# Three－dimensional optical logic devices using spatial multiwaveguide system 

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#### Abstract

Based on the weakly coupled－mode theory，the coupled－mode equations of the spatial multiwaveguide system are presented in general．The intensity distribution in each waveguide is determined by numerical method．Optical logic devices based on spatial multiwaveguide system are proposed．The analysis results show that the spatial multiwaveguide system permits different Boolean logic states obtained by phase modulation．Applications of the logic devices include optical calculation，optical interconnection，and spatial optical signal processing．


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Up to now，most of integrated optical waveguide de－ vices are realized in planar structures ${ }^{[1,2]}$ ，which means that these devices can only be applied to process the input／output signal through one－dimensional（1D）in－ terconnect channels．In fact，many applications，such as large data streams，image manipulation，artificial vision，nerve networks and optical calculation，need two－dimensional（2D）spatial optical signal process－ ing．To meet these demands，we should develop inte－ grated optical devices employing three－dimensional（3D） structures ${ }^{[3,4]}$ ，which exhibit a higher level of integra－ tion．Several 3D integrated optical devices have been successfully implemented，such as vertical waveguide power splitters ${ }^{[4]}$ ，3D optical switches ${ }^{[5]}$ ，modulators ${ }^{[6]}$ ， spatial logic coders ${ }^{[7]}$ by using thermo－optic and electro－ optic effects．

Waveguides on different planes，which are called spa－ tial multiwaveguide system in this paper，have been incorporated into many 3D integrated optics devices ${ }^{[5,6]}$ ． The coupled－mode theory ${ }^{[8]}$ presented a general ana－ lytical means for traditional planar directional couplers composed of two waveguides in the same plane，and the coupled properties of planar three－waveguide and multi－ waveguide systems were also discussed in Refs．［9］and ［10］，respectively．

In this paper，we apply the weakly coupled－mode theory to analyze the modulation properties of spatial


Fig．1．Schematic diagram of spatial three－waveguide system．
multimode－waveguide system and describe the power transfer among these spatial waveguides by simultaneous differential equations．Optical spatial logic devices are designed based on the analytical results．We consider a spatial 3D system formed by three waveguides，as shown in Fig．1．The coupled－mode equations for the spatial three－waveguide with propagation along the $z$－direction can be described as follows ${ }^{[9]}$ ：

$$
\begin{align*}
\frac{\partial A_{1}}{\partial z}= & i K_{12} A_{2} \exp \left(-i \Delta \beta_{1} z\right) \\
& +i K_{31} A_{3} \exp \left(i \Delta \beta_{3} z-i \Delta \beta_{1} z\right)  \tag{1a}\\
\frac{\partial A_{2}}{\partial z}= & i K_{12} A_{1} \exp \left(i \Delta \beta_{1} z\right)+i K_{23} A_{3} \exp \left(i \Delta \beta_{3} z\right)  \tag{1b}\\
\frac{\partial A_{3}}{\partial z}= & i K_{31} A_{1} \exp \left(i \Delta \beta_{1} z-i \Delta \beta_{3} z\right) \\
& +i K_{23} A_{2} \exp \left(-i \Delta \beta_{3} z\right) \tag{1c}
\end{align*}
$$

where $A_{i}(i=1,2,3)$ is the amplitude of the field in each waveguide．$K_{j l}(j, l=1,2,3)$ is the coupling coefficient between waveguide $j$ and waveguide $l$ ，which is a func－ tion of the separation distance between the waveguides and the distribution of refractive index distribution．$\beta_{i}$ （ $i=1,2,3$ ）is the propagation constant of each waveg－ uide．$\Delta \beta_{1}$ and $\Delta \beta_{3}$ are detuning of $\beta_{1}$ and $\beta_{3}$ from $\beta_{2}$ ， respectively．

$$
\begin{align*}
& \Delta \beta_{1}=\beta_{1}-\beta_{2}  \tag{2a}\\
& \Delta \beta_{3}=\beta_{3}-\beta_{2} \tag{2b}
\end{align*}
$$

