Real-time restoration of rotational blurred image using gradient-loading

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The key to the restoration of rotational motion blurred image is how to restore the image under a low cost and to correct the irreversibility of the degradation function matrix. Based on the special qualities of degradation function matrix and precise deduction in space-domain, we present a new approach using gradient-loading for restoration of rotational blurred image. By easily adding a gradient operator, the irreversibility of the original matrix is corrected and can be applied for inverse filtering then. Gradient-loading is the optimized approach which combines the advantages of both the approaches using constrained least square filtering and traditional diagonal-loading. Compared with the approach using least square filtering, its peak signal-to-noise ratio (PSNR) is improved from 3.18 to 6.46 dB, while the computing time is reduced to $1/2-1/3$. Experimental results demonstrate the effectiveness, noise-resistibility, robustness, and low complexity of this approach, which make it more suitable for real-time environment.

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Rotational motion blur often occurs in the field of machine vision and television guidance. The rotational motion between the camera and the target during the image capturing will result in serious degradation of image. Such degradation will cause great trouble for the succeeding jobs such as video stabilization and image matching. So to remove the rotational motion blur is an inevitable problem. Li *et al.* presented the approach of restoration of blurred images with phase diversity-based blind deconvolution[1]. Hu *et al.* introduced the increamental Wiener filters into the image deconvolution of wavefront sensing[2]. These approaches are proved effective in the restoration of common motion blur. However, rotational motion blur is much more complex than common one, because it is space-variant. The extent of blur becomes more and more serious along the radius from the center.

To solve this problem, Bonmassar *et al.* presented a solution using geometric transformation[3]. They try to transform the special problem of rotational blur to the common motion blur by gray-level interpolation. But huge amount of gray-level interpolation causes much deviation and noise, complexity of algorithm, and costs too much computing time for real-time environment. Based on the constrained least square filtering[4,5], Hong *et al.* introduced the approach for rotational motion blurred image by the means of deconvolution along the blur paths[6]. Through constraint of least square filtering, the irreversibility in the restoration is corrected and more satisfied result is got with less cost. But there is a kind of hypercorrectness so that the restored image is oversmoothened and the complexity is still too high for real-time environment.

Based on the study on the degradation function matrix of rotational blur, we present a new approach for restoration of rotational blur image using gradient-loading. Compared with the approach using constrained least square filtering, its peak signal-to-noise ratio (PSNR) is improved from 3.18 to 6.46 dB, while the computing time is reduced to $1/2-1/3$. This approach further improves the visual effect, anti-capability, and robustness with great reduction in the complexity of algorithm, which makes it more suitable for real-time environment.

In the process of rotational motion, the total exposure at any point is obtained by integrating the instantaneous exposure over the time interval during which the shutter is open. According to the distance r from the rotational center, the whole image can be divided into a series of concentric circles, along which the intensity values of some pixels are accumulated. Such circles are defined as the blur-path to the pixels with same radius. So the problem becomes how to remove the blur along the particular blur-path.

Let $f(x,y)$ be the original value of the pixel and $g(x,y)$ be the degraded one, then in the exposure time T and angular velocity ω, there is[5]

$$g\left(x,y\right)=\frac{1}{T}\int_0^T f\left(x-r\cos(\omega t),y-r\sin(\omega t)\right)\mathrm{d}t,\qquad r=\sqrt{x^2+y^2}. \tag{1}$$

Using polar coordinate, it becomes

$$g\left(r,\theta\right)=\frac{1}{T}\int_0^T f\left(r,\theta-\omega t\right)\mathrm{d}t. \tag{2}$$

Setting $s=r\omega t$, $a=r\omega T$, $l=r\theta$, Eq. (2) becomes

$$g\left(r,l\right)=\frac{1}{a}\int_0^a f\left(r,l-s\right)\mathrm{d}s. \tag{3}$$

For particular blur-path, Eq. (3) is revised as

$$g_r\left(l\right)=\frac{1}{a_r}\int_0^{a_r} f_r\left(l-s\right)\mathrm{d}s. \tag{4}$$

After unwinding the blurring path and using discrete coordinate i to express l, the formula in the discrete form

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can be presented as

$$g_r(i) = \frac{1}{a_r}\sum_{x=0}^{a_r-1} f_r(i-x), \tag{5}$$

where $i = 0, 1, \cdots, N_r - 1$; $g_r(i)$ and $f_r(i)$ are the sequences of original and degraded gray values of the pixels along blurring path with the period of N_r.

