Normalized intensity correlation function of single-mode laser system driven by colored cross-correlation noises

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Considering a single-mode laser system with cross-correlated additive colored noise and multiplicative colored noise, we study the effects of correlation among noises on the normalized intensity correlation function $C(s)$. $C(s)$ is derived by means of the projection operator method. The effects of the self-correlation time τ_1 of the additive colored noise, τ_2 of the multiplicative colored noise, and the effect of the cross-correlation time τ_0 between the two noises on $C(s)$ are discussed by numerical calculation. For the case of positive correlation ($\lambda > 0$), it is found that when $a_0 > 0$ the normalized intensity correlation function $C(s)$ increases with the increase of τ_0 or τ_2, and with value of τ_0 or τ_2 becoming larger, $C(s)$ comes to saturation. With increasing the self-correlation time τ_1 of the additive noise, a minimum and a maximum will appear on curve of $C(s)$ as $a_0 > 0$. If $a_0 < 0$, $C(s)$ decreases with the increase of τ_0, τ_1, and τ_2.

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Recently, the statistical properties of intensity fluctuation in a single-mode laser have attracted much attention[1,2]. In most of the existing theoretical studies, the additive noise (quantum noise) and the multiplicative noise (pump noise) are both modeled as Gaussian white noises, and these two noises are treated as without correlation. In 1991, Fulinski *et al.* proposed that the additive noise and the multiplicative noise are correlated under certain conditions[3]. Since then, many researchers considered the cross-correlation between the two noises when studying the statistical fluctuation properties, and the results were in better agreement with the experimental results[4−10]. In 2002, Liang *et al.* studied the stationary intensity distribution of the single-mode laser cubic model with correlated pump noise and quantum noise which were colored cross-correlation[11].

It is known that the normalized intensity correlation function is an important physical quantity to characterize the dynamic behavior of stochastic systems[12,13]. In this letter, the case of a full account of the saturation effects with cross-correlations between an additive colored noise and a multiplicative colored noise on the normalized intensity correlation function are investigated.

A fuller account of the saturation proprieties of laser may be obtained from the Langevin equation as[14]

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -KE + \frac{F_1 E}{1 + A\left|E\right|^2/F_1} + \tilde{p}(t)E + \tilde{q}(t), \quad (1)$$

by setting the electric field $E = re^{i\varphi}$, Eq. (1) can be transformed into two coupling Langevin equations about the field-amplitude r and phase φ. Decoupling them and setting the laser intensity $I = r^2$, we get the Langevin equation about I,

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2F_1 I}{1 + AI/F_1} + 2D + 2I^{1/2}q(t) + 2Ip(t), \quad (2)$$

where K is the cavity decay rate for the electric field, $F_1 = a_0 + K$ is the gain parameter, a_0 and A are real and represent net gain and self-saturation coefficients, respectively, $q(t)$ is the additive noise and $p(t)$ is the multiplicative noise. $q(t)$ and $p(t)$ are considered as Gaussian-type noises

$$\langle q(t)\rangle = \langle p(t)\rangle = 0, \quad (3)$$

$$\langle q(t)q(t')\rangle = \frac{D}{\tau_1}\exp(|t-t'|/\tau_1), \quad (4)$$

$$\langle p(t)p(t')\rangle = \frac{Q}{\tau_2}\exp(|t-t'|/\tau_2), \quad (5)$$

$$\langle q(t)p(t')\rangle = \langle p(t)q(t')\rangle = \frac{\lambda\sqrt{DQ}}{\tau_0}\exp(|t-t'|/\tau_0), \quad (6)$$

where D is the additive noise strength, Q is the multiplicative noise strength, λ is the correlation intensity between $q(t)$ and $p(t)$.

