Analysis of the positive or negative lateral shift of the reflected beam in Otto configuration under grazing incidence

Haichun Zhou (周海春)[1], Chunfang Li (李春芳)[1,2], and Xi Chen (陈 玺)[1]

[1]*Department of Physics, Shanghai University, Shanghai 200444*

[2]*State Key Laboratory of Transient Optics and Photonics, Xi'an Institute of Optics and Precision Mechanics, Chinese Academy of Sciences, Xi'an 710119*

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We investigate the lateral shift of a TM-polarized light beam reflected from Otto configuration under grazing incidence. It is found that the lateral shift is strongly dependent on the thickness of the air-gap layer. By employing the pole-null representation, we demonstrate that the lateral shift is closely related to the null of the reflection function. The numerical simulations for a Gaussian beam are performed to demonstrate the validity of our theoretical analysis.

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It is well known that a totally reflected beam experiences a lateral shift, which is referred to as Goos-Hänchen (GH) effect[1] from the position predicted by the geometrical optics, because each of its plane wave components undergoes a different phase shift[2]. Since the investigations of the GH shift were extended to the lossy multilayered structures, the pole-null representation (PNR)[3−6] has been developed. Shah *et al.*[3,4] pointed out that the incident beam may be minimally reflected in a much broader class of layered configurations. Near the reflectivity minima, it is interesting to study the behaviors of the GH shift in various structures, such as absorbing dielectric structures[3−7], metallic structures[8,9], and waveguide structures[10,11]. Recently, Shkerdin *et al.*[12] demonstrated that the lateral shift can be described by the polls and the nulls, which are associated with the reflectivity minima and represent the eigenmodes for a given structure.

Lukosz *et al.*[13] showed that the reflectivity may have two exact zeros in Otto configuration for two different angles of incidence and thickness of the central layer. The first zero reflectivity is due to surface plasmon resonance (SPR) and the corresponding property of the lateral shift has been studied in Refs. [8,14]. The second one has been found in Refs. [13,15] near grazing incidence and with a small value of the thickness of the interlayer. However, the features of the lateral shift in the latter case have not been reported.

The purpose of this paper is to investigate the lateral shift of the reflected beam near grazing incidence in Otto configuration. Our calculations show that the magnitude and the sign (positive or negative) of the GH shift are strongly dependent on the thickness of the air-gap layer. By utilizing the PNR of the reflection function, we demonstrate that the GH shift is only associated with the null, because no pole exists in this case.

The geometrical configuration considered here is shown in Fig. 1. It consists of a prism with the relative dielectric constant $\varepsilon_1 = 2.34$, an air-gap layer with the thickness d, and a metal such as silver with the relative dielectric constant $\varepsilon_3 = -5.19 + i0.28$ at the wavelength $\lambda = 435.8$ nm[14]. Suppose that a TM-polarized beam is incident from the prism at an angle θ on the interface $z = 0$, the reflection function in this structure can be obtained by solving Maxwell's equations and the boundary conditions[16]

$$r(k_x) = \frac{r_{12} + r_{23}\exp(2ik_{2z}d)}{1 + r_{12}r_{23}\exp(2ik_{2z}d)}, \tag{1}$$

with

$$r_{ab} = \frac{k_{az}/\varepsilon_a - k_{bz}/\varepsilon_b}{k_{az}/\varepsilon_a + k_{bz}/\varepsilon_b}, \quad (a, b = 1, 2, 3), \tag{2}$$

where $k_x = k_0\sqrt{\varepsilon_1}\sin\theta$ represents the propagation constant along the interface with $k_0 = 2\pi/\lambda$ being the wave vector in vacuum, ε_a is the relative dielectric permittivity of medium a, $k_{az} = k_0(\varepsilon_a - \varepsilon_1\sin^2\theta)^{1/2}$ is the corresponding component of the wave vector normal to the interface in medium a, and r_{ab} represents the Fresnel reflection coefficient for the interface separating media a and b.

