Real-time measurement of the fast axis angle of a quarter-wave plate based on simultaneous phase shifting technique

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Real-time measurement of the fast axis angle of a quarter-wave plate based on simultaneous phase shifting technique is presented. The simultaneous phase shifting function is realized by an orthogonal grating, a diaphragm, an analyzer array, and a 4-quadrant detector. The intensities of the light beams from the four analyzers with different azimuths are measured simultaneously. The fast axis angle of the quarter-wave plate is obtained through the four light intensity values. In this method, rotating elements are not required, so real-time measurement is achieved.

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The quarter-wave plate is one of the most important optical elements. It can change the light polarization state and is widely used in the field of polarization measurement[1−4]. Fast axis angle is the main parameter of the quarter-wave plate. In the measurements, the fast axis angle of the quarter-wave plate directly affects the measured results. But for many commercial quarter-wave plates, fast axes are not identified, so the determination of the quarter-wave fast axis is necessary before use. The reported methods for determining the fast axis include wave plate rotating method[5], right-angle prism method[6,7], Fresnel rhomb method[8], heterodyne interferometric method[9], phase modulation method[10,11] and so on. In these methods, the fast axis angle is achieved by rotating the quarter-wave plate to be measured, so these methods are not suitable to the real-time measurement. Simultaneous phase shifting technique based on a grating is applied in interferometry[12−14]. In this letter, real-time measurement of the fast axis angle based on simultaneous phase shifting technique is presented. The simultaneous phase shifting function is realized by an orthogonal grating, a diaphragm, an analyzer array and a 4-quadrant detector. The intensities of the light beams from the four analyzers with different azimuths are measured simultaneously. The fast axis angle is obtained through the four light intensity values.

The optical arrangement for real-time measuring the fast axis angle of the quarter-wave plate is illustrated in Fig. 1. The coordinate system is shown in Fig. 1. For convenience, the $+z$ axis is chosen to be along the light propagation direction, the x axis is parallel to the horizontal direction and the y axis is vertical to the x axis. The optical system is composed of a laser, a polarizer, a standard quarter-wave plate, the quarter-wave plate, an orthogonal grating, a diaphragm, an analyzer array, and a 4-quadrant detector. The azimuth of the polarizer is 45° ("0°" position is defined as the horizontal direction for the reference axis of the quarter-wave plate. The signs of the angles are positive for counter-clockwise rotation and negative for clockwise rotation.). The fast axis angle of the standard quarter-wave plate is 0°. The diffractive beams of 0th order and ($\pm m$, $\pm n$) orders are generated through the orthogonal grating, where m and n are positive integers. The diaphragm is put behind the grating. The diaphragm is composed of four pinholes, and the pinholes are arranged in square. The analyzer array is composed of four separate analyzers arranged in a 2×2 grid, the polarization axes of which are set to be 0°, 45°, 90° and 135°. A diaphragm allows only four diffracted beams of the fist order, having the same optical intensities, to pass. Each beam is incident on one of the analyzers. The intensities of the four beams are simultaneously detected by a quadrant detector.

The Stokes vector of the polarizer with the azimuth 45° is

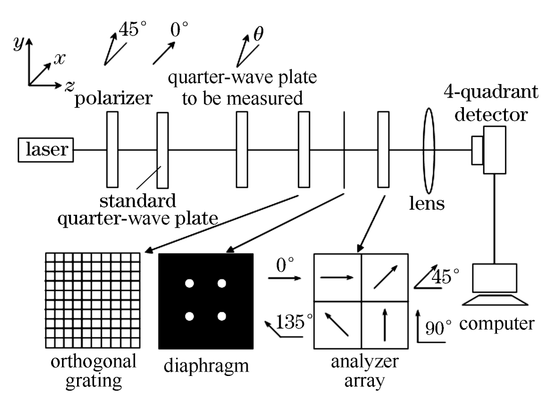

Fig. 1. Optical arrangement for measuring fast axis angle of the quarter-wave plate.

