Study on an improved five-interferogram phase-shifting algorithm

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Using traditional five-interferogram algorithm to unwrap phase for length measurement, the phase steps must be equal to $\pi/2$ exactly, but it is almost impossible to achieve in nanometer positioning technique. Aiming to overcome this defect of traditional five-interferogram algorithm, an improved five-interferogram algorithm is presented. This improved algorithm not only keeps the high accuracy of traditional five-interferogram algorithm, but also does not need absolute equal step to unwrap phase. Instead, this algorithm only needs measuring phase-shifting. With the numerical simulation, the improved five-interferogram algorithm shows high accuracy, high reliability, and feasibility in practice. It is very valuable for accurate length measurement with Fizeau interferometer and Fabry-Perot interferometer.

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Phase shifting algorithm is a high accurate and efficient algorithm to unwrap the phase difference between two interference beams. It has been applied to various situations, such as optical testing, surface profilometry, surface roughness estimation, and surface displacement measurement etc.[1−7]. The fundamental concept of phase shifting algorithm is that the phase difference in one period $(0 - 2\pi)$ can be determined by acquiring intensity signal from multiple interferograms[8−11]. Substantial results have been made in recent years to establish accurate error-compensating phase extraction algorithm[12,13]. A considerable number of these algorithms have been derived using various approaches, such as exact solutions, error minimization date filtering characteristic polynomials, least square estimation[6,8,14] and Fourier analysis techniques which represent interference fringes with non-cosine profile by a fundamental and certain higher harmonics[15]. Most of those approaches are based on mathematical model of two-beam interference. Generally speaking, the algorithm error can be ignored when these algorithms are used in the multiple-beam interference case (due to multiply reflection of standard plates)[16,17], such as the accurate length measurement with Fizeau interferometer and Fabry-Perot interferometer. According to numerical simulation, the three-interferogram algorithm or four-interferogram algorithm is much more sensitive than the traditional five-interferogram algorithm to the measuring error of interferogram intensity and phase error of shifting step[14,16]. But some strict condition must be met when using traditional five-interferogram algorithm. It means that the five shifting steps must be absolutely equal to one another, which is impossible to be realized by positioning technique[17]. In this paper, we will establish the mathematical model of multiple-beam interference, develop a novel algorithm, and study algorithm error due to ignore multiple-beam interference and other error sources by theoretical analysis and numerical simulation.

The mathematical model of intensity function is a base to various phase shifting algorithms. We will derive two kinds of intensity function models, which correspond with interferogram of two-beam interference and multiple-beam interference respectively. Figure 1 is principle schematic for length measurement using laser interference method.

In Fig. 1, P_1 is the standard plate with two reflection surfaces and P_2 is the surface to be measured. Between P_1 and P_2 is air, so the refraction index between the two plates is almost equal to 1. An incident light beam with incident angle i (it means the refraction angle is also equal to i) is reflected many times by the inner surfaces of P_1, P_2. The reflection ratio of the standard plate P_1 and the surface P_2 is ρ and ρ', respectively, $\rho = \left(\frac{A_1}{A_0}\right)^2$, $\rho' = \left(\frac{A_1'}{A_0'}\right)^2$, where A_0 is the amplitude of original incident beam, A_1 is the amplitude of first reflection beam from P_1, A_0' is the amplitude of transparent beam from P_1 to P_2, A_1' is the amplitude of first reflection beam from P_2. As shown in Fig. 1, one part of the incident beam is reflected from P_1, another part passes through P_1 and is reflected by P_2, then passes through P_1 and so on. The optical path difference between each two neighbor beams reflected by or passing through P_1 is equal to $2mh\cos(i)$ (m is air refraction index). For approximate vertical incident beam, the half wave loss due to reflect from P_2 must be taken into account. The amplitude of each reflection beam can be expressed in complex number

$$\begin{gathered}
\sqrt{\rho'(1-\rho)^2}A_0\mathrm{e}^{i(\omega t-\varphi-\pi)}, \\
(\rho',\rho)^{\frac{1}{2}}\sqrt{\rho'(1-\rho)^2}A_0\mathrm{e}^{i(\omega t-2\varphi-\pi)}, \\
\rho'\rho\sqrt{\rho'(1-\rho)^2}A_0\mathrm{e}^{i(\omega t-3\varphi-\pi)}, \\
\cdots\cdots \\
(\rho'\rho)^{\frac{n-1}{2}}\sqrt{\rho'(1-\rho)^2}A_0\mathrm{e}^{i(\omega t-n\varphi-\pi)}, \\
\cdots\cdots
\end{gathered}$$