The reflectivity R versus the incidence angle θ for

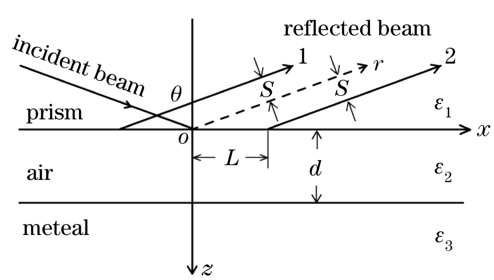

Fig. 1. Schematic diagram of a TM-polarized light beam incident upon Otto configuration at grazing angle, where $\varepsilon_1 = 2.34$, $\varepsilon_2 = 1$, $\varepsilon_3 = -5.19 + i0.28$. The lateral shift S may be negative (Ray 1) or positive (Ray 2), the dashed line (Ray r) is the path predicted by geometric optics. $L = S/\cos\theta$ represents the longitudinal shift along the interface $z = 0$.

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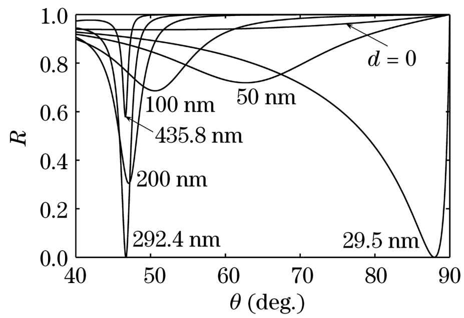

Fig. 2. Reflectivity R versus the incidence angle θ with different thicknesses d, where all the other parameters are the same as Fig. 1.

several thicknesses d is presented in Fig. 2. It is clearly seen that if the incidence angle θ and the thickness d satisfy a certain condition, exact zero reflectivity will be obtained. This condition is determined by the requirements of the phase and the amplitude of the numerator on the right-hand side of Eq. (1), that is,

$$\phi_{12} + \pi = \phi_{23}, \tag{3}$$

and

$$d = \frac{1}{2\kappa} \ln \left| \frac{r_{23}}{r_{12}} \right|, \tag{4}$$

where $\kappa = k_0(\varepsilon_1 \sin^2\theta - \varepsilon_2)^{1/2}$, ϕ_{12} and ϕ_{23} respectively represent the phase of r_{12} and r_{23}. The solutions of Eq. (3) determine at which angles of incidence the reflectivity will be zero, if the air-gap layer has the proper thickness d given by Eq. (4).

The calculated results show that only two angles meet Eq. (3). The first one $\theta_1 = 46.705°$ is very close to the SPR angle $\theta_{\mathrm{SPR}} = 46.657°$, the corresponding thickness d_1 is 292.4 nm, which is well known as the optimal thickness[17] for the excitations of surface plasmon, and the reflected beams may undergo large positive or negative lateral shifts around d_1[8]. In detail, above this optimal thickness negative lateral shifts are found, while below it the lateral shifts are positive[9,14]. The second one θ_2 is 87.936° and the corresponding thickness d_2 is 29.55 nm, meanwhile, the corresponding curve R takes on a relatively broader minimum.

Here we concentrate on the second case. By utilizing PNR of the reflection function in the complex wave vector plane, we will demonstrate that there exists a close link between the lateral shift and the null which is determined by the physical parameters of the structure, and will show that the thickness d also plays an important role. About how to evaluate the pole k_{p} and the null k_{n}, one can see Ref. [18]. Because $d_2 = 29.55$ nm is far more less than the cut-off thickness, the pole k_{p} no longer exists[15]. The solutions of the null k_{n} for several thicknesses d are figured out: $k_{\mathrm{n}}/k_0 = 1.52898 + i0.00082$ for $d = 29$ nm, $k_{\mathrm{n}}/k_0 = 1.52872 + i0.00007$ for $d = 29.5$ nm, $k_{\mathrm{n}}/k_0 = 1.52871 - i0.00009$ for $d = 29.6$ nm, and $k_{\mathrm{n}}/k_0 = 1.52877 - i0.00078$ for $d = 30$ nm. It can be clearly seen that the null crosses the real axis as the thickness d is changed from 29 to 30 nm.

