Semi-blind image restoration based on Chan-Vese denoising model

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Received September 14, 2007

A semi-blind image restoration algorithm is proposed based on reduced non-convex approximation of Luminita Vese and Tony Chan's (C-V) denoising model. Compared with C-V denoising model, we modify the fidelity term and add a term on point spread function (PSF). The function depends on two variables: the image function to be restored u and the standard deviation of Gaussian kernel to be estimated σ. Then the problems consist in solving a system with two coupled equations. Compared with the Leah Bar's semi-blind image restoration model which must solve three coupled equations, our method only needs to solve two equations. Furthermore, the estimation of f by our algorithm is superior to Leah Bar's algorithm. The experimental results demonstrate that the proposed method is effective.

OCIS codes: 100.1830, 100.3020, 100.3190.

It often happens that an acquired image has worse quality than the desired one due to various imperfections and/or physical limitations in the imaging and transmission processes. It usually looks blurry due to, for example, relative motion between the camera and the object, a camera that is out of focus, or atmospheric turbulence. Noise may be also introduced into the image owing to measurement errors, quantization, and imperfection in the recording and transmission media, etc.. This kind of degradation occurs in a variety of domains of applied science and engineering, such as visual communications, robot guidance, medical diagnostics, atmospheric turbulence, remote sensing[1−3].

In most cases, a linear model for the degradation process is denoted as

$$g = h * f + n, \tag{1}$$

where f, g, h, and n represent the original image, observed image, point spread function (PSF), and noise, respectively; $*$ denotes two-dimensional (2D) convolution.

The goal of image restoration is to recover the original image from a given degraded observed image. It is well known that image restoration is an ill-posed problem, and it often needs regularization term in the solution process. Regularization can be realized by considering image as Gibbs random field. In such cases, much relevant information of image such as line and boundary can be determined by analysis of neighbor structure. When the PSF is unknown, this case is blind deconvolution. Chan and Wong proposed a blind deconvolution method based on total variation[4]. But the PSF is Gaussian type, and the image restoration is sensitive to the PSF recovery error[5]. So Leah Bar proposed a kind of semi-blind image restoration[5]. It is assumed that the image is blurred by Gaussian PSF and the Gaussian deviation is unknown. In addition to Leah Bar semi-blind image restoration, Money and Kang used simple shock filters and proposed another semi-blind deconvolution[6].

Based on Chan-Vese (C-V) denoising model[7], we propose a novel method to solve semi-blind image restoration in this paper. The corresponding Euler-Lagrange equation involves two variables, the image function to be restored f, and the Gaussian PSF deviation σ.

The Mumford-Shah piecewise smooth segmentation[8] is defined by

$$G^{\mathrm{MS}}(f,\Gamma) = \beta \int_{\Omega} (f - f_0)^2 \mathrm{d}x\mathrm{d}y + \alpha \int_{\Omega\setminus\Gamma} |\nabla f|^2 \mathrm{d}x\mathrm{d}y + \mathrm{length}(\Gamma), \tag{2}$$

where $\Omega \subset R^2$ is a connected, bounded, and open subset representing an image domain; f_0 is an image defined on Ω; $\Gamma \subset \Omega$ is the edge set segmenting Ω; f is the piecewise smooth approximation of u_0; $\alpha > 0$, $\beta > 0$ are the scale space parameters of the model. It allows segmenting an image into some disjoint homogenous regions. Each of the regions has smoothly varying intensities, and their boundaries have sharp varying intensities. This model has been used extensively in image segmentation, denoising, inpainting, and computer vision.

Based on Mumford-Shah model[8], Vese and Chan proposed the reduced model[7]. Their model can be reduced only one equation with only one unknown function, while still being able to extract the edges. The energy function of C-V denoising model is

$$G_{\rho}^{\mathrm{CV}}(f) = \int_{\Omega} (\alpha \frac{|\nabla f|^2}{1 + 4\alpha\rho |\nabla f|^2} + \beta |f - f_0|^2) \mathrm{d}x\mathrm{d}y, \tag{3}$$

where $\alpha > 0$, $\beta > 0$, $\rho > 0$, and $|\nabla f|$ is the gradient norm of f, f_0 is a noise image. The C-V denoising method only needs to solve one equation[7].

Based on this C-V denoising model, we propose a kind of semi-blind image restoration algorithm. That is to say, PSF type is assumed to be known, such as Gaussian type, but the parameter deviation σ is unknown. This case is incurred in many conditions of real application.

