General optical scintillation in turbulent atmosphere

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A general expression of the scintillation index is proposed for optical wave propagating in turbulent atmosphere under arbitrary fluctuation conditions. The expression depends on extreme behaviors of the scintillation indices under both weak and strong fluctuations. The maximum scintillation index in the onset region and the corresponding Rytov index can be evaluated from the general expression. Plane and spherical waves in the cases of zero and non-zero turbulence inner scale are given as examples for illustration of the general behaviors of scintillation indices.

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Scintillation is the main effect of light propagation in turbulent atmosphere[1]. Efforts of theoretical analysis, experimental studies, and numerical simulations have been carried out on the problem. However, the scintillation theory is established satisfactorily only in the weak fluctuation region[2]. Some analytic asymptotic formulations were also obtained for the scintillation index in the strong fluctuation region[3,4]. In many practical applications, the light propagation condition is in the onset region, also called the strong focusing region, from weak to strong fluctuation. In this region the light intensity fluctuates in an extreme extent, and it is very difficult to make any approximation for theoretical analysis. There is a practical difficulty on experiments under strong fluctuation condition, and only limited experiment events present valuable results for the public reference[5]. Some numerical simulations were done and quantitative results were obtained for several values of the Fresnel size and the turbulence inner size[6−8]. An explicit expression is necessary for the scintillation index in the onset region in many applications. Andrews *et al.* proposed a scintillation theory, usually referred to as the modified Rytov approximation, based on heuristic reasoning[9,10]. The expression of scintillation index in the theory is rather complicated, especially in the case of a finite inner or outer scale of turbulence, and thus cannot be easily used.

We note that main contributions to the scintillation under weak fluctuation damp out when the fluctuation becomes stronger, and main contributions to the scintillation under strong fluctuation damp out when the fluctuation becomes weaker. Thus an assumption is made: the general expression of the scintillation index β_I^2 under arbitrary fluctuation condition depends on its behaviors β_0^2, β_∞^2 under both weak and strong fluctuation conditions through a role of reciprocal addition. That is,

$$1/\beta_I^2 = 1/\beta_0^2 + 1/\beta_\infty^2. \quad (1)$$

Thus we get a much simplified general expression for the scintillation index under arbitrary fluctuation condition,

$$\beta_I^2 = \beta_0^2\beta_\infty^2/(\beta_0^2 + \beta_\infty^2). \quad (2)$$

This general expression coincides with the Rytov approximation result and the asymptotic formula under weak and strong fluctuation conditions respectively, and has a maximum in the strong focusing region. It generally represents the characteristics of the scintillation behavior.

The scintillation index under weak fluctuation condition is well established by Rytov approximation as $\beta_I^2 = \beta_0^2$ ($\beta_0^2 \ll 1$), where the Rytov index $\beta_0^2 = \alpha C_n^2 k^{7/6} L^{11/6}$, $\alpha = 1.23$ for plane wave, and 0.5 for spherical wave. If we know the correct behavior of the scintillation index in the strong fluctuation region, we could built the general form in the whole fluctuation region. Current research results indicate that the scintillation index $\beta_I^2 = \beta_\infty^2$ ($\beta_0^2 \gg 1$) under strong fluctuation condition presents a behavior as a function of the Rytov index[4,8],

$$\beta_\infty^2 = 1 + a(\beta_0^2)^{-b}. \quad (3)$$

Therefore we have the general scintillation index as a function of the Rytov index,

$$\beta_I^2 = \frac{a\beta_0^2 + \left(\beta_0^2\right)^{1+b}}{a + \left(\beta_0^2\right)^b + \left(\beta_0^2\right)^{1+b}}. \quad (4)$$

From the general expression of scintillation index, we can find out the Rytov index at which the scintillation index reaches its maximum. Letting the derivative of Eq. (4) be zero, we can get

$$\left(\beta_0^2\right)^{2b} - ab\left(\beta_0^2\right)^{b+1} + 2a\left(\beta_0^2\right)^b + a^2 = 0. \quad (5)$$

It can be further transformed as

$$\left(\beta_0^2\right)^b - \sqrt{ab}\left(\beta_0^2\right)^{(b+1)/2} + a = 0. \quad (6)$$

The numerical solution of this nonlinear equation can be obtained. Through the solution, the relation between the maximum scintillation index (or the related Rytov index) and constants a, b could be built.

Below some examples are used for illustration. The coefficients of an analytic asymptotic formula[9] and those of a numerical result with non-zero inner scale[8] for a plane wave and a spherical wave are listed in Table 1. Corresponding general scintillation indices as functions of the Rytov index are plotted in Figs. 1 and 2, respectively. For the non-zero inner scale case, scintillation indices with five values 0.2, 0.4, 0.6, 0.8, and 1.0 for $l_0/l_{\rm Fr}$

Table 1. Coefficients of Scintillation Index Formula in the Strong Fluctuation Region

l_0/l_{Fr}	Plane Wave		Spherical Wave	
	a	b	a	b
Zero	0.86	0.4	1.9	0.4
Non-Zero	$2.2085(l_0/l_{\mathrm{Fr}})^{0.45}$	0.33	$20.923(l_0/l_{\mathrm{Fr}})^{0.8}$	0.45

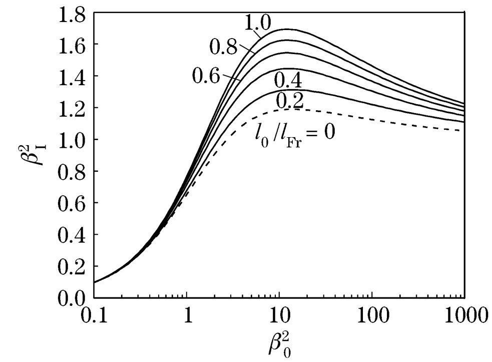

Fig. 1. General scintillation index of a plane wave as a function of Rytov index for various values of l_0/l_{Fr}.