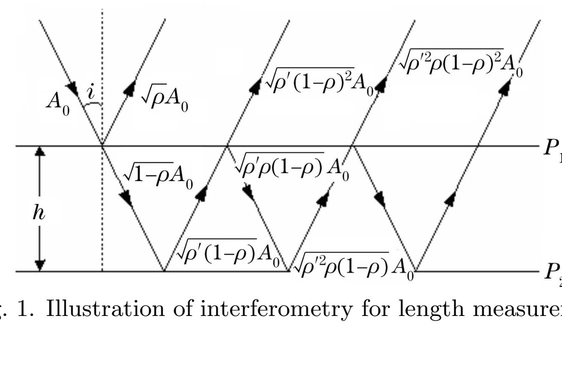

Fig. 1. Illustration of interferometry for length measurement.

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where φ is the phase difference between two-neighbor beams. Using Euler's formula $\cos\varphi = \frac{e^{i\varphi}+e^{-i\varphi}}{2}$, the composite intensity I can be expressed as below

$$I = A_0^2\left\{\rho + \frac{\rho'(1-\rho)^2[1+(\rho\rho')^n - 2(\rho\rho')^{n/2}\cos(n\varphi)] - 2\sqrt{\rho\rho'}(1-\rho)[\cos(\varphi) - \sqrt{\rho\rho'} - (\rho\rho')^{n/2}(\cos((n+1)\varphi) - \sqrt{\rho\rho'}\cos(n\varphi))]}{1+\rho\rho' - 2\sqrt{\rho\rho'}\cos(\varphi)}\right\}. \tag{1}$$

Equation (1) is the exact expression for intensity distribution of interferogram. When $n = 1$, that is, only two light beams are taken into account, the composite intensity I' can be expressed as

$$I' = A_0^2[\rho + \rho'(1-\rho)^2 - 2\sqrt{\rho\rho'}(1-\rho)\cos\varphi]. \tag{2}$$

Equation (2) shows a strict cosine dependent relationship between the interferogram intensity and the phase difference φ of two-neighbor beams. When $n = \infty$, that means, all light beam reflected from and passing through standard plate is taken into account, the composite intensity I can be expressed as

$$I = A_0^2\left[1 - \frac{(1-\rho)(1-\rho')}{1+\rho\rho' - 2\sqrt{\rho\rho'}\cos\varphi}\right]. \tag{3}$$

For unwrapping the phase φ caused by optical path difference between two neighbor interference beams, we neglect the minor interference beams and suppose the interferogram is formed only by interference of two main beams. Let intensity signal of five interferograms be I_1, I_2, I_3, I_4, I_5, respectively, the phase in each interferogram can be expressed as $\varphi - 2\varepsilon - k_1$, $\varphi - \varepsilon - k_2$, φ, $\varphi + \varepsilon + k_3$, $\varphi + 2\varepsilon + k_4$. The ε is shifting step, and k_1, k_2, k_3, k_4 are positioning deviation of shifting step. According to Eq. (2), we have the following equation groups:

$$\begin{aligned} I_1 &= A + B\cos(\varphi - 2\varepsilon - k_1), \\ I_2 &= A + B\cos(\varphi - \varepsilon - k_2), \\ I_3 &= A + B\cos\varphi, \\ I_4 &= A + B\cos(\varphi + \varepsilon + k_3), \\ I_5 &= A + B\cos(\varphi + 2\varepsilon + k_4), \end{aligned}$$

where $A = A_0^2[\rho + \rho'(1-\rho)^2]$, $B = A_0^2[-2\sqrt{\rho\rho'}(1-\rho)]$. Unwrapping this equation groups, we have

$$\tan\varphi = \frac{2k + k(\cos k_1 + \cos k_4) + \sin k_2 - \sin k_3}{\cos k_2 + \cos k_3 + k(\sin k_4 - \sin k_1)}\sin\varepsilon. \tag{4}$$

