Enhanced spontaneous emission factor for microcavity lasers

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The microcavity and the influence of nonradiative recombination can control spontaneous emission. An analytic resolution of rate equation is studied for microcavity lasers. The relationship between output properties and structural parameters of multi-quantum wells (MQWs) is obtained. One of the most important consequences of the increased spontaneous emission factor is the reduction of laser threshold. It is found that the characteristic curve of a "thresholdless" laser is strongly nonradiative depopulation-dependent. The light output is increased by the enhanced well number and the reduced width. In particular, there is an optimal well number corresponding to the lowest threshold current density for MQW structure in the microcavity lasers.

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It is well known that placing an emitter within a confined geometry not only alters the radiation pattern of the emitter but also modifies its spontaneous emission decay rate. This was noted by Purcell[1] and is embodied within Fermi's golden rule[2]. Recently, its novel engineering applications such as low noise oscillators, filters, and opto-electronic switches in high-speed communications[3] are found.

One of the major microacvity lasers is the microdisk laser which was pioneered by McCall *et al.* and Levi *et al.* at AT&T Bell laboratories in 1992[4,5]. Another novel structure with the disks grown on a glass substrate was fabricated by the Optoelectronic Research Group at University College, Cork, Ireland. The group achieved room temperature, continuous-wave (CW) lasing with these devices for the first time in 1998[6]. Recent advances in microfabrication technology enabled researchers to control the spontaneous emission rate of an emitter by coupling it to a semiconductor microcavity[7]. Enhanced spontaneous emission rate was used to demonstrate microcavity semiconductor lasers with thresholds dramatically reduced relative to conventional lasers[8]. Thresholds can be further reduced by using quantum dot lasers[9]. A recent experiment demonstrated direct modulation rates can far exceed 100 GHz in quantum well photonic crystal (PC) lasers[10]. The progress in microcavity research is remarkably rapid on all fronts[11−13].

In this letter, we present an approach to get the analytic resolution of rate equation for multi-quantum well microcavity lasers. Based on the concept of spontaneous emission enhancement in a microcavity, we introduce the structural parameter of multi-quantum well (MQW) into the rate equation and consider the influence of nonradiative processes, and obtain a good agreement with the literature. Furthermore, we show analytically that there is an optimal well number corresponding to the lowest threshold in the microcavity MQW lasers.

Because spontaneous emission is a major source of energy loss, speed limitation and noise in lasers, the capability to control spontaneous emission is expected to improve laser's performance. Two important parameters for laser's performance are the spontaneous emission factor and the rate of the radiation from the cavity, depending on the cavity geometry and the spectral emission width of the quantum well (QW). New phenomena are expected if the radiation pattern is highly directed toward a lasing mode and the faction of spontaneous emission coupled into the lasing mode is made close to 1. The fraction of the total spontaneous emission that is coupled into the single lasing mode is defined to be the spontaneous emission factor β, which is written as in a conventional semiconductor laser[14,15]

$$\beta = \lambda^4/4\pi^2 V\Delta\lambda n^3, \tag{1}$$

where $\Delta\lambda$ is the emission linewidth, n is the refractive index, and V is the mode volume. This expression accounts for the number of modes in the system and the number of modes within the emission linewidth, with the former roughly equal to the number of cubic wavelengths in the system, and the latter proportional to $\Delta\lambda/\lambda$. It is clear that β is proportional to the inverse of the mode volume and the emission linewidth. On the other hand, the radiation rate R in a cavity of volume V is

$$R = R_0 Q\lambda^3/V, \tag{2}$$

where R_0 is the probability of photon emission per unit time in free space, or more familiarly called the Einstein A-coefficient, which is proportional to the square of the Rabi frequency in vacuum and to the mode density. It is given by the Fermi's "golden rule"[2,16]. The quality factor Q, given by $\Delta\lambda/\lambda$, describes the cavity bandwidth. Therefore, comparing Eqs. (1) and (2), both V and $\Delta\lambda$ can be reduced to obtain a larger spontaneous emission factor and radiation rate. The regime of very high Q manifests totally new behavior. The radiation remains in the cavity so long that there is a high probability to be reabsorbed by the atom before it dissipates. Spontaneous emission becomes reversible as the atom and the field exchange excitation at the rate of the vacuum Rabi frequency $\Omega_{\rm ef}$, at which the atom and the field would exchange energy if there is only a single mode of the field. In principle, β value can be increased to close to 1. Recently, various cavity resonator geometries including micro-Fabry-Perot (F-P) cavities, "Whispering gallery" and a defect mode of condensed matter optical microcavities have been fabricated. Planar F-P microcavities