For mathematical simplicity，we use

$$
\begin{align*}
& a_{1}=A_{1} \exp \left(i \Delta \beta_{1} z\right),  \tag{3a}\\
& a_{2}=A_{2}  \tag{3b}\\
& a_{3}=A_{3} \exp \left(i \Delta \beta_{3} z\right), \tag{3c}
\end{align*}
$$

which yield

$$
\frac{\partial}{\partial z}\left[\begin{array}{l}
a_{1}  \tag{4}\\
a_{2} \\
a_{3}
\end{array}\right]=i\left[\begin{array}{ccc}
\Delta \beta_{1} & K_{12} & K_{31} \\
K_{12} & 0 & K_{23} \\
K_{31} & K_{23} & \Delta \beta_{3}
\end{array}\right]\left[\begin{array}{l}
a_{1} \\
a_{2} \\
a_{3}
\end{array}\right]
$$

The matrix equation can be used to solve the weakly coupled problem of the spatial three-waveguide system and predict the intensity distribution in each waveguide. Assuming the eigenvalues of the matrix $\left[\begin{array}{ccc}\Delta \beta_{1} & K_{12} & K_{31} \\ K_{12} & 0 & K_{23} \\ K_{31} & K_{23} & \Delta \beta_{3}\end{array}\right]$ are $\beta_{1}, \quad \beta_{2}$, and $\beta_{3}$, respectively, and the corresponding eigenvectors are $\left(\begin{array}{l}v_{11} \\ v_{12} \\ v_{13}\end{array}\right)$, $\left(\begin{array}{l}v_{21} \\ v_{22} \\ v_{23}\end{array}\right),\left(\begin{array}{l}v_{31} \\ v_{32} \\ v_{33}\end{array}\right)$, respectively. The solution to Eq. (4) can be written as

$$
\begin{align*}
{\left[\begin{array}{l}
a_{1}(z) \\
a_{2}(z) \\
a_{3}(z)
\end{array}\right]=} & \left(\begin{array}{lll}
v_{11} & v_{21} & v_{31} \\
v_{12} & v_{22} & v_{32} \\
v_{13} & v_{23} & v_{33}
\end{array}\right) \\
& \times\left[\begin{array}{ccc}
e^{i \beta_{1} z} & 0 & 0 \\
0 & e^{i \beta_{2} z} & 0 \\
0 & 0 & e^{i \beta_{3} z}
\end{array}\right]\left[\begin{array}{l}
B_{1} \\
B_{2} \\
B_{3}
\end{array}\right], \tag{5}
\end{align*}
$$

where $B_{1}, B_{2}$, and $B_{3}$ are the undetermined coefficients, which can be determined by the following equation:

$$
\left[\begin{array}{l}
B_{1}  \tag{6}\\
B_{2} \\
B_{3}
\end{array}\right]=\left(\begin{array}{lll}
v_{11} & v_{21} & v_{31} \\
v_{12} & v_{22} & v_{32} \\
v_{13} & v_{23} & v_{33}
\end{array}\right)^{-1} \cdot\left[\begin{array}{l}
a_{1}(0) \\
a_{2}(0) \\
a_{3}(0)
\end{array}\right]
$$

where $\left[\begin{array}{lll}a_{1}(0) & a_{2}(0) & a_{3}(0)\end{array}\right]^{\mathrm{T}}$ is the initial condition. The intensity distribution in each waveguide under different initial conditions and detuning parameters can be determined by Eqs. (5) and (6).

If the light power is injected into waveguide 1, the initial conditions will be

$$
\begin{equation*}
A_{1}(0)=1, \quad A_{2}(0)=A_{3}(0)=0 \tag{7}
\end{equation*}
$$

Assuming $K_{12}=0.001, K_{23}=0.0015, K_{31}=0.0015$, the length of three identical waveguides $z$ is $\pi / 0.003$. Solving the simultaneous differential equations, we can get seven different logic states under different modulation parameters. In the case of $\Delta \beta_{1}=-0.018, \Delta \beta_{3}=0.03$, the input optical power is completely switched to the waveguide 1. For

$$
\left[\begin{array}{l}
\left|a_{1}(z)\right|^{2}  \tag{8}\\
\left|a_{2}(z)\right|^{2} \\
\left|a_{3}(z)\right|^{2}
\end{array}\right]=\left[\begin{array}{c}
0.9999 \\
0.0000 \\
0.0001
\end{array}\right]
$$

this case corresponds to the logic state [100]. We define the extinction ratio ER between the final outputs as

$$
\begin{equation*}
\mathrm{ER}=10 \log \left[\frac{P_{\text {OON"state }}^{\min }}{P_{\text {"OFF"state }}^{\max }}\right], \tag{9}
\end{equation*}
$$

where $P_{\text {"ON"state }}^{\text {min }}$ is the minimal optical power and $P$ "OFF"state $\max$ is maximal optical power. Using the above definition, the extinction ratio $\mathrm{ER}=40 \mathrm{~dB}$.

