Generation of three-mode W-type entangled coherent states in free-travelling optical fields

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We propose a scheme for generation of three-mode W-type entangled coherent states (ECSs) in free-travelling optical fields by using a single-photon source, coherent state sources, beam splitters, photodetectors, and three-mode cross-Kerr media. The scheme consists of a Mach-Zehnder interferometer (MZI) in which each arm contains a cross-Kerr medium. We calculate the success probability of the generated W-type ECSs, and the total success probability is unity under the ideal conditions.

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In the last ten years, much attention has been paid to understandings and applications of entangled states in the quantum optics and quantum information fields in connection with quantum[1−5], cryptography[6−9], quantum computing[10−12], and quantum dense coding[13], hence, quantum entanglement has been viewed as an essential resource for quantum information processing. Dür *et al.*[14] have shown that there were two inequivalent classes of tripartite entanglement states under stochastic local operation and classical communication (LOCC). One is GHZ-type, which is represented as

$$|\mathrm{GHZ}\rangle = c_1|000\rangle + c_2|111\rangle, \tag{1}$$

by taking an appropriate basis $|0\rangle$, $|1\rangle$ for each qubit or logical qubit. Here $c_{1,2}$ are the normalization coefficients. The other is W-type, which is represented as

$$|W\rangle = a_1|101\rangle + a_2|011\rangle + a_3|110\rangle, \tag{2}$$

where $a_{1,2,3}$ are the normalization coefficients. The W state has some interesting properties. On the one hand, it retains bipartite entanglement when any one of these three qubits is traced out and thus it is much more robust than the GHZ state. On the other hand, there exist some tasks for which the GHZ-type states are not suitable but the W-type states are, for example, the remote symmetric entangling problem[5]. W states can also be used to test quantum nonlocality without inequality[15]. For the above reasons a few schemes to produce W states have been produced[16−26]. Scalable generation of multi-atom W states with a single resonant interaction was proposed by Zheng[16]. Guo *et al.*[17] showed that such states could also be generated via step-by-step resonant interaction of atoms with a cavity field. Fidio *et al.*[18] gave an approach to prepare W-type states of three distant atoms. Yu *et al.*[19] also proposed a scheme to generate GHZ and W states of three distant atoms using the effects of quantum statistics of indistinguishable photons. Generation of the polarization entangled W state from photons emitted by parametric down-conversion was discussed by Yamamoto *et al.*[20]. Zou *et al.*[21] also proposed a scheme to generate four-photon W states. Experimental realization of a three-qubit entangled W state was reported in Ref. [25].

On the other hand, entangled coherent states (ECSs)[2,11,27−31] have been suggested not only to implement some schemes for quantum information processing, such as quantum teleportation[2,5,11,29−31], and to test some fundamental problems in quantum mechanics[32,33], but also to improve the weak-force detection[34]. Ba[5] has shown an optimal quantum information processing via multimode W-type ECSs. In particular, it was shown that there exists a quantum information protocol which can be done only with W-type ECSs while GHZ-type ECSs fail to accomplish such a task. Recently, Jeong and Ba[24] proposed a scheme to generate three-mode W-type ECSs in free-traveling optical fields via two-mode cross-Kerr media. However, the success probability of their scheme is no more than 3/5. We also proposed a scheme to generate four-mode W-type ECSs via two-mode cross-Kerr media[26], the total success probability is unity under the ideal conditions. While the scheme cannot be used to generate three-mode W-type ECSs. In this letter, we suggest an optical scheme to generate three-mode W-type ECSs in free-travelling optical fields via three-mode cross-Kerr media with the total success probability approaching to unity under the ideal conditions.

Figure 1 is the proposed device of generation of W-type ECSs. It consists of a Mach-Zehnder interferometer

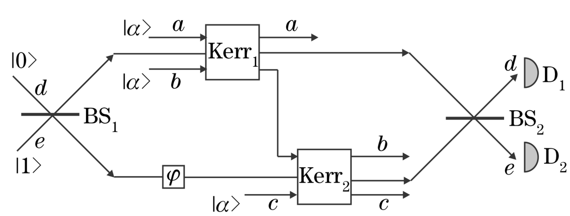

Fig. 1. Schematic diagram to generate W-type ECSs. A MZI, injected with a single photon in the e mode, with three-mode cross-Kerr media placed in each arm. The Kerr media are fed into $|1\rangle_e|\alpha\rangle_a|\alpha\rangle_b$ and $|0\rangle_d|\alpha\rangle_b|\alpha\rangle_c$, respectively. D_1 and D_2 are the detectors at output.