1671-7694/2008/060405-03

The model is as follows:

$$F_{\rm new}(f,v,\sigma)=\frac{1}{2}\int_\Omega (h_\sigma * f-g)^2{\rm d}x{\rm d}y$$
$$+G_{\rm new}(f,v)+\gamma\int_\Omega |\nabla h_\sigma|^2{\rm d}x{\rm d}y, \quad (4)$$

$$G_{\rm new}(f,v)=\beta\int_\Omega v^2\,|\nabla f|^2\,{\rm d}x{\rm d}y$$
$$+\alpha\int_\Omega \frac{(v-1)^2}{4\varepsilon}{\rm d}x{\rm d}y, \quad (5)$$

$$v=\frac{\alpha}{4\beta\varepsilon\,|\nabla f|^2+\alpha}. \quad (6)$$

Substituting Eqs. (5) and (6) into Eq. (4), we got

$$F_{\rm new}(f,\sigma)=\frac{1}{2}\int_\Omega (h_\sigma * f-g)^2{\rm d}x{\rm d}y$$
$$+\int_\Omega \frac{\alpha\beta\,|\nabla f|^2}{4\beta\varepsilon\,|\nabla f|^2+\alpha}{\rm d}x{\rm d}y+\gamma\int_\Omega |\nabla h_\sigma|^2{\rm d}x{\rm d}y, \quad (7)$$

where the first term is fidelity term, h_σ, f, and g denote Gaussian PSF, original image, and degraded image respectively. The second term is regularization term on image, α, β, ε, and γ are all positive, $|\nabla\cdot|$ is the gradient norm. The third term is regularization term on PSF.

From Eq. (7), the Euler-Lagrange equations are deduced as

$$\frac{\partial F_{\rm new}}{\partial f}=(h_\sigma * f-g)*h_\sigma(-x,-y)$$
$$-2\beta{\rm Div}((\frac{\alpha}{4\beta\varepsilon\,|\nabla f|^2+\alpha})^2\nabla f)$$
$$=0, \quad (8)$$

$$\frac{\partial F_{\rm new}}{\partial \sigma}=\int_\Omega [(h_\sigma * f-g)(\frac{\partial h_\sigma}{\partial\sigma}*f)+\gamma\frac{\partial}{\partial\sigma}\,|\nabla h_\sigma|^2\,]{\rm d}x{\rm d}y$$
$$=0. \quad (9)$$

Here h_σ, $\frac{\partial h_\sigma}{\partial\sigma}$, and $\frac{\partial}{\partial\sigma}\,|\nabla h_\sigma|^2$ are defined respectively as

$$h_\sigma=\frac{1}{2\pi\sigma^2}{\rm e}^{\frac{x^2+y^2}{-2\sigma^2}}, \quad (10)$$

$$\frac{\partial h_\sigma}{\partial\sigma}=\frac{1}{2\pi\sigma^2}{\rm e}^{-\frac{x^2+y^2}{2\sigma^2}}\cdot(\frac{x^2+y^2}{\sigma^3}-\frac{2}{\sigma}), \quad (11)$$

$$\frac{\partial}{\partial\sigma}\,|\nabla h_\sigma|^2=\frac{1}{2\pi^2\sigma^4}{\rm e}^{-\frac{x^2+y^2}{2\sigma^2}}\cdot(\frac{x^2+y^2}{\sigma^7}-\frac{4}{\sigma^5})\cdot(x^2+y^2). \quad (12)$$

Equations (8) and (9) are solved by "lagged diffusivity fixed point iteration"[9] and bisection method, respectively. Then we adopt alternate minimization numerical scheme[4,10].

Our experiments show that such kind of non-convex image semi-blind restoration algorithm is very effective. Consider the images of Lena and Cameraman obtained by blurring the original images with a Gaussian kernel with $\sigma=2.1$. We compared the Leah Bar method[5] and ours with the same parameters such as $\alpha=10^{-8}$, $\beta=10^{-4}$, $\gamma=40$, $\varepsilon=10^{-3}$, the initial value $\sigma=0.5$. For Lena image, the estimation values of σ are 2.04 in Ref. [5] and 2.09 by our model, respectively (Fig. 1). For Cameraman image, the estimation values of σ are 1.97 in Ref. [5] and 2.08 by our model, respectively (Fig. 2). Compared with Leah Bar method, our results are better. For Boat image and Pepper image, the estimation values of σ are 2.01 and 1.99 respectively in Ref. [5], and the corresponding values are 2.09 and 2.04 by our new model (Figs. 3 and 4). Compared with Leah Bar method, our results are also superior.

Furthermore, we test the algorithm of Ref. [5] and our method under the noise and blur condition. In this case, $\alpha=10^{-6}$, $\beta=10^{-3}$, $\gamma=60$, $\varepsilon=10^{-3}$, the initial value $\sigma=0.5$ and the real value $\sigma=2.6$. The estimation values of σ are 2.25 in Ref. [5] and 2.51 by our algorithm, respectively. In Fig. 5, Leah Bar method causes noise amplified, but our method does not lead to such result.