The corresponding Fokker-Planck equation for the probability function $P(I,t)$ of the laser intensity I is given by[13]

$$\frac{\partial P(I,t)}{\partial t} = L_{\mathrm{FP}}P(I,t), \quad (7)$$

$$L_{\mathrm{FP}} = -\frac{\partial}{\partial I}f(I) + \frac{\partial^2}{\partial I^2}G(I), \quad (8)$$

in which

$$f(I) = -2KI + \frac{2F_1 I}{1 + AI/F_1} + 2D + 2B_1 + 6B_0 I^{1/2} + 4IB_2, \quad (9)$$

$$G(I) = 4IB_1 + 8B_0 I^{3/2} + 4I^2 B_2, \quad (10)$$

and

$$B_0 = \frac{\lambda\sqrt{DQ}}{1+2K\tau_0(1-K/F_1)}, \tag{11}$$

$$B_1 = \frac{D}{1+2K\tau_1(1-K/F_1)}, \tag{12}$$

$$B_2 = \frac{Q}{1+2K\tau_2(1-K/F_1)}. \tag{13}$$

Equation (7) should meet $1+2K\tau_0(1-K/F_1) > 0$, $1+2K\tau_1(1-K/F_1) > 0$, and $1+2K\tau_2(1-K/F_1) > 0$. When a_0 is above the threshold ($a_0 > 0$), there is not any limit on τ_0, τ_1 and τ_2. When $a_0 < 0$, τ_0, τ_1 and τ_2 have to meet $0 < \tau_0, \tau_1, \tau_2 < -(K+a_0)/2Ka_0$. The steady-state probability density function $P_{\rm st}(I)$ can be obtained by Eq. (7),

$$P_{\rm st} = N_0(B_2I + 2B_0I^{1/2} + B_1)^{\alpha_1}(\frac{AI}{F_1}+1)^{\frac{F_1\alpha_2}{2A}} I^{\frac{D-B_1}{2B_1}}$$
$$\times \exp(\alpha_3 {\rm arctg}(\frac{B_2\sqrt{I}+B_0}{\sqrt{|B_1B_2-B_0^2|}}) + \alpha_4\sqrt{\frac{F_1}{A}}{\rm arctg}(\sqrt{\frac{AI}{F_1}}), \tag{14}$$

where N_0 is the normalization constant and

$$\alpha_1 = \frac{(\alpha_4B_1+\alpha_6A/F_1)B_2F_1B_1 - 2B_0B_2A(D-B_1) - 2A(K+2B_2)B_0B_1}{4AB_0B_1B_2}, \tag{15}$$

$$\alpha_2 = -\frac{(B_1F_1 - B_1A)AF_1}{F_1^2B_2^2 - 2AF_1B_1B_2 + A^2B_1^2 + 4AF_1B_0^2}, \tag{16}$$

$$\alpha_3 = \frac{-B_0B_2 + B_1^2B_2\alpha_4 - B_0B_1\alpha_5 - B_0B_1B_2 + B_0B_1K}{B_0B_1\sqrt{|B_1B_2-B_0^2|}}, \tag{17}$$

$$\alpha_4 = \frac{2AF_1^2B_0}{F_1^2B_2^2 - 2AF_1B_1B_2 + A^2B_1^2 + 4AF_1B_0^2}, \tag{18}$$

$$\alpha_5 = \frac{(\alpha_4B_1 + N_1A/F_1)B_2F_1}{2B_0A}. \tag{19}$$

The normalized intensity correlation function $C(s)$ is defined as[12]

$$C(s) = \frac{\langle\delta I(t+s)\delta I(t)\rangle_{\rm st}}{\langle(\delta I)^2\rangle_{\rm st}}. \tag{20}$$

In terms of the adjoint operator $L^+_{\rm FP}$ of the operator given by Eq. (8), $\delta I(t+s)$ can be expressed as $\delta I(t+s) = \exp(L^+_{\rm FP}s)\delta I(t)$. Thus, we can rewrite Eq. (20) and get the associated Laplace transform

$$\tilde{C}(\omega) = \int_0^\infty \exp(-\omega s)C(s){\rm d}s$$
$$= \frac{1}{\langle(\delta I)^2\rangle_{\rm st}}\left\langle \delta I\frac{1}{\omega - L^+_{\rm FP}}\delta I\right\rangle_{\rm st}. \tag{21}$$