$$S = I_0 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad (1)$$

where I_0 is the intensity of the light from the polarizer. The Mueller matrix of the standard quarter-wave plate with the angle 0° of the fast axis is

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \quad (2)$$

The Mueller matrix of the quarter-wave plate with a retardation of δ and an angle θ of the fast axis is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \sin^2 2\theta \cos\delta & (1-\cos\delta)\sin 2\theta \cos 2\theta & -\sin 2\theta \sin\delta \\ 0 & (1-\cos\delta)\sin 2\theta \cos 2\theta & \sin^2 2\theta + \cos^2 2\theta \cos\delta & \cos 2\theta \sin\delta \\ 0 & \sin 2\theta \sin\delta & -\cos 2\theta \sin\delta & \cos\delta \end{pmatrix}. \quad (3)$$

For the diffractive beam of the first order, the Mueller matrix of the orthogonal grating is

$$G = \beta \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

where β is the diffraction efficiency of the first order beams. The Mueller matrix of the analyzer is

$$A = \frac{1}{2} \begin{pmatrix} 1 & \cos 2\alpha & \sin 2\alpha & 0 \\ \cos 2\alpha & \cos^2 2\alpha & \sin 2\alpha \cos 2\alpha & 0 \\ \sin 2\alpha & \sin 2\alpha \cos 2\alpha & \sin^2 2\alpha & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (5)$$

where α is the azimuth of the analyzer. The Stokes vector of the emerging beam from the analyzer is

$$S' = AGMBS = \frac{\beta I_0}{2} \times \begin{pmatrix} 1+\cos 2\alpha \sin 2\theta \sin\delta - \sin 2\alpha \cos 2\theta \sin\delta \\ \cos 2\alpha + \cos^2 2\alpha \sin 2\theta \sin\delta - 0.5\sin 4\alpha \cos 2\theta \sin\delta \\ \sin 2\alpha + 0.5\sin 4\alpha \sin 2\theta \sin\delta - \sin^2 2\alpha \cos 2\theta \sin\delta \\ 0 \end{pmatrix}. \quad (6)$$

Thus, the light intensity signal reaching the detector is

$$I = \frac{\beta I_0}{2}(1+\cos 2\alpha \sin 2\theta \sin\delta - \sin 2\alpha \cos 2\theta \sin\delta). \quad (7)$$

Four intensity values measured at $\alpha = 0°$, 45°, 90° and 135° can be proved as

$$I(0°) = \frac{\beta I_0}{2}[1+\sin 2\theta \sin\delta], \quad (8)$$

$$I(45°) = \frac{\beta I_0}{2}[1-\cos 2\theta \sin\delta], \quad (9)$$

$$I(90°) = \frac{\beta I_0}{2}[1-\sin 2\theta \sin\delta], \quad (10)$$

$$I(135°) = \frac{\beta I_0}{2}[1+\cos 2\theta \sin\delta]. \quad (11)$$

According to Eqs. (8)—(11), we can obtain

$$k_1 = \frac{I(0°) - I(90°)}{I(0°) + I(90°)} = \sin 2\theta \sin\delta, \quad (12)$$

$$k_2 = \frac{I(135°) - I(45°)}{I(135°) + I(45°)} = \cos 2\theta \sin\delta. \quad (13)$$

Using $\sin^2 2\theta + \cos^2 2\theta = 1$, we can obtain

$$\sin 2\theta = \frac{k_1}{\sqrt{k_1^2 + k_2^2}}, \quad (14)$$

$$\cos 2\theta = \frac{k_2}{\sqrt{k_1^2 + k_2^2}}. \quad (15)$$

The fast axis angle θ has a range from 0° to 180°. So θ can be obtained according to Eqs. (14) and (15).