Let $h(x) = \begin{cases} 1/a & 0 \le x \le a-1 \\ 0 & a \le x \le N-1 \end{cases}$, where $h(x)$ is the point spreading function (PSF) of the blurring path, Eq. (2) can be expressed as[5]

$$g(i) = \sum_{x=0}^{N-1} f(x)\, h(i-x). \tag{6}$$

Thus, in matrix form, it can be expressed as $g = Hf$, where g and f are the $N \times 1$ vectors of gray values corresponding to $f(i)$ and $g(i)$ $(i = 0, 1, \cdots, N_r - 1)$. Here H is exactly the matrix form of degradation function. As an $N \times N$ matrix resulting from the PSF, H is changeable for different blurring-path. Because its inverse matrix does not have to exist, there is irreversibility in the inverse filtering. On the other hand, simply using inverse filtering will cause instability if it contains noise in the process of capturing image.

Hunt presented the constrained least square filtering in the restoration of common motion blur by adding Laplace operator as constrainer item to solve the irreversibility of matrix. The final result is[6]

$$f = \left(H^{\mathrm{T}}H + \lambda D^{\mathrm{T}}D\right)^{-1} H^{\mathrm{T}}g. \tag{7}$$

But it is still not suitable for real-time environment because of too much computation with high-order matrix. It seems to be a kind of hypercorrectness because the restored image is something over-smoothed, which may cause some trouble in the succeeding job.

As mentioned before, degradation function matrix H is a cyclic matrix, thus it has some particular characteristics that can be taken advantage of. Next we will make precise deduction on it to find these characteristics that can be applied in our new approach.

Define the basic cyclic matrix as

$$D = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & 1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & 1 \\ 1 & 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}. \tag{8}$$

Obviously, $D, D^2, D^3, \cdots, D^n$ are all cyclic matrices, and $D^n = I$. Thus, for general cyclic matrix

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & \cdots & a_{n-1} \\ a_n & a_0 & \cdots & \cdots & \cdots & a_{n-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_1 & a_2 & \cdots & \cdots & a_{n-1} & a_0 \end{bmatrix}, \tag{9}$$

according to Eq. (8), it can be expressed as $A = a_0 I + a_1 D + a_2 D^2 + \cdots + a_{n-1} D^{n-1}$.

Setting $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1}x^{n-1}$, then we can get

$$A = f(D). \tag{10}$$

It is proved that the basic cyclic matrix D can be diagonalized[7−9], and it is similar to the diagonal matrix $\Lambda = \mathrm{diag}(\lambda_0, \lambda_1, \lambda_2, \cdots, \lambda_{n-1})$, where $\lambda_0, \lambda_1, \lambda_2, \cdots, \lambda_{n-1}$ are the eigenvalues of D, that is,

$$D = P^{-1}\Lambda P,$$

$$\lambda_k = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n} = \mathrm{e}^{i\frac{2k\pi}{n}},$$

$$k = 0, 1, 2, \cdots, n-1. \tag{11}$$

Thus, combining Eqs. (10) and (11), it yields

$$\begin{aligned} A = f(D) = f(P^{-1}\Lambda P) &= a_0 P^{-1}P + a_1 P^{-1}\Lambda P \\ &+ a_2 P^{-1}\Lambda^2 P + \cdots + a_{n-1}P^{-1}\Lambda^{n-1}P \\ &= P^{-1}\mathrm{diag}(f(\lambda_0), f(\lambda_1), \cdots, f(\lambda_{n-1}))P. \end{aligned} \tag{12}$$

Equations (10) − (12) prove that general cyclic matrix can also be diagonalized, and the eigenvalues are

$$\omega_k = f(\lambda_k) = \sum_{t=0}^{n-1} a_t \lambda_k^t, \tag{13}$$

where $\lambda_k = \mathrm{e}^{i\frac{2k\pi}{n}}$, $k = 0, 1, 2, \cdots, n-1$.