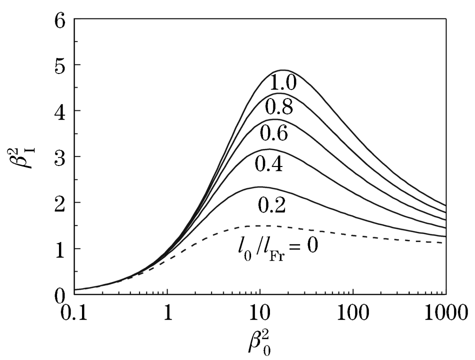

Fig. 2. General scintillation index of a spherical wave as a function of Rytov index for various values of l_0/l_{Fr}.

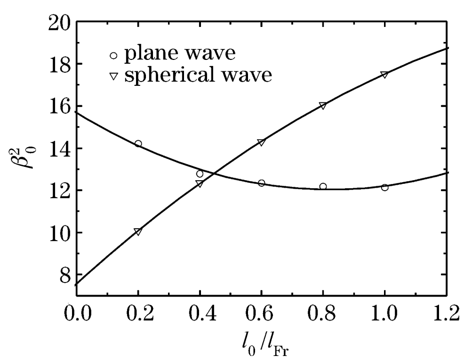

Fig. 3. Rytov index corresponding to the maximum scintillation index as a function of l_0/l_{Fr} for a plane wave and a spherical wave.

are shown, where $l_{\mathrm{Fr}} = \sqrt{L/k}$ is the Fresnel scale, k is the wavenumber, and L is the propagation length.

In the non-zero inner scale cases listed in Table 1, the constant a depends on l_0/l_{Fr}. Therefore we can determine the relations between the maximum scintillation index and related Rytov index and l_0/l_{Fr}. Results for five

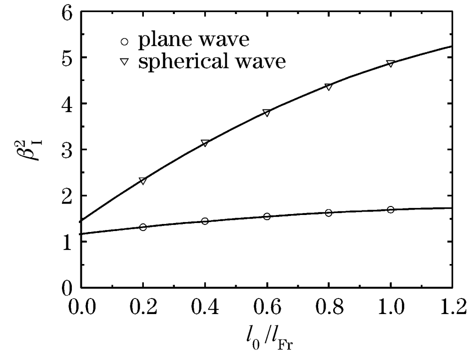

Fig. 4. Maximum scintillation index as a function of l_0/l_{Fr} for a plane wave and a spherical wave.

values of l_0/l_{Fr} from 0.2 to 1 are plotted in Figs. 3 and 4. As for the Rytov index, the behaviors of a plane wave and a spherical wave are quite different. For a plane wave, the Rytov index decreases not very significantly from 14 to 12 with l_0/l_{Fr} increasing from 0.2 to 1. For a spherical wave, the Rytov index position increases significantly from 10 to 17.5 with the same l_0/l_{Fr} increasing. Polynomial fittings of the results are as follows:

$$\beta_0^2(\mathrm{pl}) = 15.6732 - 8.8907\,(l_0/l_{\mathrm{Fr}}) + 5.4223\,(l_0/l_{\mathrm{Fr}})^2\,, \quad (7)$$

$$\beta_0^2(\mathrm{sp}) = 7.5676 + 13.1936\,(l_0/l_{\mathrm{Fr}}) - 3.2464\,(l_0/l_{\mathrm{Fr}})^2\,. \quad (8)$$

The behaviors of the maximum scintillation indices of a plane wave and a spherical wave are also quite different. The maximum scintillation index of a plane wave increases very slowly with l_0/l_{Fr}, while that of a spherical wave increases significantly from about 2 to 5 with l_0/l_{Fr} increasing from 0.2 to 1. Polynomials fitting of the results are as follows:

$$\beta_I^2(\mathrm{pl}) = 1.1677 + 0.7885\,(l_0/l_{\mathrm{Fr}}) - 0.2643\,(l_0/l_{\mathrm{Fr}})^2\,, \quad (9)$$

$$\beta_I^2(\mathrm{sp}) = 1.4575 + 4.7103\,(l_0/l_{\mathrm{Fr}}) - 1.2957\,(l_0/l_{\mathrm{Fr}})^2\,. \quad (10)$$

In summary, the general expression of the scintillation index proposed here depends only on the Rytov index and two constants which depict the asymptotic behavior under strong fluctuation, and it is thus simple in formulation. The predicted maximum scintillation index in the strong focusing region is consistent with experimental and numerical simulation results. It is expected that this expression could be applied in many applications, such as light propagation studies[11,12].

It should be noted that the general expression of the scintillation index could be applied for any propagation condition. In the given examples, only the impact of the turbulence inner scale and the Fresnel size on the scintillation are shown. Similarly, the impact of the turbulence outer scale on scintillation could also be reflected through its impact on the behavior under strong fluctuation[10].

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