

# Control of soliton interactions by use of super-Gaussian sliding-frequency filters

Yingji He (何影记)<sup>1,2</sup>, Jinping Tian (田晋平)<sup>3</sup>, Guosheng Zhou (周国生)<sup>3</sup>,  
Wenrui Xue (薛文瑞)<sup>3</sup>, Yan Xiao (肖燕)<sup>3</sup>, and Hezhou Wang (汪河洲)<sup>2</sup>

<sup>1</sup>*School of Electronics and Information, Guangdong Polytechnic Normal University, Guangzhou 510665*

<sup>2</sup>*State Key Laboratory of Optoelectronic Materials and Technologies, Sun Yat-Sen University, Guangzhou 510275*

<sup>3</sup>*Department of Electronics and Information Technology and State Key Subjects of Optics, Shanxi University, Taiyuan 030006*

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We derived the theoretical results of soliton interactions in optical fiber with super-Gaussian sliding-frequency filters. The results demonstrate that the interactions between optical fiber solitons can be effectively suppressed by super-Gaussian sliding-frequency filters. And the results also show that the super-Gaussian filter with sliding is more effective in suppressing soliton interactions than that without sliding.

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One of the main limitations of soliton-based fiber telecommunications is the interaction between adjacent pulses. Thus, many researchers have made their efforts to reduce soliton interaction. For example, it was found that a temporal synchronous active modulation can effectively overcome the interactions of solitons<sup>[1]</sup>. Optical fibers with random varying birefringence were suggested to cancel the interaction of solitons<sup>[2]</sup>. In photonic crystal fibers, the interaction between solitons with a dispersive wave was analyzed experimentally<sup>[3]</sup>. In addition, by use of modified group-velocity dispersion, the interaction of solitons can be changed and overcome<sup>[4]</sup>. Recently, the interactions of chirped and chirp-free similaritons have been researched<sup>[5]</sup>.

The use of Gaussian filters has been demonstrated to be an effective method to control soliton interactions<sup>[6]</sup>. In particular, the filters with the center frequency slid along the transmission line can greatly suppress soliton interactions<sup>[7]</sup>. The sliding-frequency filters are also effective in reducing soliton timing jitter in overcoming the self-frequency shift<sup>[7-9]</sup>. The mechanism is that soliton follows the filter sliding while linear narrow-band noise does not<sup>[10]</sup>. Hence the filter can create a transmission line that is opaque to noise and transparent to soliton<sup>[10-13]</sup>. However, the low-order filter (e.g., Gaussian filter) has the drawback that its distributed transfer function contains higher-order dispersion terms that lead to a strong asymmetry in the sideband spectrum and an increase in temporal jitter.

When the order of the filter increases, the damping coefficient of system decreases, so that an excess gain is smaller, which is beneficial to stabilize the system<sup>[14]</sup>. Therefore, higher-order filter (e.g., Butterworth filter) was proposed to control soliton propagation<sup>[14]</sup>. However, Butterworth filters introduce additional losses that negatively affect the soliton stability. Another higher-order filter is super-Gaussian filter which can produce dramatic power enhancement of optical solitons<sup>[15]</sup>. This allows smaller timing jitter without sacrificing the signal-

to-noise ratio (SNR)<sup>[15,16]</sup>. Recently, it has also shown that super-Gaussian filters can reduce timing jitter and phase jitter more effectively than the Gaussian filters do<sup>[16,17]</sup>. Super-Gaussian filters can be implemented with holographic fiber gratings<sup>[18]</sup> and their design is performed by means of the inverse scattering technique<sup>[19,20]</sup>. The transfer function of super-Gaussian filters in the form of holographic fiber grating does not introduce any phase distortion and optimizes the jitter reduction<sup>[18]</sup>. Thus, super-Gaussian filters are better candidates for soliton transmission control than the previous filters.

In this letter, by means of two-soliton perturbation theory, we obtain the results of controlling soliton interactions by use of super-Gaussian sliding-frequency filters (SGSFs). The results show that the interactions between solitons can be effectively controlled by SGSFs. It is found that super-Gaussian filter with sliding is more effective in suppressing soliton interactions than that without sliding. The numerical examples confirming the analytical results are given.

In standard soliton units the simplified propagation equation in the presence of Gaussian and super-Gaussian sliding-frequency filters is<sup>[21]</sup>

$$\frac{\partial u}{\partial z} = \frac{i}{2} \frac{\partial^2 u}{\partial t^2} + i|u|^2 u + \frac{1}{2}[\alpha - \eta(i\frac{\partial}{\partial t} - \omega_f)^{2n}]u, \quad (1)$$

where  $n$ , a positive integer, determines the order of the filter,  $t$  is the time,  $\alpha$ ,  $\eta$  and  $\omega_f$  are the excess gain of the amplifiers, filter strength, and filter peak frequency, respectively. For  $n \geq 2$ , the filter is super-Gaussian, and the sliding frequency rate  $\omega'_f = d\omega_f/dz$ .