By using the definition in Eq. (8), we can use basic cyclic matrix to express the degradation function matrix,

$$H = \frac{1}{a}(I + D + D^2 + \cdots + D^{a-1}). \tag{14}$$

According to Eq. (13), the eigenvalues of H are

$$\xi_k = \frac{1}{a}\sum_{n=0}^{a-1} \mathrm{e}^{i\frac{2\pi kn}{N}}, \quad k = 0, 1, 2, \cdots, N-1. \tag{15}$$

Only when all of the eigenvalues of H are nonzero, will the matrix be reversible. For any of $k = 0, 1, 2, 3, \cdots, N-1$, it requires ka/N not to be integer, otherwise, H will be irreversible, which will cause the irreversibility of inverse filtering. In fact, such qualification is too hard to meet when the order of H is high, so there must be a solution with amendment.

Diagonal-loading is exactly such a solution, which is firstly used in beam-forming to correct the irreversibility of covariance matrix and suppress the interference of side lobes[10,11]. Elnashar *et al.* proved the robustness of diagonal-loading in correcting the invalidation of matrix[12].

Though the restoration approach based on traditional diagonal-loading avoids the irreversibility of degradation function matrix under such a low cost, the direct use of traditional diagonal-loading in image processing is still a problem. By adding a positive λ, the small eigenvalues of system are pulled approximate to it with little influence on the main eigenvalues relatively. Thus the side lobes are restrained and could be recognized clearly from the main ones. But for image processing, such high-frequency emphasis is not permitted because it will make the noise "whitened" and cause some salt-pepper noise.

Such phenomenon is not allowed in image restoration because it will cause the reduction of system anti-noise capability.

According to the above analysis, gradient-loading shall be a perfect solution. Define a gradient operator

$$\nabla f = f(x+1) - f(x). \tag{16}$$

Let ∇ be the matrix of gradient operator, that is,

$$\nabla = \begin{bmatrix} 1 & 0 & \cdots & 0 & -1 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & \cdots \\ \cdots & \cdots & -1 & 1 & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}. \tag{17}$$

Obviously, ∇ is also a cyclic matrix.

Thus, according to Eqs. (9) and (10), ∇ can be expressed as

$$\nabla = 1 - D^{n-1}. \tag{18}$$

Similar to the processing of diagonal-loading, let

$$\hat{H} = H + \lambda\nabla, \tag{19}$$

then $\hat{H}$ is also a cyclic matrix and can be expressed by the basic cyclic matrix as

$$\hat{H} = \frac{1}{a}(I + D + D^2 + \cdots + D^{a-1} + \lambda\nabla). \tag{20}$$

According to Eqs. (10) – (12), eigenvalues of $\hat{H}$ can be expressed as

$$\xi_k = f(\lambda_k) = \frac{1}{a}\sum_{n=0}^{a-1} \mathrm{e}^{i\frac{2\pi kn}{N}} + \lambda = M + \lambda\nabla, \tag{21}$$

where $M = \frac{1}{a}\sum\limits_{n=0}^{a-1} \mathrm{e}^{in\theta}$ denotes the eigenvalues of H, $\theta = \frac{k}{N}2\pi$, $k = 0, 1, 2, \cdots, N-1$.