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have been studied theoretically and experimentally in some details. Because a common semiconductor laser has β value of only 10^{-5}, the cavity should be quite small, and in order to avoid the photon lifetime decreasing, the reflectivity of cavity mirrors should be quite high. The microcavity laser has an ultrashort cavity length which is more than two orders of magnitude shorter than the cavity length of an edge-emitting laser (EEL). A plane mirrors F-P configuration can provide a rather large value for $\beta \sim 0.1$. By employing spherical mirror to create a F-P resonator, Q can be enhanced substantially. The β value of microdisks is near 0.2, compared with the theoretically predicted value of 0.3. Full confinement of spontaneous emission into the cavity mode might be realized with microsphere or microcube cavity structures.

Although enhanced spontaneous emission is not the necessary condition for the absence of threshold, the consequent increase in the spontaneous emission factor has some great advantages in the view of device. On the other hand, because quantum well lasers are very sensitive to internal losses, the surface and defect recombination may influence the current-carrier relationship, and Auger recombination is the most dominant effect for the high temperature-sensitivity of the threshold current[16]. Considering the effects of these nonradiative processes and the above analysis, we employ a three-level rate-equation model adapted from Ref. [17]. The MQW microcavity lasers steady-state rate equations can be written as

$$\frac{\eta J}{eML_z} - v_{\rm g} g(N)S - \frac{N}{\tau_{\rm sp}} - \frac{N}{\tau_{\rm nr}} = 0, \quad (3)$$

$$M\Gamma_0 v_{\rm g} g(N)S + \beta\frac{N}{\tau_{\rm sp}} - \frac{S}{\tau_{\rm ph}} = 0, \quad (4)$$

where S is the photon density of intracavity, N is the carrier density, $\tau_{\rm sp}$ is the spontaneous emission lifetime, $\tau_{\rm nr}$ is the nonradiative depopulation lifetime, $\tau_{\rm ph}$ is the photon lifetime, $v_{\rm g}$ is the group velocity, J is the injection current density, η is the current-injection efficiency, Γ_0 is the single-quantum well (SQW) light confinement factor, and e is the electron charge. The pair of parameters are introduced in equations for M and L_z (the well number and width of MQW), respectively. We assume a linear gain model

$$g(N) = a(N - N_{\rm tr}), \quad (5)$$

where a is the gain coefficient, $N_{\rm tr}$ is the optical transparency density, and the spontaneous emission rate is $N/\tau_{\rm sp}$. The linear gain model is chosen and gives a good quantitative agreement when the lasers are operated around the threshold, but the model will overestimate the gain in case of well above threshold. A more sophisticated gain model is necessary to model the modulation response of the lasers when gain saturation effects become significant.

Omitting the derivation process, an analysis solution for the photon density S and the carrier density N of the MQW microcavity lasers can be given directly as

$$N = \frac{-B - \sqrt{B^2 - 4AC}}{2A}, \quad (6)$$

where

$$A = v_{\rm g} a[M\Gamma_0 Q - \frac{\beta}{\tau_{\rm sp}}],$$

$$B = -[\frac{\Gamma_0 v_{\rm g} a\eta J}{eL_z} + PQ - \frac{\beta v_{\rm g} a N_{\rm tr}}{\tau_{\rm sp}}],$$