Set

$$u(t, z) = v(t + \omega'_f z^2/2, z) \times \exp(-i\omega'_f zt - i\omega'^2_f z^3/3), \quad (2)$$

where  $v$  obeys

$$\frac{\partial v}{\partial z} = \frac{i}{2} \frac{\partial^2 v}{\partial \tau^2} + i|v|^2 v + \frac{1}{2}[\alpha - \eta(i\frac{\partial}{\partial \tau})^{2n}]v + i\omega'_f \tau v, \quad (3)$$

where  $\tau = t + \omega'_f z^2/2$ . In Eq. (3) one can separate the effect of the sliding from that of the filter. For  $n = 2$  (here we consider only  $n = 2$ ), the usual ansatz for the soliton  $v = A \operatorname{sech}(A\tau - q) \exp(-i\Omega\tau + i\sigma)$  is introduced into Eq. (3), and a pair of couple equations for the soliton amplitude  $A$  and frequency  $\Omega$  is obtained. The results are

$$\frac{dA}{dz} = \alpha A - \eta \left( \frac{7A^4}{15} + 2A^2\Omega^2 + \Omega^4 \right) A, \quad (4a)$$

$$\frac{d\Omega}{dz} = \omega'_f - \frac{4}{15} \eta (7A^2 + 5\Omega^2) \Omega A^2. \quad (4b)$$

To have  $A = 1$  (equilibrium point at  $A = 1$ ), from Eq. (4), we require that

$$\alpha = \eta \left( \frac{7}{15} + 2\Omega^2 + \Omega^4 \right), \quad (5a)$$

$$\omega'_f = \frac{4}{15} \eta (7 + 5\Omega^2) \Omega. \quad (5b)$$

By means of two-soliton perturbation theory, we consider the two pulses in Eq. (3):

$$v_j(z, \tau) = A_j \operatorname{sech}[A_j(\tau - q_j)] \exp[-i\Omega_j(\tau - q_j) + i\sigma_j], \quad (6)$$

$$j = 1, 2.$$

By combining Eq. (4) with the results of Ref. [22], we obtain

$$\frac{dA}{dz} = \alpha A - \eta \left( \frac{7A^4}{15} + 2A^2\Omega^2 + \Omega^4 \right) A, \quad (7a)$$

$$\frac{d\Omega}{dz} = \omega'_f - \frac{4}{15} \eta (7A^2 + 5\Omega^2) \Omega A^2, \quad (7b)$$

$$\frac{dp}{dz} = 4A^3 \exp(-A\Delta) \sin \delta + p \left[ \alpha - \eta \left( \frac{7A^4}{3} + 6A^2\Omega^2 + \Omega^4 \right) \right] - 4\eta A \Omega k (A^2 + \Omega^2), \quad (7c)$$

$$\frac{dk}{dz} = 4A^3 \exp(-A\Delta) \cos \delta - \frac{4}{15} \eta A \Omega p (28A^2 + 10\Omega^2) - \frac{4}{15} \eta A^2 k (7A^2 + 15\Omega^2), \quad (7d)$$

$$\frac{d\Delta}{dz} = -2k, \quad (7e)$$

$$\frac{d\delta}{dz} = 2Ap - \frac{4}{15} \eta (7A^2 + 5\Omega^2) \Omega A^2 \Delta, \quad (7f)$$

where  $A = (A_1 + A_2)/2$ ,  $\Omega = (\Omega_1 + \Omega_2)/2$ .  $p = (A_1 - A_2)/2$  is the amplitude difference,  $k = (\Omega_1 - \Omega_2)/2$  is the frequency difference,  $\Delta = q_1 - q_2 > 0$  is the pulse separation, and  $\delta = \Delta\Omega + \sigma$  is the phase difference,  $\sigma = \sigma_1 - \sigma_2$ .