In our experiment, the light source was a He-Ne laser, and the laser wavelength was 632.8 nm. The polarizer was a Glan-Tayor prism with an extinction ratio greater than 10^5:1. The analyzers in the analyzer array were manufactured using the same polaroid. The standard quarter-wave plate and the quarter-wave plate were all fused silica wave plate. The groove density of the grating was 200 lines/mm. The minimum reading of rotation stage was 0.2°, and the resolution was 0.002°. The 4-quadrant detector was Si Pin detector manufactured by Chongqing Optoelectronic Research Institute, China. The data acquisition card was 4-channel NuDAQ-9812, and the analog input resolution was 12 bit.

In our experiment, the calibrations included the following steps:

1) The intensities of the four beams from the analyzer array are made equal through calibration procedure. In this procedure, the polarizer, the standard quarter-wave plate, and the quarter-wave plate were removed.

2) The polarizer was positioned according to the Fig. 1. The azimuth of the polarizer was set to be 45° and the analyzer array was set to be 0° in a straight-through arrangement through null method. So the azimuths of the four units of the analyzer array were 0°, 45°, 90°, and 135°, respectively.

3) The standard quarter-wave plate was positioned according to the Fig. 1, and its azimuth was set to be 0°.

The fast axis angles were measured when the quarter-wave plate was rotated at 15° intervals from 0° to 180°. The measured results are shown in Fig. 2. According to Fig. 2, relation between the fast axis angle and the reference axis angle is linear. This is according with our expectation. Thus the feasibility of the method is verified by the experiments.

The intensity values of the four channels were measured every three minutes at the same fast axis angle. The results of ten consecutive measurements at the same position of the quarter-wave plate are shown in Table 1.

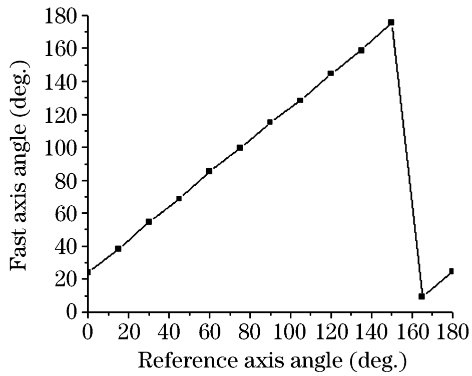

Fig. 2. Measured results with the quarter-wave plate rotated.

Table 1. Repeated Measurements at the Same Fast Axis Angle

Number of Measurement	1	2	3	4	5
Fast Axis Angle (deg.)	24.21	23.84	23.89	24.15	24.23
Number of Measurement	6	7	8	9	10
Fast Axis Angle (deg.)	23.96	24.08	24.31	24.18	24.06

The average and standard deviation for the ten measurements are 24.09° and 0.15°, respectively.

The main error sources in the method includes: 1) Retardation error of the standard quarter-wave plate; 2) Fast axis angle error of the standard quarter-wave plate; 3) Azimuth errors of four analyzers in the analyzer array.

Assuming that the retardation of the standard quarter-wave plate is δ_1, the Mueller matrix of the standard quarter-wave plate with the angle of the fast axis at 0° is

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta_1 & \sin\delta_1 \\ 0 & 0 & -\sin\delta_1 & \cos\delta_1 \end{pmatrix}. \tag{16}$$

The detected intensity is obtained through the similar analysis method in the measurement principle part. We obtain

$$\begin{aligned} I = \frac{\beta I_0}{2}\{1 &+ [0.5\cos 2\alpha \sin 4\theta(1-\cos\delta) \\ &+ \sin 2\alpha(\sin^2 2\theta + \cos\delta\cos^2 2\theta)]\cos\delta_1 \\ &+ (\cos 2\alpha \sin 2\theta \sin\delta - \sin 2\alpha \cos 2\theta \sin\delta)\sin\delta_1\}. \end{aligned} \tag{17}$$

According to error analysis theory, the resulting fast axis error is

$$\begin{aligned} \Delta\theta = [&-0.25\frac{\cos 2\theta \sin 4\theta(1-\cos\delta)}{\sin\delta} \\ &-0.5\frac{\sin^3 2\theta - \cos\delta\cos^2 2\theta \sin 2\theta}{\sin\delta}]\Delta\delta_1. \end{aligned} \tag{18}$$

If the retardation of the quarter-wave plate is 90° and the retardation error of the standard quarter-wave plate is less than 1°, the resulting fast axis angle error is less than 0.5° according to Eq. (18).