Let the shifting step be equal to $\pi/2$[12], then

$$\tan\varphi = \frac{2k + k(\cos k_1 + \cos k_4) + \sin k_2 - \sin k_3}{\cos k_2 + \cos k_3 + k(\sin k_4 - \sin k_1)}, \tag{5}$$

where $k = \frac{I_2 - I_4}{2I_3 - I_5 - I_1}$. According to Eq. (5), it is easy to unwrap phase φ. The phase error of φ only comes from two error sources: the measuring error of interferogram intensity and the measuring error of shifting step. Let's pay attention to a special case: supposing the positioning errors of shifting step k_1, k_2, k_3, k_4 are equal to 0, then the phase shifting of each step is $\pi/2$ exactly. According to Eq. (5), we have

$$\tan\varphi = \frac{2(I_2 - I_4)}{2I_3 - I_5 - I_1}. \tag{6}$$

Equation (6) is the classical formula of traditional five-interferogram algorithm[17]. The most significant difference between Eqs. (5) and (6) is that the exact shifting step of $\pi/2$ is demanded in Eq. (6) which is almost impossible to positioning technique. The following is an example using Eqs. (5) and (6) to unwrap φ, which suppose that each positioning error of shifting step is 0.8 nm, $\rho = 0.035$ (quartz glass material) and $\rho' = 0.35$ (silicon crystal).

In Fig. 2, the dot curve represents phase error in phase range of $0 - \pi/2$, which is produced by unwrapping phase by using Eq. (6). It means the phase error due to positioning error of shifting step with 0.8 nm reaches to 1% phase period and will strongly influence accuracy of traditional five-interferogram algorithm; The solid line represents phase error produced by Eq. (5) which is completely covered with horizontal axis, it means that the phase error is 0 in phase range of $0 - \pi/2$.

When shifting step ε is not equal to $\pi/2$ or the shifting step is not equal to each other exactly, the traditional five-interferogram algorithm will cause a significant phase error. In this case the improved five-interferogram algorithm (IFIA) is a reasonable choice. As follows, we will discuss the phase error of IFIA due to ignorance multiple-beam interference and other error sources, such as intensity error and shifting step error by theoretical analysis and numerical simulation.

Get local derivative of I_1, I_2, I_3, I_4, I_5 with respect to φ of Eq. (5), let $\varepsilon = \pi/2$, we have sub-uncertainty u_1 due to ignorance of multiply beam interference

$$\begin{aligned} u_1 = \Delta\varphi &= \sum_{i=1}^{5}\frac{\partial\varphi}{\partial I_i}\Delta I_i \\ &= F\left\{\frac{\sin\varphi}{4}(\Delta I_1 - 2\Delta I_3 + \Delta I_5) + \frac{\cos\varphi}{2}(\Delta I_2 - \Delta I_4)\right], \end{aligned} \tag{7}$$

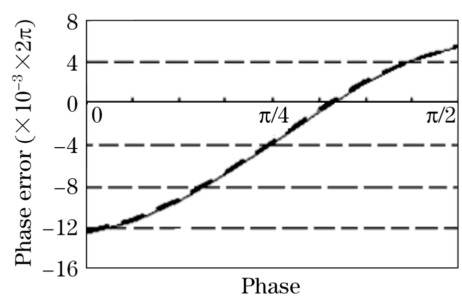

Fig. 2. Phase error in phase range of $0 - \pi/2$.

where $F = \frac{2(\cos k_2+\cos k_3)+\cos(k_1-k_2)+\cos(k_1+k_3)+\cos(k_2+k_4)+\cos(k_4-k_3)}{2[\cos k_2+\cos k_3+k(\sin k_4-\sin k_1)]^2}$. ΔI_i ($i = 1, 2, 3, 4, 5$) can be calculated by Eqs. (2) and (3), that is,

$$\frac{\Delta I_i}{A_0^2} = \rho + \rho'(1-\rho)^2 - 1 - 2\sqrt{\rho\rho'}(1-\rho)\cos\varphi_i + \frac{(1-\rho)(1-\rho')}{1+\rho\rho'-2\sqrt{\rho\rho'}\cos\varphi_i}. \quad (8)$$

According to Eq. (7), let value of k_1, k_2, k_3, k_4 randomly, for example, be equal to 5 nm, the reflection ratio of surface to be measured $\rho' = 0.35$ (surface of crystal silicon sphere), and the reflection ratio of the standard plate ρ changes from 0.035 to 0.35, then the sub-uncertainty u_1 of phase error is less than 0.01%, which can be ignored even in high precision measurement[18]. When the reflection ratio of standard plate is equal to 0.35, the phase error is up to 0.5% which is a significant phase error and cannot be ignored in an accurate measurement. The standard plate is usually made from quartz material, and the reflection ratio of ρ is much less than 0.1. So the sub-uncertainty u_1 is much less than 0.01%, which can be ignored.