Then the eigenvalues of $\hat{H}$ can be expressed as

$$\xi_k = M + \lambda(1 - \mathrm{e}^{i(N-1)\theta}). \tag{22}$$

Suitable coefficient shall be determined for gradient-loading to keep ξ_k nonzero. As shown in Fig. 1, the vector of M must be on the circle whose center is $(\frac{1}{2a}, \frac{1}{2a\tan\frac{\theta}{2}})$ and radius is $\frac{1}{2a\sin\frac{\theta}{2}}$. Thus ξ_k will be nonzero if the loading item is out of the circle that is center symmetric to the one of M. Meanwhile, λ should also keep positive so that the restored image will be smoothed instead of sharpened. Combining all of the restrictions above, the final form of estimate value can be expressed as

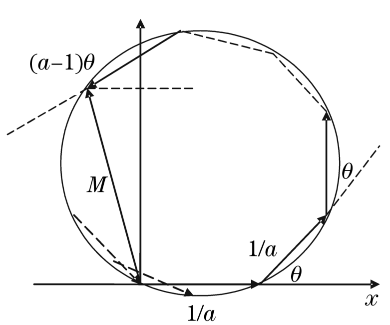

Fig. 1. Analysis of the eigenvalues in the form of vectors.

$$\hat{f} = (H + \lambda\nabla)^{-1}g, \tag{23}$$

where $\lambda > 0$ and $\lambda \neq \frac{1}{2a\tan\frac{\theta}{2}}$.

By such a simple form, the irreversibility of the degradation function matrix H is corrected. Compared with diagonal-loading, gradient-loading has the totally same form without any rise of algorithm complexity. Compared with the traditional solution using constrained least square filtering as Eq. (7), it avoids most of the multiplication of high-order matrix, which makes it more suitable for real-time environment.

We have applied the new approach to restore the rotational blurred images, as shown in Fig. 2. Let the original image be rotationally blurred by 15°, and the rotational center is exactly the center of image. Figures 2(c)—(e) are the restoration results by different approaches.

Besides the visual effect, we also use PSNR and improvement of signal-to-noise ratio (ISNR) as objective measures. ISNR can be defined in decibels as

$$\mathrm{ISNR} = 10\log_{10}\frac{\sum\limits_{m}\sum\limits_{n}|f-g|^2}{\sum\limits_{m}\sum\limits_{n}\left|f-\hat{f}\right|^2}, \tag{24}$$

where m, n are the size of the image.

And we also use the PSNR defined as

$$\mathrm{PSNR} = 10\log_{10}\frac{\sum\limits_{m}\sum\limits_{n}255^2}{\sum\limits_{m}\sum\limits_{n}\left|f-\hat{f}\right|^2}. \tag{25}$$

The result using diagonal-loading gets a best visual effect with high sharpness (Fig. 3(d)). But as analyzed before, it shows the effect of high-frequency emphasis. The image is corrupted by salt-pepper noise lightly. Its ISNR = 31.36 dB and PSNR = 3.18 dB. Figure 3(c) is the result using constrained least squares. Its visual effect is not so good, but it get a much better result in signal-to-noise ratio (SNR), for its ISNR = 33.81 dB and PSNR = 5.63 dB. Still the result is not satisfying because it is over smoothened as proved in the previous analysis.

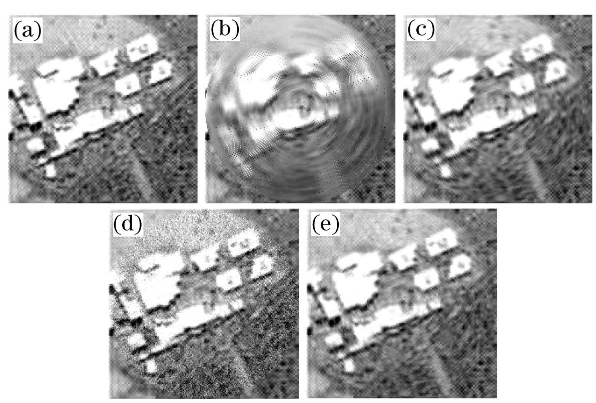

Fig. 2. Restoration of practical image. (a) Original image; (b) rotationally blurred image by 15°; (c) result of constrained least square filtering; (d) result of diagonal-loading filtering; (e) result of gradient-loading filtering.