$$C = \frac{\eta JP}{eML_z},$$

$$P = M\Gamma_0 v_{\rm g} a N_{\rm tr} + \frac{1}{\tau_{\rm ph}},$$

$$Q = \frac{1}{\tau_{\rm sp}} + \frac{1}{\tau_{\rm nr}},$$

and

$$S = \frac{-B' + \sqrt{B'^2 + 4A'C'}}{2A'}, \quad (7)$$

where

$$A' = \frac{v_{\rm g} a}{\tau_{\rm ph}},$$

$$B' = -[\frac{\Gamma_0 v_{\rm g} a\eta J}{eL_z} - PQ + \frac{\beta v_{\rm g} a N_{\rm tr}}{\tau_{\rm sp}}],$$

$$C' = \frac{\eta J\beta}{eML_z\tau_{\rm sp}}.$$

These equations will be used in the following part to discuss the parameter characteristics of microcavity lasers. Considering the gain-saturation effect of MQW, with Eqs. (3) and (4), the threshold current density is calculated by

$$J_{\rm th} = \frac{eML_z N_{\rm tr}}{\eta\tau_{\rm sp}} \exp\frac{1}{M\Gamma_0 g_0 \tau_{\rm ph}}, \quad (8)$$

$dJ_{\rm th}/dM = 0$, and we obtain the optimal well number

$$M_{\rm opt} = \frac{v_{\rm g}}{\Gamma_0 g_0}(\alpha_{\rm int} + \frac{1}{L}\ln\frac{1}{R}), \quad (9)$$

where g_0 is the gain coefficient per QW, $\alpha_{\rm int}$ is internal loss, L is the cavity length, and R is the facet reflectivity. There is an optimal well number corresponding to the lowest threshold current density,

$$J_{\rm th}^{\rm min} = \frac{ev_{\rm g} L_z N_{\rm tr}}{\eta\tau_{\rm sp}\Gamma_0 g_0}(\alpha_{\rm int} + \frac{1}{L}\ln\frac{1}{R}) \times \exp\frac{1}{v_{\rm g}\tau_{\rm ph}(\alpha_{\rm int} + \frac{1}{L}\ln\frac{1}{R})}. \quad (10)$$

The calculation is based on the values given in Table 1. Figure 1 shows how the spontaneous emission factor β affects microcavity lasers output properties. It is seen

Table 1. Typical Parameter Values for Microcavity Lasers[17]

Description	Parameter	Value
Group Velocity	v_g	8.8×10^9 cm/s
Gain Coefficient	a	3×10^{-16} cm^{-3}
Optical Transparency Density	N_{tr}	1.2×10^{18} cm^{-3}
Optical Confinement Factor	Γ_0	0.025
Gain Coefficient per QW	g_0	1.5×10^3 cm^{-1}
Cavity Length	L	1 μm
Facet Reflectivity	R	0.995
Internal Loss	α_{int}	30 cm^{-1}
Current-Injection Efficiency	η	0.6
Spontaneous Emission Lifetime	τ_{sp}	2×10^{-9} s
Photon Lifetime	τ_{ph}	1×10^{-12} s
Nonradiative Depopulation Lifetime	τ_{nr}	1×10^{-10} s

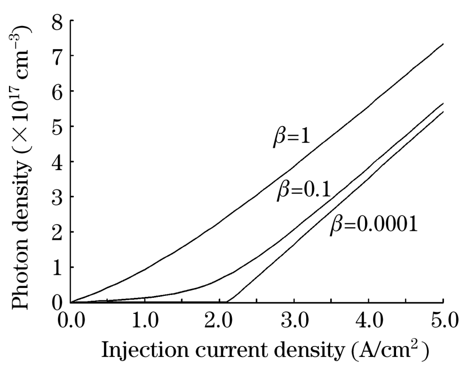

Fig. 1. Relationship between the photon density S and injection current density J.

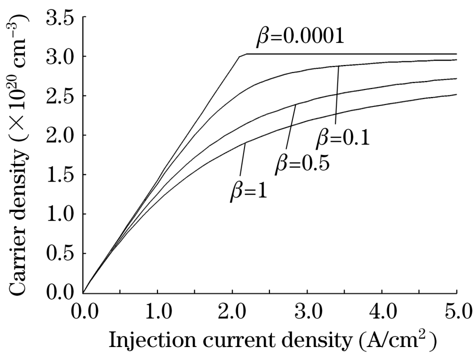

Fig. 2. Relationship between the carrier density N and injection current density J.

that the photon density S increases, the threshold decreases when β increases, and at the same time transfer efficiency increases. The transition from the nonlasing to the lasing state becomes softer with an increase of β. This is because of the substantial increase in the amount of spontaneous emission coupled into the cavity mode. Numerical simulations indicate that, when nonradiative depopulation is not zero, the light output properties is not linearly even for an ideal closed microcavity.