(MZI) containing in each arm three-mode cross-Kerr media which are fed into coherent states $|\alpha\rangle_a$, $|\alpha\rangle_b$, and $|\alpha\rangle_c$. A vacuum and single photon state, $|0\rangle_d|1\rangle_e$, is fed into the beam splitter BS_1 of the MZI firstly, where d and e represent the modes internal to the MZI, and then are fed into the beam splitter BS_2 after passing through the cross-Kerr media.

As is well known, a three-mode cross-Kerr interaction is described by the Hamiltonian[35]

$$\hat{H} = -\hbar\chi\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}\hat{c}^\dagger\hat{c}, \quad (3)$$

where χ is proportional to the fifth-order nonlinear susceptibility, $\chi^{(5)}$, of the medium. $\hat{a}^\dagger(\hat{b}^\dagger, \hat{c}^\dagger)$ and $\hat{a}(\hat{b}, \hat{c})$ are the creation and annihilation operators for modes $a(b, c)$, respectively. The cross-Kerr interaction leads to the following unitary evolution operator:

$$\hat{K}_{abc}(\tau) = \mathrm{e}^{i\tau\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}\hat{c}^\dagger\hat{c}}, \quad (4)$$

where $\tau = \chi t$ with t being the evolution time. It is easy to see that when $\chi t = \pi$, the action of the three-mode cross-Kerr unitary evolution operator on one mode with Fock state $|n\rangle_i$ ($n = 0$ or 1) and two modes with the coherent state input $|\alpha\rangle_j|\alpha\rangle_k$ is given by

$$\hat{K}_{ijk}(\pi)|1\rangle_i|\alpha\rangle_j|\alpha\rangle_k = \frac{1}{2}|1\rangle_i(N_+|\alpha\rangle_j|\alpha_+\rangle_k + N_-|-\alpha\rangle_j|\alpha_-\rangle_k),$$

$$\hat{K}_{ijk}(\pi)|0\rangle_i|\alpha\rangle_j|\alpha\rangle_k = |0\rangle_i|\alpha\rangle_j|\alpha\rangle_k, \quad (5)$$

where $|\alpha_\pm\rangle$ are two normalized Schrödinger cat states, i.e., even and odd coherent quantum superposition states defined by

$$|\alpha_\pm\rangle = \frac{1}{N_\pm}(|\alpha\rangle \pm |-\alpha\rangle), \quad N_\pm = \sqrt{2(1 \pm \mathrm{e}^{-2|\alpha|^2})}. \quad (6)$$

The evolution operator for the two cross-Kerr interactions in our scheme is then

$$\hat{U}_{ck}(\pi, \pi) = \hat{K}_{dbc}(\pi)\hat{K}_{eab}(\pi). \quad (7)$$

The phase shift φ in the counter-clockwise beam of the MZI is generated by the operator $\hat{P}(\varphi) = \exp(i\varphi\hat{d}^\dagger\hat{d})$, where we choose $\varphi = -\pi/2$.

We assume that the input state is given by the product state $|0\rangle_d|1\rangle_e|\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c$. After the BS_1 with $\hat{B}_{BS_1} = \exp[i\theta(\hat{a}_d^\dagger\hat{a}_e + \hat{a}_d\hat{a}_e^\dagger)]$ and the phase shifter and just before the cross-Kerr interactions the state of our system is the superposition

$$|\psi\rangle_1 = (\cos\theta|0\rangle_d|1\rangle_e + \sin\theta|1\rangle_d|0\rangle_e)|\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c. \quad (8)$$

Then the mode e couples with external modes a, b, and the mode d couples with external modes b, c through the cross-Kerr media, which produce the state

$$|\psi\rangle_2 = \hat{U}_{ck}(\pi, \pi)|\psi\rangle_1$$
$$= \frac{\cos\theta}{2}|0\rangle_d|1\rangle_e(N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c + N_-|-\alpha\rangle_a|\alpha_-\rangle_b|\alpha\rangle_c)$$
$$+ \frac{\sin\theta}{2}|1\rangle_d|0\rangle_e(N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c + N_-|\alpha\rangle_a|\alpha_-\rangle_b|-\alpha\rangle_c). \quad (9)$$