For a given value of the thickness d, we assume that

$$k_{\mathrm{n}} = \beta + i\alpha, \tag{5}$$

where β and α are the real and imaginary parts of k_{n}, respectively. We then use a Taylor expansion to express the reflection function $r(k_x)$ at the null k_{n}:

$$r(k_x) = r(k_{\mathrm{n}}) + r(k_x)'|_{k_x=k_{\mathrm{n}}}(k_x - k_{\mathrm{n}}) + \frac{1}{2} r(k_x)''|_{k_x=k_{\mathrm{n}}}(k_x - k_{\mathrm{n}})^2 + \cdots, \tag{6}$$

where $r(k_{\mathrm{n}}) = 0$. Near the reflectivity minima, we may take into account only the first two terms of Eq. (6), so that the following expression is found,

$$r(k_x) \approx r(k_x)'|_{k_x=k_{\mathrm{n}}}[(k_x - \beta) - i\alpha]. \tag{7}$$

Thus the phase of $r(k_x)$, $\phi_{\mathrm{r}} = \mathrm{Im}[\ln r(k_x)]$ can be approximated as

$$\phi_{\mathrm{r}} \approx C + \arctan \frac{-\alpha}{k_x - \beta}, \tag{8}$$

where C is a constant. According to stationary-phase method[2], the lateral shift of the reflected beam can be calculated analytically as $S = -\frac{1}{k_0\sqrt{\varepsilon_1}} \frac{\mathrm{d}\phi_{\mathrm{r}}}{\mathrm{d}\theta}$. Thus the following expression for the lateral shift S is obtained,

$$S \approx \frac{-\alpha}{(k_x - \beta)^2 + \alpha^2} \cos\theta. \tag{9}$$

Formula (9) describes the approximate relation between the GH shift S and the null k_{n} and shows that the sign of GH shift is opposite to the sign of α. Furthermore, it is interesting to find that the longitudinal shift L approaches a maximum value at $k_x = \beta$ which can be written as

$$L_{\max} \approx -\frac{1}{\alpha}. \tag{10}$$

To demonstrate the validity of the above analysis, we now consider a two-dimensional (2D) Gaussian beam with amplitude independent on the y-coordinate (the y-axis directs out the plane of the paper). The incident magnetic field is assumed to be Eq. (11) on the $z = 0$ plane,

$$H_{\mathrm{in}}(x, z)|_{z=0} = \exp[-(\frac{x}{W_x})^2 + ik_{x0}x], \tag{11}$$

where $W_x = W/\cos\theta_0$, W is the beam width and θ_0 represents the incident angle for the center of the bounded beam, $k_{x0} = k_0\sqrt{\varepsilon_1}\sin\theta_0$. The time dependence $\exp(-i\omega t)$ is implied and suppressed.

Using the plane-wave spectrum of the incident field at $z = 0$, the magnetic field has the Fourier integral in the following form:

$$H_{\mathrm{in}}(x, 0) = \int_{-\infty}^{\infty} A(k_x) \exp(ik_x x) \mathrm{d}k_x, \tag{12}$$

where $A(k_x) = \frac{W_x}{2\sqrt{\pi}} \exp[-(\frac{W_x}{2})^2(k_x - k_{x0})^2]$, the reflected magnetic field can be written as

$$H_{\mathrm{r}}(x, z) = \int_{-\infty}^{\infty} r(k_x) A(k_x) \exp(ik_x x - ik_{1z}z) \mathrm{d}k_x, \tag{13}$$

where $z \leq 0$. The numerically calculated GH shift S is obtained by performing Eq. (13) at $z = 0$ and searching the position x at which $|H_r(x, 0)|$ is maximal[19]. If the profile of the reflected beam remains closely to the incident Gaussian profile, then $S = x \cos\theta_0$.