We compared the estimation of σ and the improvement of signal noise ratio (ISNR) by the Leah Bar method and

Fig. 1. Lena images before and after semi-blind restoration. (a) Blurred image; (b) restored by Leah Bar method; (c) restored by the method in this paper.

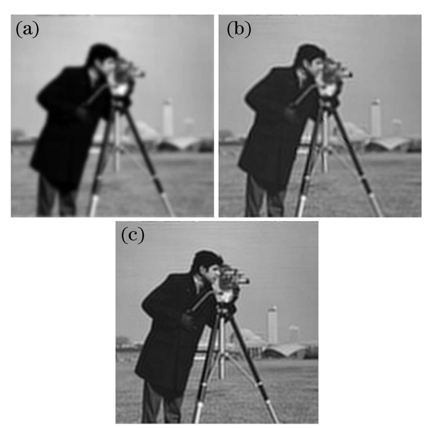

Fig. 2. Cameraman images before and after semi-blind restoration. (a) Blurred image; (b) restored by Leah Bar method; (c) restored by the method in this paper.

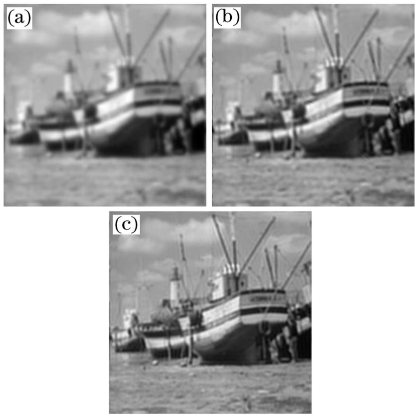

Fig. 3. Boat images before and after semi-blind restoration. (a) Blurred image; (b) restored by Leah Bar method; (c) restored by the method in this paper.

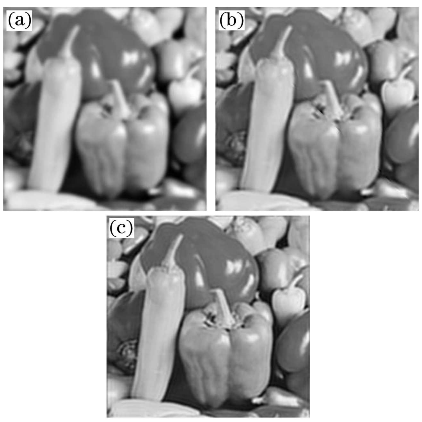

Fig. 4. Pepper images before and after semi-blind restoration. (a) Blurred image; (b) restored by Leah Bar method; (c) restored by the method in this paper.

Fig. 5. Noisy Lena image and the results of semi-blind restoration (SNR = 30). (a) Blurred image; (b) restored by Leah Bar method; (c) restored by the method in this paper.

our method in Table 1. It is obviously seen that our method works better than that in Ref. [5].

Image restoration based on varitional method is a focus in the recent years. Based on C-V denosing model, we proposed a new non-convex semi-blind image restoration algorithm. Compared with the method proposed by Leah Bar[5], our method shows the following advantages. Firstly, the solved equations are reduced to two, one is the image and the other is for PSF. Secondly, the estimated parameter σ is much more accurate than Leah Bar method. Thirdly, the restoration result by our method is superior to Leah Bar method. Furthermore, our method does not lead to noise amplification. Still much improvement is possible, such as the parameters α, β, γ chosen adaptively. This will be studied further.

Table 1. Deviation Estimation and ISNR by the Leah Bar Method[5] and Our Method

Image	Real σ	Estimated σ		ISNR	
		Leah Bar	Our	Leah Bar	Our
Lena	2.1	2.04	2.09	2.86	4.13
Lena	2.6	2.51	2.58	1.50	3.45
Boat	2.1	1.97	2.08	3.82	4.57
Boat	2.6	2.39	2.55	2.60	4.57
Cameraman	2.1	1.94	2.05	2.72	4.39
Cameraman	2.6	2.31	2.42	1.56	3.41
Pepper	2.1	2.01	2.04	−1.72	1.18
Pepper	2.6	2.47	2.54	−0.81	0.01
Lena (Noisy)	2.1	1.79	1.92	−4.38	0.73
Lena (Noisy)	2.6	2.25	2.41	−5.48	0.24

This work was supported by the Knowledge Innovation Program of Chinese Academy of Sciences (No. 07A1210101). Y. Tang is the author to whom the correspondence should be addressed, his e-mail address is ytang@sina.cn.

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