Using the projection operator method used by Fujisaka and Grossmann[15] to deal with the Laplace resolvent $\omega - L^+_{\rm FP}$ in Eq. (21), we have[15,16]

$$\tilde{C}(\omega) = \frac{1}{\omega+\mu_0+\frac{\eta_1}{\omega+\mu_1+\frac{\eta_2}{\omega+\mu_2+\cdots}}}, \tag{22}$$

in which

$$\mu_i = -\frac{\langle\delta I_iL^+_{\rm FP}\delta I_i\rangle_{\rm st}}{\langle(\delta I_i)^2\rangle_{\rm st}}, \tag{23}$$

$$\eta_i = -\frac{\langle(\delta I_i)^2\rangle_{\rm st}}{\langle(\delta I_{i-1})^2\rangle_{\rm st}}, \tag{24}$$

$$\delta I_{i+1} = S_{i+1}L^+_{\rm FP}\delta I_i, \tag{25}$$

with starting $\delta I_0 = \delta I$ and $S_0 = 1$, the operator S_i is determined by

$$K_{i-1} = S_{i-1} - S_i = \frac{\delta I_{i-1}}{\langle(\delta I_{i-1})^2\rangle_{\rm st}}\langle\delta I_{i-1}|, \tag{26}$$

where the operator $\langle\delta I_i|$ acting on $\varphi(I)$ means the scalar product

$$\langle\delta I_i|\varphi(I) = \langle(\delta I_i\varphi(I))\rangle_{\rm st} = \int_0^\infty P_{\rm st}(I)\delta I_i\varphi(I){\rm d}I. \tag{27}$$

The projection operator K_i projects $\varphi(I)$ onto the subspace associated with the variable δI_i. The projector S_i projects onto the space orthogonal to the space containing δI_i. Setting $\eta_2 = 0$, the approximation of the intensity correlation function is

$$\tilde{C}(\omega) = \frac{\omega+\mu_1}{(\omega+\mu_0)(\omega+\mu_1)+\eta_1}, \tag{28}$$

$$\mu_0 = \frac{\langle G(I)\rangle_{\rm st}}{\langle(\delta I)^2\rangle_{\rm st}}, \tag{29}$$

$$\eta_1 = \frac{\langle G(I)f'(I)\rangle_{\rm st}}{\langle(\delta I)^2\rangle_{\rm st}} + \mu_0^2, \tag{30}$$

$$\mu_1 = -\frac{\left\langle G(I)\left[f'(I)\right]^2\right\rangle_{\rm st}}{\eta_1\langle(\delta I)^2\rangle_{\rm st}} + \frac{\mu_0^3}{\eta_1} - 2\mu_0. \tag{31}$$

Performing the Laplace converse transformation of Eq. (28), we get

$$C(s) = \beta\exp(-\alpha_-s) + (1-\beta)\exp(-a_+s), \tag{32}$$

in which

$$\alpha_\pm = \frac{\mu_0+\mu_1}{2} \pm \frac{1}{2}\sqrt{(\mu_1-\mu_0)^2 - 4\eta_1}, \tag{33}$$

and
$$\beta = \frac{\mu_1 - \alpha_-}{\alpha_+ - \alpha_-}. \quad (34)$$

By virtue of the expression of the normalized intensity correlation function Eq. (32), the effects of the self-correlation time and the cross-correlation time on $C(s)$ are discussed in Figs. $1-3$. The solid line is the result of the projection operator method, the open circles is the result of numerical simulations from Eqs. (2) to (6).

Figure 1 shows the $C(s)$ as a function of the cross-correlation time τ_0 for different values of a_0. It is known that $C(s)$ is a measure of correlation between laser intensity fluctuation at time t and that of $t+s$. We find that $C(s)$ increases with the increase of τ_0 if $a_0 > 0$. In other words, the decay rate of the intensity fluctuation becomes slower and slower with the increase of the cross-correlation time τ_0. In the case of $a_0 > 0$, we also find that at larger value of τ_0, there is almost no change for $C(s)$ when τ_0 changes. However, the decay rate of the intensity fluctuation becomes faster and faster with the increase of τ_0 when $a_0 < 0$.

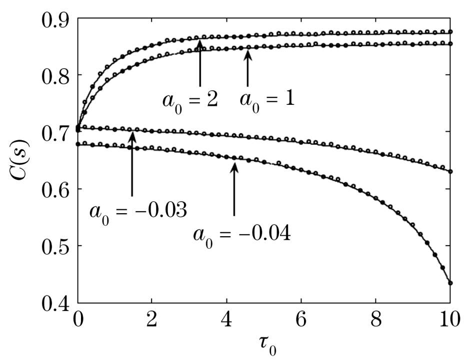

Fig. 1. $C(s)$ as a function of τ_0 for different values of a_0; $Q = 3$, $D = 2$, $\tau_1 = 3.5$, $\tau_2 = 3.5$, $K = 1$, $A = 3$, $\lambda = 0.8$.