The numerical results for the GH shifts and the profiles of the reflected magnetic field are shown in Fig. 3, where the width W of the incident beam is chosen to be 0.1 mm. The theoretical results for the GH shifts S depicted in formula (9) as a function of the incidence angle θ depending on the thickness d are shown by the solid curves, and the corresponding numerical results are shown by the dotted curves. Suppose that the peak of the incident Gaussian beam depicted in Eq. (11) is unity, the profiles of the reflected magnetic field on the plane $z = 0$ under different conditions are shown in the insets of Fig. 3. It can be seen from Fig. 3 that the theoretical results coincide well with the numerical results. When the thickness d is above 29.55 nm, positive lateral shifts can be obtained, while below it the lateral shifts are negative. These results are good agreement with formula (9). Figure 3 also shows that the lateral shift is a peak when the incidence angle θ nears 87.936°, which also fits well with the requirement of formula (9). Although the intensities of the reflected magnetic field are relatively weak, calculation results show that the profiles of the reflected beam remain substantially Gaussian. Note that, if the intensity of the reflected beam can be availably detected, the lateral shifts S can be also photodetected by a position-sensitive detector[9,11,20].

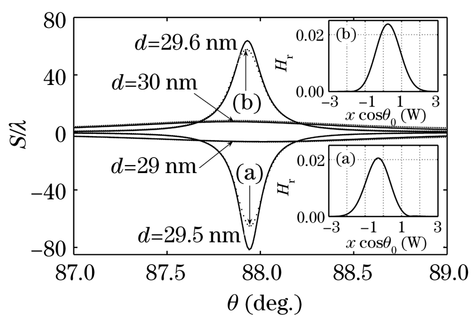

Fig. 3. Dependence of the GH shifts S on the angle of incidence θ for several values of the thickness d, and all the other parameters are the same as Fig. 1, where the solid curves represent the theoretical results, the dotted curves represent the corresponding numerical results for $W = 0.1$ mm. The inset (a) shows the profile of the reflected magnetic field on the plane $z = 0$ when $\theta_0 = 87.94°$ and $d = 29.55$ nm, the inset (b) corresponds to $\theta_0 = 87.93°$ and $d = 29.56$ nm.

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References

1. V. F. Goos and H. Hänchen, Ann. Physik **1,** 333 (1947).
2. V. K. Artmann, Ann. Physik **2,** 87 (1948).
3. V. Shah and T. Tamir, Opt. Commun. **37,** 383 (1981).
4. V. Shah and T. Tamir, J. Opt. Soc. Am. **73,** 37 (1983).
5. C. W. Huse and T. Tamir, J. Opt. Soc. Am. A **2,** 978 (1985).
6. T. Tamir, J. Opt. Soc. Am. A **3,** 558 (1986).
7. L.-G. Wang and S.-Y. Zhu, J. Appl. Phys. **98,** 043522 (2005).
8. P. Mazur and B. Djafari-Rouhani, Phys. Rev. B **30,** 6759 (1984).
9. X. Yin, L. Hesselink, Z. Liu, N. Fang, and X. Zhang, Appl. Phys. Lett. **85,** 372 (2005).
10. X. Liu, Z. Cao, P. Zhu, Q. Shen, and X. Liu, Phys. Rev. E **73,** 056617 (2006).
11. L. Chen, Z. Cao, F. Ou, H. Li, Q. Shen, and H. Qiao, Opt. Lett. **32,** 1432 (2007).
12. G. Shkerdin, J. Stiens, and R. Vounckx, J. Opt. A: Pure Appl. Opt. **5,** 386 (2003).
13. W. Lukosz and H. Wahlen, Opt. Lett. **3,** 88 (1978).
14. S. L. Chuang, J. Opt. Soc. Am. A **3,** 593 (1986).
15. M. Nevière and P. Vincent, J. Optics **11,** 153 (1980).
16. M. Born and E. Wolf, *Principle of Optics* (Cambridge University Press, London, 7th ed., 1999), p.758.
17. E. Kretschmann, Z. Physik **241,** 313 (1971).
18. J. J. Burke, G. I. Stegeman, and T. Tamir, Phys. Rev. B **33,** 5186 (1986).
19. C.-F. Li and X.-Y. Yang, Chin. Phys. Lett. **21,** 485 (2004).
20. H. Gilles, S. Girard, and J. Hamel, Opt. Lett. **27,** 1421 (2002).