By setting  $\eta = 0.15$  and  $\omega'_f = 0.05^{[7,11]}$ , one obtains  $\Omega_a = 0.18$  and  $\alpha = 0.08$  from Eq. (5) for a fixed point of the one-soliton solution with  $A_a = 1$ . Figure 1 shows examples of solutions of Eq. (7) with the same initial values of  $\Omega = p = k = \sigma = 0$ ,  $A = 1$ , and different initial pulse separations  $\Delta_0 = 7.6$  (solid curve),  $\Delta_0 = 6$  (dash

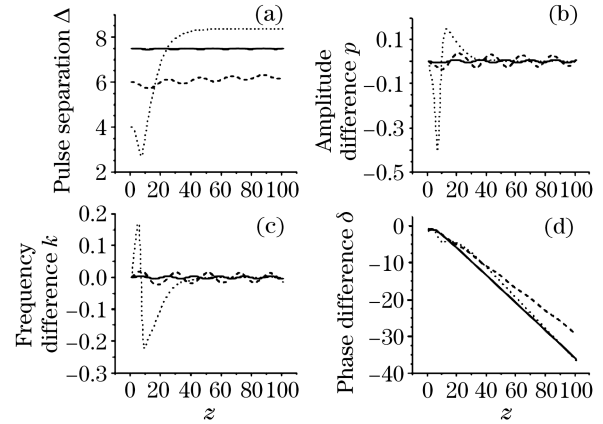


Fig. 1. Perturbative evolution of (a) two-soliton separation  $\Delta$ , (b) amplitude difference  $p$ , (c) frequency difference  $k$ , and (d) phase difference  $\delta$ .  $\Delta_0 = 7.6$  (solid curve),  $\Delta_0 = 6$  (dashed curve) and  $\Delta_0 = 4$  (dotted curve).

curve) and  $\Delta_0 = 4$  (dotted curve) at  $z = 0$ , in the distance range of  $z = 0 - 100$ . As can be seen from Fig. 1(a), the pulse separation  $\Delta$  remains almost constant for  $\Delta_0 = 7.6$ . But for the input pulse separation smaller than a certain critical value (here  $\Delta_0 = 4$ ), the initial interaction is too strong, and the pulse separation settles into the equilibrium value  $\Delta \approx 8.3$ . In fact, Figs. 1(b) and (c) show that the amplitude differences  $p$  and the frequency difference  $k$  rapidly oscillate around zero and the phase difference  $\delta$  changes linearly with  $z$  [see Fig. 1(d)], which further provides a demonstration to the control of soliton interactions as shown in Fig. 1(a). Since the two-soliton interaction is attractive for in-phase soliton and repulsive for  $\pi$  out-of-phase soliton, the rapid linear change of  $\delta$  in Fig. 1(d) averages the soliton interaction to zero. Thus, the soliton interaction is effectively suppressed.

To further confirm the above theoretical results, we perform the numerical simulations for initial separations  $\Delta_0 = 7.6$  and 4. We consider the initial condition

$$u(t, z = 0) = \omega_f [\operatorname{sech}(t - \Delta_0/2) + \operatorname{sech}(t + \Delta_0/2)]. \quad (8)$$

By setting the typical value of other parameters as<sup>[8]</sup>:  $\lambda = 1.55 \mu\text{m}$ ,  $D = -1 \text{ ps}/(\text{km}\cdot\text{nm})$ ,  $1.763T_0 = 20 \text{ ps}$ , the amplifier spacing  $z_a = 50 \text{ km}$ , the loss is  $0.2 \text{ dB}/\text{km}$ , we get the dispersion length  $L_D = 201 \text{ km}$ . Figure 2 shows the evolutions of two-soliton interaction with SGSFs for  $\Delta_0 = 7.6$  and  $\Delta_0 = 4$ . From Fig. 2, we find that the theoretical results based on Eq. (7) shown in Fig. 1(a) are well confirmed by the numerical simulations displayed in Figs. 2(a) and (b), respectively for initial separations  $\Delta_0 = 7.6$  and 4.

Finally we also give the evolution of two-pulse interaction for  $\Delta_0 = 7.6$  with super-Gaussian filters without sliding (i.e.,  $\omega'_f = 0$ ). The parameters are  $\eta = 0.15$ ,  $\omega'_f = 0$  and  $\alpha = 0.07$ . Figure 3(a) shows the evolution of two-soliton interaction without sliding for  $\Delta_0 = 7.6$  basing on Eq. (7), which is in agreement with the simulation shown in Fig. 3(b). From Fig. 3, we realize that the collision distance is approximately at  $z = 80$  for  $\Delta_0 = 7.6$  with super-Gaussian filter without sliding. This shows super-Gaussian filter with sliding can suppress soliton interactions more effectively than that without sliding.

