringes were monitored at a screen with grids. For exactly locating and accounting the fringes flashed on the screen as tilting the crystal, a photodiode was just put barely at the margin of the fringe.

The thickness d should be enough to separate the birefringence on the emerging surface in a measurable size and to have a remarkable $\Delta\delta$. In this letter we make $d = 10 \pm 0.001$ mm.

Before measuring the index of the e-ray, we have to determine the azimuth of the optic axis. For this, the system was somewhat detuned in order to observe high order fringes like a comb. Fixing the polarizors and rotating YVO_4, as a result, we observed that the row of the fringes is always perpendicular to the separation between the e-ray and the o-ray, which shows the azimuth of the axis of the crystal.

There are two following situations in measuring the index of the e-ray: at normal incidence and at oblique incidence.

For the situation at normal incidence, as a prerequisite of the experiments, we must measure the separation of the double refractions on the emerging surface of the crystal for further giving rise to the angle α of the e-ray separated to the o-ray. In these measurements the telephoto system TP was taken away, and a reading microscope was directly aimed at the emerging surface. The reading accuracy is in the order of magnitude of micrometer.

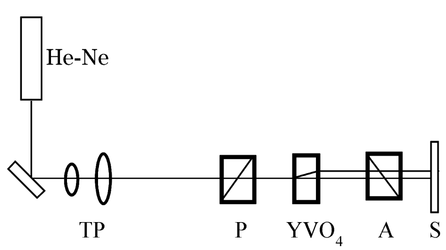

Fig. 1. Apparatus for measuring the index of the e-ray.

For getting a large variation of the path difference, we always install the crystal in such a way that the e-ray is above or beneath the o-ray. Relocating the telephoto and carefully collimating the system, we can observe the "zero order" fringes. Then slightly rotating the deck to make the crystal incline in the direction in consistence with the separation of the birefringence by carefully twisting the micrometer, at the same time we monitor the fringes in dark and bright alternatively. The inclination corresponds to deviation of the incidence from the normal. The varied periods of the fringes indicate the difference of $\Delta\delta$, and the corresponding rotated angle of the crystal equals the incidence i.

For the situation at oblique incidence[6,7], the spatial relations between the rays, the normal and the optic axis are more dazzling than those of normal incidence. It leads to complicated calculations.

Let the incidence incline to the crystal at $\theta_{\rm i}$ and give rise to the refractive angle $\theta_{\rm r}$ in horizontal plane. Looking into triangle $\Delta OEO'$ shown in Fig. 2, we inferred the path of the o-ray

$$OO' = \frac{d}{\cos\theta_{\rm r}}, \tag{6}$$

and the path of the e-ray

$$OE = \frac{d}{\cos\theta_{\rm r}}\left[\cos\alpha \pm \sqrt{\left(O'E\frac{\cos\alpha}{d}\right)^2 - \sin^2\alpha}\right]. \tag{7}$$

So the phase difference in this case

$$\begin{aligned}\delta &= \frac{2\pi}{\lambda}(n_{\rm o}OO' - n_{\rm e}OE) \\ &= \frac{2\pi d}{\lambda\cos\theta_{\rm r}} \\ &\quad \times \left\{n_{\rm o} - n_{\rm e}(\beta)\left[\cos\alpha \pm \sqrt{\left(O'E\frac{\cos\alpha}{d}\right)^2 - \sin^2\alpha}\right]\right\}.\end{aligned} \tag{8}$$

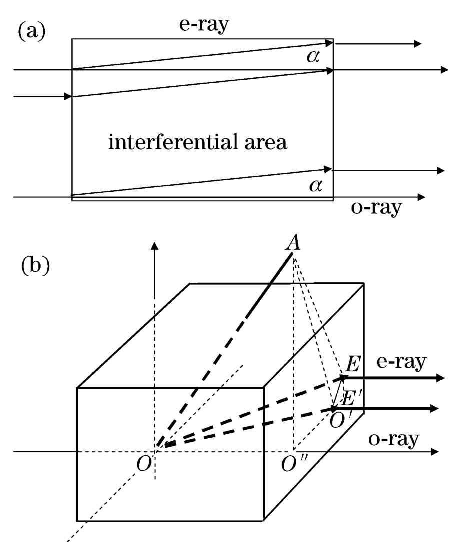

Fig. 2. Paths of the two rays in birefringence in uniaxial crystal. (a) At normal incidence; (b) at oblique incidence.