The phase error also depends on the intensity measurement error. For IFIA, the phase sub-uncertainty u_2 due to the intensity measurement error can be calculated with Eq. (5) and be expressed as

$$u_2 = F\sqrt{[(\Delta I_2)^2 + (\Delta I_4)^2][\frac{\cos\varphi}{2}]^2 + [(\Delta I_1)^2 + (2\Delta I_3)^2 + (\Delta I_5)^2][\frac{\sin\varphi}{4}]^2}. \quad (9)$$

Equation (9) shows the relationship between the phase error and the measuring error of interferogram intensity. For more simple and easy to understand this relationship, suppose that the measuring error of each shifting step is exact equal to each other, and the maximal error range of relative intensity $\frac{\Delta I_{i\max}}{I_{\max}}$ changes from 0.001 to 0.035 when $\rho = 0.035$, $\rho' = 0.35$. Figure 3 expresses the relationship schematic of the phase error of this algorithm in one phase period versus intensity measuring error (let value of k_1, k_2, k_3, k_4 be 5 nm randomly).

Figure 3 shows, the increase of phase error goes together with the increase of intensity measuring error. When the intensity measuring error is less than 1% which is easy to realize, the phase error is better than 0.1% phase period. According to this simulation, if measuring error of interference intensity is 0.5%, we have u_2=0.04% phase period.

When using the phase shifting algorithm to unwrap phase for length measurement, it is almost impossible to control shifting step to $\pi/2$ exactly. In the previous discussion of this article, the phase in each step can be expressed as '$\varphi - 2\varepsilon - k_1$, $\varphi - \varepsilon - k_2$, φ, $\varphi + \varepsilon + k_3$, $\varphi + 2\varepsilon + k_4$'. Obviously, it can be allowed to let ε equal to $\pi/2$ by changing the value of k_1, k_2, k_3, k_4. But for

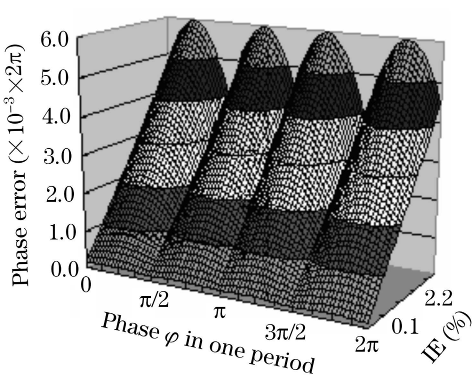

Fig. 3. Phase error of the improved algorithm versus intensity measuring error. IE: intensity error.

decreasing the absolute value of k_1, k_2, k_3, k_4 to improve accuracy of phase shifting algorithm, we usually let the shifting step be equal to ε, close to $\pi/2$. Here we will discuss how the positioning deviation of shifting step $\Delta\varepsilon$ ($\Delta\varepsilon$ is equal to $\varepsilon - \pi/2$) influences the sub-uncertainty u_3 of IFIA. Get derivative of ε with respect to Eq. (4), then,

$$u_3 = F\frac{\sin(2\varphi)\cos(\varepsilon)}{2}\Delta\varepsilon. \quad (10)$$

Because $\Delta\varepsilon$ is a very small quantity, and $\varepsilon = 90° + \Delta\varepsilon$, approximately, we have $\sin(\Delta\varepsilon/2) = \Delta\varepsilon/2$, Eq. (10) can be changed to

$$u_3 = F\frac{\sin(2\varphi)}{2}\sin(\frac{\Delta\varepsilon}{2})\Delta\varepsilon. \quad (11)$$

When positioning error of each shifting step is less than 0.8 nm, the phase error of IFIA based on the two-beam interference is much less than 10^{-4} (0.01%) phase period[18]. We give a special calculation by using Eq. (11). In a precision length measuring system, when the positioning error of shifting step is 2 nm, the measuring error of length is 0.02 nm. This result shows the IFIA based on the two-beam interference is very insensitive to the positioning error of the shifting step. In fact, in most cases, there is not any positioning error of shifting step to IFIA, because we make the shifting step ε be equal to $\pi/2$ exactly. Any positioning error of shifting step is included in k_1, k_2, k_3, k_4, so the u_3 does not exist.