If the fast axis angle of the standard quarter-wave plate is not 0° precisely, assuming the fast axis angle is θ_1, the Mueller matrix of the standard quarter-wave plate is

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta_1 & 0.5\sin 4\theta_1 & -\sin 2\theta_1 \\ 0 & 0.5\sin 4\theta_1 & \sin^2 2\theta_1 & \cos 2\theta_1 \\ 0 & \sin 2\theta_1 & -\cos 2\theta_1 & 0 \end{pmatrix}. \tag{19}$$

The detected intensity can be expressed as

$$\begin{aligned} I = \frac{\beta I_0}{2}\{&1 + 0.5[\cos 2\alpha(\cos^2 2\theta + \cos\delta\sin^2 2\theta) \\ &+0.5\sin 2\alpha \sin 4\theta(1-\cos\delta)]\sin 4\theta_1 \\ &+[0.5\cos 2\alpha \sin 4\theta(1-\cos\delta) \\ &+\sin 2\alpha(\sin^2 2\theta + \cos\delta\cos^2 2\theta)]\sin^2 2\theta_1 \\ &+(\cos 2\alpha \sin 2\theta \sin\delta - \sin 2\alpha \cos 2\theta \sin\delta)\cos 2\theta_1\}. \end{aligned} \tag{20}$$

According to error analysis theory, the resulting fast axis error is

$$\begin{aligned} \Delta\theta = [&\frac{\cos 2\theta(\cos^2 2\theta + \cos\delta\sin^2 2\theta)}{\sin\delta} \\ &+\frac{\sin^2 2\theta \cos 2\theta(1-\cos\delta)}{\sin\delta}]\Delta\theta_1. \end{aligned} \tag{21}$$

If the retardation of the quarter-wave plate is 90° and the fast axis angle error of the standard quarter-wave plate is less than 0.2° (minimum reading of rotation stage is 0.2°), the resulting fast axis angle error is less than 0.2° according to Eq. (21).

If the azimuth errors of four analyzers in the analyzer array are ΔA_1, ΔA_2, ΔA_3, and ΔA_4, respectively, the resulting fast axis angle error of the quarter-wave plate is obtained from Eqs. (7)—(15).

$$\begin{aligned} \Delta\theta = &0.5(-\cos^2 2\theta + \cos^2 2\theta \sin 2\theta \sin\delta)\Delta A_1 \\ &+0.5(-\cos^2 2\theta - \cos^2 2\theta \sin 2\theta \sin\delta)\Delta A_3 \\ &-0.5(\sin^2 2\theta + \sin^2 2\theta \cos 2\theta \sin\delta)\Delta A_2 \\ &-0.5(\sin^2 2\theta - \sin^2 2\theta \cos 2\theta \sin\delta)\Delta A_4. \end{aligned} \tag{22}$$

The azimuth errors of four analyzers are less than 0.2° through null method. The quarter-wave plate to be measured and the standard quarter-wave plate were removed. Assuming that the retardation of the quarter-wave plate is 90°, the maximum fast axis angle errors of the quarter-wave plate due to ΔA_1, ΔA_2, ΔA_3, and ΔA_4 are all 0.12°.

According to Eqs. (17), (21) and (22), We can see that the measurement error $\Delta\theta$ is related to the fast axis angle θ. Compared with the phase modulation method, the measurement precision is relatively low, but real-time measurement is achieved. If we do not carry out real-time measurement, and choose a fast axis angle, the measurement precision can be reached.

In summary, the real-time measurement of the fast axis angle of the quarter-wave plate based on simultaneous phase shifting technique is presented. The simultaneous phase shifting function is realized by an orthogonal grating, a diaphragm, an analyzer array and a 4-quadrant detector. In this method, the wave plate is not required to be rotated. In addition, the measured result is free of intensity fluctuation of light source.

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