Figure 2 shows the variation of N and J with different β. In the case of an ordinary laser as shown in $\beta = 0.0001$, N proportionally increases with the increase of J. With the increase of J, N increases until to the lasing threshold level and then is clamped there. N is nearly constant in the threshold and does not increase significantly with a further increase in J. For a microcavity laser, this behavior is different from that of ordinary spontaneous emission, N slowly increases with the increase of J, and it reaches a constant value at infinite J.

The relationship between the light output and width L_z is presented in Fig. 3. It corresponds to different values of β (i.e., different mode volumes). As can be seen, the S quickly increases as β increasing. It is also found that for fixed β, S starts to decrease rapidly as L_z increasing, and then S is nearly constant in the L_z region (estimated to be between several and 10 nm) and does not decrease significantly with a further increase in L_z. The results show that MQW structures exhibit a higher S, which suggests that S has a higher sensitivity to L_z compared with M, and the sharpness of the increment depends on β. In addition to employing an analytic expression, we also consider the effect of width L_z and reflective R on the threshold current density J (A/cm^2). We can realize a higher J_{th} for SQW lasers compared with MQW lasers in the short cavity, and the effect would be different for long cavity structure.

Figure 4 shows the variation of M and S with different β. It shows that the larger M is, the higher S is. There is an optimal well number for the minimum J_{th}, thus demonstrating a good agreement with the experiment results[8,9]. It is important that the device structure is optimized with an optimal well and cavity length.

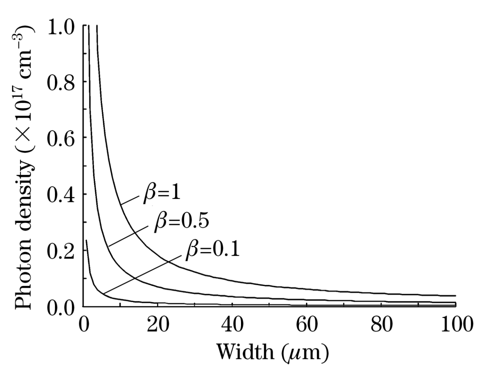

Fig. 3. Relationship between the photon density S and width L_z.

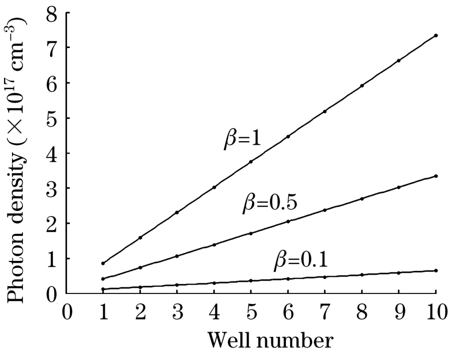

Fig. 4. Relationship between the photon density S and well number M.

In conclusion, the influence of spontaneous emission on properties and structure parameters of microcavity lasers is investigated. It is evident that such high spontaneous emission factor value can be used to enhance the efficiency of light-emitting and lower thresholds of lasers. The light output is enhanced by increasing the well number and decreasing the width, at the same time, the sharpness of those variations depends on spontaneous emission factor. In particular, we show analytically that there is a lower threshold current density for MQW lasers in the microcavity. There is an optimal well number corresponding to different structural parameter values. It is also found that the characteristic curve of a "thresholdless" laser is strongly nonradiative depopulation-dependent. When nonradiative depopulation is not zero, even for an ideal closed microcavity, the light-current characteristic is not linearly. The result shows that increasing the coupling of spontaneous emission into the cavity mode causes the lasing properties becoming quite different from those of the conventional laser having cavity dimensions much larger than the lasing wavelength. These studies are important for understanding, improving, and engineering the microcavity devices to be applied in novel engineering.

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