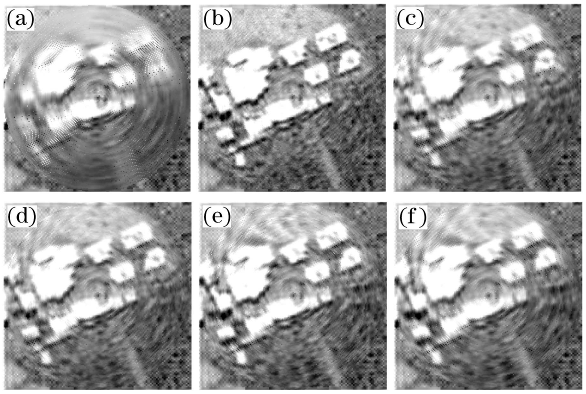

Fig. 3. Experiment on anti-noise capability and robustness. (a) 15° blurred image corrupted by Gaussian noise with SNR = 10 dB; (b) image restored using gradient-loading; (c), (d), (e), (f) are images restored by mistaking the parameters as 13°, 14°, 16°, 17°, respectively.

And its computing time is several times longer than diagonal-loading. As shown in Fig. 2(e), the approach using gradient-loading is a good compromise. It preserves the sharp transition and thus keep a high sharpness and better visual effect. On the other hand, it protects the image from being corrupted by noise as diagonal-loading does. For the result, we get ISNR = 34.64 dB and PSNR = 6.46 dB, that is the best one among the three. Thus the approach using gradient-loading perfectly balanced the anti-noise capability and sharpness and could be the best solution. And its computing time is almost the same as diagonal-loading.

For practical use, there are still many random factors such as noise. As shown in Fig. 3(a), there is a 15° blurred image, which is corrupted by Gaussian noise with SNR = 10 dB. Through approach using gradient-loading to restore the image, we restore the image in Fig. 3(b). The result shows a good visual effect with ISNR = 34.22 dB and PSNR = 6.03 dB.

On the other hand, robustness means fault-tolerance. It requires the system to work efficiently even though there is minor mistake. We simulate the instance that there are some errors about the rotational blur angle with a range of $\pm 2°$. The parameters are mistaken as 13°, 14°, 16°, and 17°, respectively, as shown in Fig. 3(c)—(f). In such situation, it will cause a strong instability if we still use inverse filtering or diagonal-loading. By using the gradient-loading filtering, the error of parameter causing some difficulty on the restoration are still acceptable. The PSNRs of the restored image are 3.71, 4.23, 4.01, and 3.57 dB, respectively.

All the experiments are processed on the platform of Matlab7.0, using a computer with Pentium4 1.4 GHz. As shown in Table 1, images of different sizes are restored using the three approaches respectively, and the whole processing time is recorded. It shows the trend that the processing time has a rise of exponential level with the increase of image size. For the approach using gradient-loading, the processing time is almost the same as the one using diagonal-loading, and both are $1/2 - 1/3$ less than the time for constrained least square filtering. Thus, we can see that gradient-loading combines the advantages of the other two and even does better. Compared with diagonal-loading, it improves the visual effect, anti-noise capability, and robustness without any rise of time complexity. Both diagonal-loading and gradient-loading have a great reduction on complexity than constrained least square filtering, which makes them more suitable for real-time environment.

Table 1. Comparison of the Processing Time

Image Size (pixel)	50×50	100×100	200×200
Constrained Least Square	0.124 s	1.100 s	17.789 s
Diagonal-Loading	0.061 s	0.364 s	5.803 s
Gradient-Loading	0.066 s	0.421 s	6.339 s

In brief, this paper presents the approach for restoration of rotational blur image based on gradient-loading. In comparison with former solutions such as constraint least square or diagonal-loading, our approach demonstrates the effectiveness, anti-noise capability, and robustness. At the same time, it greatly reduces the complexity and computing time, which makes it more suitable for real-time environment.

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