Finally, the modes d and e passing through the BS_2 with $\hat{B}_{BS_2} = \exp[i\phi(\hat{a}_d^\dagger\hat{a}_e + \hat{a}_d\hat{a}_e^\dagger)]$ produce the output state

$$|\psi\rangle_3 = \frac{1}{2}[\cos\theta(\cos\phi|0\rangle_d|1\rangle_e + i\sin\phi|1\rangle_d|0\rangle_e)$$
$$(N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c + N_-|-\alpha\rangle_a|\alpha_-\rangle_b|\alpha\rangle_c)$$
$$+ \sin\theta(i\sin\phi|0\rangle_d|1\rangle_e + \cos\phi|1\rangle_d|0\rangle_e)$$
$$(N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c + N_-|\alpha\rangle_a|\alpha_-\rangle_b|-\alpha\rangle_c)], \quad (10)$$

with some rearrangement, then

$$|\psi\rangle_3 = \frac{1}{2}\{|0\rangle_d|1\rangle_e[(\cos\theta\cos\phi$$
$$+ i\sin\theta\sin\phi)N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c$$
$$+ \cos\theta\cos\phi N_-|-\alpha\rangle_a|\alpha_-\rangle_b|\alpha\rangle_c$$
$$+ i\sin\theta\sin\phi N_-|\alpha\rangle_a|\alpha_-\rangle_b|-\alpha\rangle_c]$$
$$+ |1\rangle_d|0\rangle_e[(i\cos\theta\sin\phi + \sin\theta\cos\phi)N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c$$
$$+ i\cos\theta\sin\phi N_-|-\alpha\rangle_a|\alpha_-\rangle_b|\alpha\rangle_c$$
$$+ \sin\theta\cos\phi N_-|\alpha\rangle_a|\alpha_-\rangle_b|-\alpha\rangle_c]\}. \quad (11)$$

Now, we imagine that a von Neumann state reductive measurement is performed on the output modes d and e of the BS_2. If detector D_1 clicks, but not D_2, the resulting state is reduced to

$$|W_1\rangle = N_1[(i\cos\theta\sin\phi + \sin\theta\cos\phi)N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c$$
$$+ i\cos\theta\sin\phi N_-|-\alpha\rangle_a|\alpha_-\rangle_b|\alpha\rangle_c$$
$$+ \sin\theta\cos\phi N_-|\alpha\rangle_a|\alpha_-\rangle_b|-\alpha\rangle_c], \quad (12)$$

where the normalization constant is given by $N_1^{-2} = 4(\sin^2\theta\cos^2\phi + \cos^2\theta\sin^2\phi)$. The success probability is

$$P_1 = |_d\langle 0|_e\langle 1|\psi\rangle_3|^2 = \sin^2\theta\cos^2\phi + \cos^2\theta\sin^2\phi. \quad (13)$$

Whereas if detector D_2 clicks, but not D_1, the resulting state is reduced to

$$|W_2\rangle = N_2[(\cos\theta\cos\phi + i\sin\theta\sin\phi)N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c$$
$$+ \cos\theta\cos\phi N_-|-\alpha\rangle_a|\alpha_-\rangle_b|\alpha\rangle_c$$
$$+ i\sin\theta\sin\phi N_-|\alpha\rangle_a|\alpha_-\rangle_b|-\alpha\rangle_c], \quad (14)$$

where the normalization constant is given by $N_2^{-2} = 4(\cos^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi)$. The success probability is

$$P_2 = |_d\langle 0|_e\langle 1|\psi\rangle_3|^2 = \cos^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi. \quad (15)$$

Thus we obtain the three-mode ECSs with the adjustable superposition coefficients through adjusting the parameters θ and ϕ of the BSs. The total success probability is $P_1 + P_2 = 1$. For example, when we choose $\theta = \phi = \pi/4$, Eqs. (12) and (14) become

$$|W_1'\rangle = \frac{1}{2\sqrt{2}}[(1+i)N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c + iN_-|-\alpha\rangle_a|\alpha_-\rangle_b|\alpha\rangle_c + N_-|\alpha\rangle_a|\alpha_-\rangle_b|-\alpha\rangle_c], \quad (16)$$

and

$$|W_2'\rangle = \frac{1}{2\sqrt{2}}[(1+i)N_+|\alpha\rangle_a|\alpha_+\rangle_b|\alpha\rangle_c + N_-|-\alpha\rangle_a|\alpha_-\rangle_b|\alpha\rangle_c + iN_-|\alpha\rangle_a|\alpha_-\rangle_b|-\alpha\rangle_c], \quad (17)$$

which both have the success probability of 1/2.