In the case of $\Delta \beta_{1}=0$ and $\Delta \beta_{3}=-0.003$, the input optical power is completely switched to the waveguide 2 . This case corresponds to the logic state [010], and the
extinction ratio $\mathrm{ER}=27 \mathrm{~dB}$. For

$$
\left[\begin{array}{l}
\left|a_{1}(z)\right|^{2}  \tag{10}\\
\left|a_{2}(z)\right|^{2} \\
\left|a_{3}(z)\right|^{2}
\end{array}\right]=\left[\begin{array}{l}
0.001 \\
0.997 \\
0.002
\end{array}\right],
$$

in the case of $\Delta \beta_{1}=-0.0439$ and $\Delta \beta_{3}=-0.0439$, the input optical power is completely switched to the waveguide 3. This case corresponds to the logic state [001], and the extinction ratio $\mathrm{ER}=27 \mathrm{~dB}$. For

$$
\left[\begin{array}{l}
\left|a_{1}(z)\right|^{2}  \tag{11}\\
\left|a_{2}(z)\right|^{2} \\
\left|a_{3}(z)\right|^{2}
\end{array}\right]=\left[\begin{array}{l}
0.002 \\
0.000 \\
0.998
\end{array}\right],
$$

in the case of $\Delta \beta_{1}=0.008$ and $\Delta \beta_{3}=0.0102$, the input optical power is equally divided between the waveguide 1 and waveguide 3 . This case corresponds to the logic state [101], and the extinction ratio ER $=27 \mathrm{~dB}$. For

$$
\left[\begin{array}{l}
\left|a_{1}(z)\right|^{2}  \tag{12}\\
\left|a_{2}(z)\right|^{2} \\
\left|a_{3}(z)\right|^{2}
\end{array}\right]=\left[\begin{array}{l}
0.500 \\
0.001 \\
0.499
\end{array}\right]
$$

in the case of $\Delta \beta_{1}=0.0022$ and $\Delta \beta_{3}=-0.0215$, the input optical power is equally divided between the waveguide 1 and waveguide 2 . This case corresponds to the logic state [110], and the extinction ratio $\mathrm{ER}=24 \mathrm{~dB}$. For

$$
\left[\begin{array}{l}
\left|a_{1}(z)\right|^{2}  \tag{13}\\
\left|a_{2}(z)\right|^{2} \\
\left|a_{3}(z)\right|^{2}
\end{array}\right]=\left[\begin{array}{l}
0.498 \\
0.500 \\
0.002
\end{array}\right]
$$

in the case of $\Delta \beta_{1}=0.0018$ and $\Delta \beta_{3}=0.0004$, the input optical power is equally divided between the waveguide 2 and waveguide 3 . This case corresponds to the logic state [011], and the extinction ratio $\mathrm{ER}=7.1 \mathrm{~dB}$. For

$$
\left[\begin{array}{l}
\left|a_{1}(z)\right|^{2}  \tag{14}\\
\left|a_{2}(z)\right|^{2} \\
\left|a_{3}(z)\right|^{2}
\end{array}\right]=\left[\begin{array}{l}
0.090 \\
0.463 \\
0.447
\end{array}\right],
$$

in the case of $\Delta \beta_{1}=0.0002$ and $\Delta \beta_{3}=0.0005$, the input optical power is equally distributed among three waveguides. This case corresponds to the logic state [111], and the outputs are all "ON" states. For

$$
\left[\begin{array}{l}
\left|a_{1}(z)\right|^{2}  \tag{15}\\
\left|a_{2}(z)\right|^{2} \\
\left|a_{3}(z)\right|^{2}
\end{array}\right]=\left[\begin{array}{l}
0.327 \\
0.338 \\
0.335
\end{array}\right],
$$

the analysis results show that the spatial multiwaveguide system permits different Boolean logic states obtained by phase modulation.
In this paper, optical logic devices based on spatial multiwaveguide system are proposed. The intensity distribution in each waveguide is determined by numerical method. The analysis results show that the optical logic device permits different Boolean logic states obtained by phase modulation. The extinction ratio between the minimum power of "ON" state and the maximum power of "OFF" state can reach 7.1 dB . Applications of the logic devices include optical calculation, optical interconnection, and spatial optical signal processing.

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