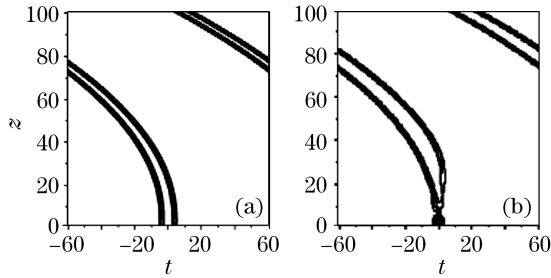


Fig. 2. Two-soliton interaction with SGSFs for (a)  $\Delta_0 = 7.6$  and (b)  $\Delta_0 = 4$ .

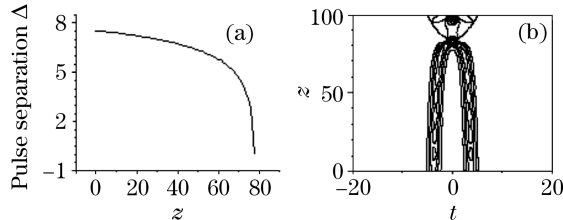


Fig. 3. Two-soliton interaction without sliding for  $\Delta_0 = 7.6$ . (a) Evolution of two-soliton interaction; (b) contour plot of computer simulation corresponding to (a). The parameters are  $\eta = 0.15$ ,  $\omega'_f = 0$ , and  $\alpha = 0.07$ .

In conclusion, we demonstrate that SGSFs can fully suppress the interactions between solitons in optical fiber. By comparing with the case of without sliding, we find that the super-Gaussian filter with sliding is more effective in suppressing soliton interactions. Since super-Gaussian filter has more merits than Gaussian filters as mentioned above, this method of controlling soliton interactions would be more promising, and permit an all-fiber system that is perfectly compatible with wavelength-division multiplexing (WDM). Our results are also expected to be applied in other types of solitons<sup>[23,24]</sup>.

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## References

1. M. Nakazawa, E. Yamada, H. Kubota, and E. Suzuki, *Electron. Lett.* **27**, 1270 (1991).
2. Y. Chung, V. V. Lebedev, and S. S. Vergeles, *Opt. Lett.* **29**, 1245 (2004).
3. A. Efimov, A. V. Yulin, D. V. Skryabin, J. C. Knight, N. Joly, F. G. Omenetto, A. J. Taylor, and P. Russell, *Phys. Rev. Lett.* **95**, 213902 (2005).
4. K. T. Stoychev, M. T. Primatarowa, and R. S. Kam-burova, *Phys. Rev. E* **73**, 066611 (2006).
5. S. A. Ponomarenko and G. P. Agrawal, *Opt. Express* **15**, 2963 (2007).
6. Y. Kodama and S. Wabnitz, *Opt. Lett.* **18**, 1311 (1993).
7. Y. Kodama and S. Wabnitz, *Opt. Lett.* **19**, 162 (1994).
8. A. Mecozzi, J. D. Moores, H. A. Haus, and Y. Lai, *Opt. Lett.* **16**, 1841 (1991).
9. P. T. Dinda, K. Nakkeeran, and A. Labruyère, *Opt. Lett.* **27**, 382 (2002).
10. A. Mecozzi, M. Midrio, and M. Romagnoli, *Opt. Lett.* **21**, 402 (1996).
11. L. F. Mollenauer, J. P. Gordon, and S. G. Evangelides, *Opt. Lett.* **17**, 1575 (1992).
12. A. Mecozzi, *J. Opt. Soc. Am. B* **10**, 2321 (1993).
13. M. F. S. Ferreira and S. C. V. Latas, *J. Lightwave Technol.* **19**, 332 (2001).
14. A. Mecozzi, *Opt. Lett.* **20**, 1859 (1995).
15. B. P. Luce, *Opt. Lett.* **23**, 765 (1998).
16. Y. J. He, G. S. Zhou, X. J. Shi, G. Wang, and W. R. Xue, *Opt. Eng.* **43**, 489 (2004).
17. Y. J. He and H. Z. Wang, *Opt. Commun.* **250**, 36 (2005).
18. E. Peral, J. Capmany, and J. Marti, *Opt. Lett.* **21**, 1894 (1996).
19. Y. Kodama and A. Hasegawa, *Opt. Lett.* **17**, 31 (1992).
20. E. Peral, J. Capmany, and J. Marti, *Electron. Lett.* **32**, 918 (1996).
21. P. V. Mamyshev and L. F. Mollenauer, *Opt. Lett.* **19**, 2083 (1994).
22. V. I. Karpman and V. V. Solovov, *Physica D* **3**, 487 (1981).
23. H. Li and D. Wang, *Chin. Opt. Lett.* **5**, 504 (2007).
24. Z. Fang, B. Shen, and Z. Zhang, *Chin. Opt. Lett.* **4**, 532 (2006).