For evaluating the phase error (sub-uncertainty u_4) due to the measuring error of shifting step, get derivative of k_1, k_2, k_3, k_4 with respect to Eq. (5), then

$$u_4 = \sqrt{\sum_{i=1}^{4}(\frac{\partial\varphi}{\partial k_i}\Delta k_i)^2}, \quad (12)$$

where

$$\frac{\mathrm{d}\varphi}{\mathrm{d}k_1}=\frac{k\cos^2\varphi(k+2k\cos k_1+k\cos(k_1+k_4)+\sin(-k_1+k_2)-\sin(k_1+k_3))}{(\cos k_2+\cos k_3-k(\sin k_1-\sin k4))^2},$$
$$\frac{\mathrm{d}\varphi}{\mathrm{d}k_2}=\frac{\cos^2\varphi(1+\cos(k_2+k_3)+2k\sin k_2-k\sin(k_1-k_2)+k\sin(k_2+k_4))}{(\cos k_2+\cos k_3-k(\sin k_1-\sin k_4))^2},$$
$$\frac{\mathrm{d}\varphi}{\mathrm{d}k_3}=\frac{\cos^2\varphi(-1-\cos(k_2+k_3)+2k\sin k_3+k\sin(k_1+k_3)+k\sin(k_3-k_4))}{(\cos k_2+\cos k_3-k(\sin k_1-\sin k_4))^2},$$
$$\frac{\mathrm{d}\varphi}{\mathrm{d}k_4}=-\frac{k\cos^2\varphi(k+2k\cos k_4+k\cos(k_1+k_4)+\sin(k_2+k_4)+\sin(-k_3+k_4))}{(\cos k_2+\cos k_3-k(\sin k_1-\sin k_4))^2}.$$

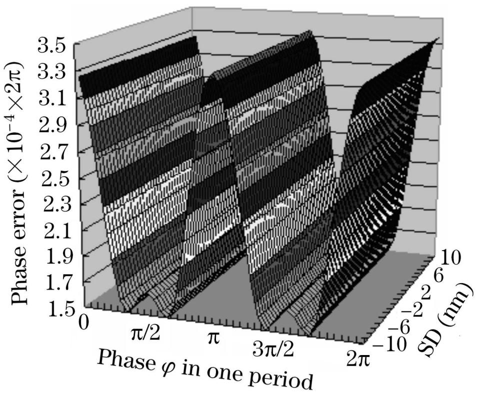

Fig. 4. 3D schematic diagram of phase error versus phase φ and deviation k_4. SD: step deviation.

Because the measuring error of shifting step is random, the "Root-Sun-Square" method is reasonable to composite the phase error u_4. As a special case in Eq. (7), if $k_1 = k_4 = 2\Delta\varepsilon$, $k_2 = k_3 = \Delta\varepsilon$, the result of $\Delta\varphi$ is completely equivalent with the evaluative result by using Eq. (6): $\Delta\varphi = \sin(2\varphi)\sin^2(\Delta\varepsilon/4)$[12]. In such a case, each positioning error of shifting step must be equal to $\pi/2$ exactly, which is almost impossible to positioning technique. For more reasonable evaluation of the phase error, the deviation value of shifting step (by actual measurement) k_1, k_2, k_3, k_4 is needed, and the measuring error of the deviation value of shifting step Δk_1, Δk_2, Δk_3, Δk_4 should be added to Eq. (12). Within the range of $-10-10$ nm, we suppose the value of k_1, k_2, k_3 is randomly −5, 6, 10 nm, respectively, and the absolute value of measuring error Δk_1, Δk_2, Δk_3, Δk_4 are less than 0.8 nm, and thus take 0.8 nm to put in Eq. (12). Figure 4 is the three-dimensional (3D) schematic diagram of phase error versus phase φ and positioning deviation of shifting step k_4 in the range $-10-10$ nm.

In conclusion, when the measuring error of shifting step is less than 0.8 nm, the phase error of IFIA is about 10^{-4} phase period (take $u_4 = 0.035\%$). The phase error of IFIA is almost irrelevant to the change of deviation k_4 within the range of $-10-10$ nm (in fact, we have done large number of numerical calculation, all of which produce the same result). This result indicates that the randomly supposed value of k_1, k_2, k_3 is reasonable.

If measuring error of interferogram intensity is 0.5% and all of measuring error of shifting step Δk_1, Δk_2, Δk_3, Δk_4 are less than 0.8 nm (this is easy to realize with nanometer measurement technique), the composite standard uncertainty U of IFIA can be expressed as follows:

$$U=\sqrt{u_1^2+u_2^2+u_3^2+u_4^2}=0.053\%\ \text{(phase period)}.$$

In order to overcome the defects of the traditional five-interferogram algorithm, which demands the equal shifting step strictly and is almost impossible to achieve, a new IFIA is developed. This algorithm not only keeps the high accuracy of traditional five-interferogram algorithm, but also does not need absolute equal step to unwrap phase. Instead, this algorithm only needs measuring phase-shifting. With the results of numerical simulation, the improved algorithm shows high accuracy of 0.05% phase period. It means, for length measurement with 633 laser, the uncertainty of length can reach to 0.2 nm when using IFIA to unwrap phase.

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