After the crystal is rotated at a tiny angle in direction vertical to the incident plane, the paths are $OE' = \frac{OE}{\cos r}$ for the e-ray and $OO'' = \frac{OO'}{\cos r}$ for the o-ray. The phase difference in this case

$$\delta' = \frac{2\pi d}{\lambda \cos r}\left[n_o OO' - n_e(\beta) OE\right]. \quad (9)$$

Expanding it, we have

$$\delta' = \frac{2\pi d}{\lambda \cos\theta_r \cos r} \times \left\{ n_o - n_e(\beta)\left[\cos\alpha \pm \sqrt{\left(O'E\frac{\cos\alpha}{d}\right)^2 - \sin^2\alpha}\right]\right\}. \quad (10)$$

Then the difference

$$\Delta\delta = \delta - \delta' = \frac{2\pi d}{\lambda\cos\theta_r} \times \left\{ n_o - n_e(\beta)\left[\cos\alpha \pm \sqrt{\left(O'E\frac{\cos\alpha}{d}\right)^2 - \sin^2\alpha}\right]\right\} \times \left(1 - \frac{1}{\cos r}\right). \quad (11)$$

Finally, the index of the e-ray

$$n_e(\beta) = \frac{n_o - \frac{\Delta\delta\lambda\cos\theta_r\cos r}{2\pi d(\cos r - 1)}}{\cos\alpha \pm \sqrt{\left(O'E\frac{\cos\alpha}{d}\right)^2 - \sin^2\alpha}}, \quad (12)$$

and the angle of the e-ray subtended to the optic axis

$$\cos\beta = \frac{2d^2 + O'E^2 - AE^2}{2\sqrt{2}d \times O'E}, \quad (13)$$

with

$$AE^2 = AO'^2 + EO'^2 - 2AO' \times EO' \cos\gamma, \quad (14)$$

$$AO' = OO' = \frac{d}{\cos r}, \quad (15)$$

$$\gamma = \pi - \sin^{-1}(\cos r) - \angle EO'E', \quad (16)$$

where $\angle EO'E'$ can be measured in experiments.

According to Eq. (3), δ would be very sensitive to d when d is thick enough. So as long as the crystal is slightly tilted, δ goes to experience a number of periods. Thus the main error comes from accurately locating the fringes. For example as rising $\Delta\delta = 10$ to $\Delta\delta = 10.1$, the measured index $n_e(\beta)$ would increase 3.0×10^{-3} (see Table 1 error analysis). We have ensured the location in the measurements as precisely as less than a tenth of an order.

Table 1. Error Analysis of the Measured $n_e(\beta)$

Factor	Variation	Error in $n_e(\beta)$
$\Delta\delta$	±0.1	$\pm3\times10^{-3}$
d	$\pm5\times10^{-3}$ mm	$\pm5\times10^{-4}$
α	±0.1 mrad	$\pm3\times10^{-3}$
r	±1 mrad	$\pm2\times10^{-3}$
β	±1 mrad	$\pm10^{-4}$

The error of α affects $n_e(\beta)$ relatively weak. As $n_e(\beta)$ varies from the minimum $n_o = 1.9929$ to the maximum $n_e = 2.2514$, β increases from 0 to $\pi/2$. Then β varies 1 mrad, $n_e(\beta)$ varies about 10^{-4} according to Eq. (1).

Above error analysis makes us reasonably believe that the accuracy achieved attains to the order of magnitude in 10^{-3}.

In this letter we have given the new equation to measure the index of the e-ray in uniaxial crystal with the axis at an arbitrary orientation. To assure the accuracy of the measurements, one should pay attention to the following key techniques.

1) The parallelism of the optical surfaces of the crystal should be less than $5''$, which assures error within $\lambda/10$. And the crystal should be arranged at normal incidence.

2) The crystal should be thick enough to assure a large separation of the birefringence on the emerging surface of the crystal, then the deviation angle and the phase difference could be accurately measured.

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