If we denote $|\alpha\rangle$ ($|\alpha_-\rangle$) as logic qubit $|1\rangle$ and $|-\alpha\rangle$ ($|\alpha_+\rangle$) as logic qubit $|0\rangle$, then Eqs. (16) and (17) become

$$|W_1'\rangle = \frac{1}{2\sqrt{2}}[(1+i)N_+|1\rangle_a|0\rangle_b|1\rangle_c + iN_-|0\rangle_a|1\rangle_b|1\rangle_c + N_-|1\rangle_a|1\rangle_b|0\rangle_c], \quad (18)$$

$$|W_2'\rangle = \frac{1}{2\sqrt{2}}[(1+i)N_+|1\rangle_a|0\rangle_b|1\rangle_c + N_-|0\rangle_a|1\rangle_b|1\rangle_c + iN_-|1\rangle_a|1\rangle_b|0\rangle_c]. \quad (19)$$

We call states (18) and (19) W-type states because of their formal resemblance with the discrete-variable W state[5]. From Eqs. (16) and (17) we can see that mode b is a discrete variable, while modes a and c are continuous variables, so the W-type states obtained are hybrid W-type ECS, which can serve as a valuable resource in quantum information processing since it build up a bridge between quantum information protocols of discrete and continuous variables.

Now we consider the influence of the imperfections of photon detectors. For unit quantum efficiency, the action of the photon detector is described by the two-value positive operator valued measure (POVM) $\{\Pi_0 = |0\rangle\langle 0|, \Pi_1 = I - \Pi_0\}$, which represents a partition of the Hilbert space of the signal. In the realistic case, an incoming photon can not be detected with unit probability. If the efficiency of the detector is η, the POVM is given by[36]

$$\Pi_0(\eta) = \sum_{i=0}(1-\eta)^i|i\rangle\langle i|, \quad \Pi_1(\eta) = I - \Pi_0(\eta). \quad (20)$$

In this situation, the success probabilities of generation of states $|W_1'\rangle$ and $|W_2'\rangle$ can be rewritten as

$$P_1 = P_2 = \eta^2/2. \quad (21)$$

We can see that the inefficiency of photon detectors might reduce the success probability of the generated W-type ECSs, but it will not affect the quality of the W-type ECS's generated[24]. Similar analysis can indicate the inefficiencies of the single-photon source only reduce the success probability but not affect the quality of the W-type ECS's to be generated.

Finally, we discuss the feasibility of the present scheme. In our scheme, except the beam splitters, we need the three-mode cross-Kerr media. It is a greater challenge to experimentally produce large Kerr nonlinearities. However, significant progresses are being made in this area. In particularly, recent progresses on atomic quantum coherence[37−40] indicate that it is possible to prepare Kerr medium with the giant Kerr nonlinearities through using the electromagnetically induced transparency (EIT)[41−45] technology. Recently, Zubairy *et al.*[35] have shown the resonant enhancement of $\chi^{(5)}$ and higher-order nonlinearities keeping the losses at a low level. Based on the existence of coherent population trapping (CPT) in six-level media, they obtained the three-mode cross-Kerr interaction, expressed as Eq. (1), and they estimated the coupling constant $\chi \simeq 3 \times 10^{11}$ s^{-1} under the typical experimental parameters, which means that we can get the phase shift of the order of π over a distance of $\sim$ 3.1 mm. Kuang *et al.*[39] showed that the fifth nonlinear susceptibilities is $\sim 10^{22}$ times greater than those measured for other materials[46]. We also discussed the enhanced fifth nonlinear susceptibilities in triple-EIT. On the other hand, even though available Kerr nonlinearities are weak, very recently, Jeong[47] have shown that the use of strong coherent fields may circumvent this problem even under the realistic assumption of decoherence.

In conclusion, we have proposed a scheme for generation of the W-type ECSs in free-traveling optical fields by using the optical elements, such as a single-photon source, coherent state sources, beam splitters, photodetectors, and Kerr media. We have calculated the success probability to produce W-type ECSs, the total success probability of the scheme is 1.

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