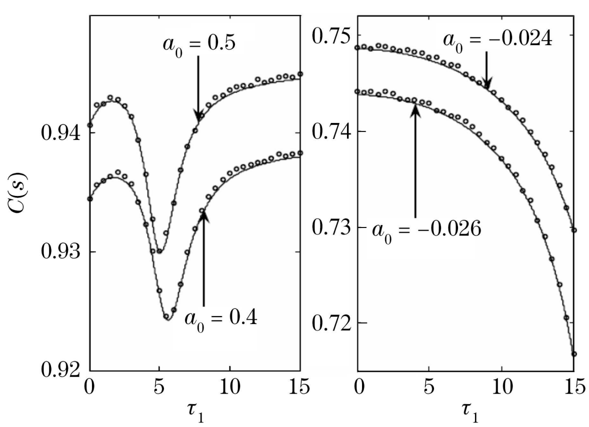

Fig. 2. $C(s)$ as a function of τ_1 for different values of a_0; $Q = 3$, $D = 2$, $\tau_0 = 3.5$, $\tau_2 = 3.5$, $K = 1$, $A = 3$, $\lambda = 0.1$.

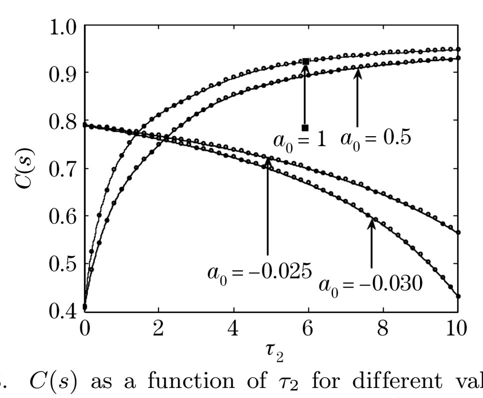

Fig. 3. $C(s)$ as a function of τ_2 for different values of a_0; $Q = 3$, $D = 2$, $\tau_0 = 3.5$, $\tau_1 = 3.5$, $K = 1$, $A = 3$, $\lambda = 0.25$.

In order to show the effects of the self-correlation time τ_1 of the additive noise on the statistical properties of single-mode laser, $C(s)$ versus τ_1 with different values of a_0 are plotted in Fig. 2. A minimum and a maximum $C(s)$ appear with the increase of τ_1 when $a_0 > 0$; But if $a_0 < 0$, $C(s)$ decreases with the increase of τ_1. This means that the decay rate of the intensity fluctuation become faster and faster with the increase of τ_1 in the case of $a_0 < 0$.

Figure 3 shows the $C(s)$ as a function of the self-correlation time τ_2 of the multiplicative noise for different values of a_0. It is shown that $C(s)$ increases with the increase of τ_2 if $a_0 > 0$. We can find that at larger value of τ_2, there is almost no change for $C(s)$ when τ_2 changes as $a_0 > 0$. When $a_0 < 0$, $C(s)$ decreases with the increase of τ_2. It is obvious that the effect of τ_2 is similar with the effect of τ_0. From Figs. $1-3$, we can see that the larger the a_0 is, the larger the $C(s)$ becomes, whenever $a_0 > 0$ or $a_0 < 0$. From Figs. $1-3$, we can also find that the results of the projection operator method are in agree with the results of numerical simulations.

In conclusion, the effects of the cross-correlation time τ_0 and the self-correlation time τ_1 and τ_2 on the statistical properties of single-mode laser are investigated. It is found that when $a_0 > 0$, the normalized intensity correlation function $C(s)$ increases with the increase of τ_0 and τ_2; when $a_0 < 0$, $C(s)$ decreases with the increase of τ_0, τ_1, and τ_2. With increasing the self-correlation time τ_1, a minimum and a maximum will appear on the curve of $C(s)$ as $a_0 > 0$.

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