Propagation of light in (2+1)-dimensional nonlinear optical media with spatially inhomogeneous nonlinearities

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The (2+1)-dimensional nonlinear Schrödinger (NLS) equation with spatially inhomogeneous nonlinearities is investigated, which describes propagation of light in (2+1)-dimensional nonlinear optical media with inhomogeneous nonlinearities. New types of optical modes and nonlinear effects in optical media are presented numerically. The results reveal that the regular split of beam can be obtained in (2+1)-dimensional nonlinear optical media with inhomogeneous nonlinearities, by adjusting the guiding parameter. Furthermore, the stability of beam regular split is discussed numerically, and the results reveal that the beam regular split is stable to the finite initial perturbations.

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During the past years, much attention in nonlinear optics has been focused on the wave propagation in nonlinear optical media with the transverse harmonic modulation because of their extensive applications. Discrete spatial solitons, as spatially localized modes of periodic modulation, have been introduced and studied theoretically[1,2], and have been observed experimentally[3]. Also a lot of practical applications have been considered[2−11]. On the other hand, all kinds of new opportunities are offered by the intermediate regime constituted by continuous or quasi-continuous nonlinear media with an imprinted transverse modulation of the refractive index[12−17]. Moreover, the problem can be extended to many branches of physics and applied mathematics, including nonlinear condensed matter.

At present, the theory of the nonlinear wave propagation is developed in (1+1)-dimensional nonlinear optical media with homogeneous nonlinearities. However, the theoretical study for the propagation of light in (2+1)-dimensional nonlinear optical media with spatially inhomogeneous nonlinearities has not been noted widely. In fact, there have been many theoretical studies on nonlinear waves in Bose-Einstein condensates (BECs) with spatially inhomogeneous nonlinearities[18−21]. Therefore, the study about the propagation of light in (2+1)-dimensional nonlinear optical media with spatially inhomogeneous nonlinearities could have important consequences for the further development of nonlinear optics and for the other branches of physics.

In this letter, we investigate (2+1)-dimensional nonlinear Schrödinger (NLS) equation with spatially inhomogeneous nonlinearities, which describes propagation of light in (2+1)-dimensional nonlinear optical media with inhomogeneous nonlinearities, and new types of optical modes and nonlinear effects in optical media are presented numerically. Furthermore, these results are useful not only in some experiments of intentional problems, but also in the possible explanation of some interesting phenomena.

We start the analysis in the electromagnetic waves propagating along the z direction in (2+1)-dimensional waveguide with transverse modulation of the linear refractive index, and focus on the inhomogeneous Kerr-type nonlinearity. The problem can be described in two transverse dimensions by the following NLS equation:

$$iq_z + \frac{1}{2}(q_{xx} + q_{yy}) + g(x, y)\,|q|^2\, q + pR(x, y)q = 0, \quad (1)$$

where $q(x, y, z) = (L_{\rm dif}/L_{\rm nl})^{1/2} A(x, y, z) I_0^{-1/2}$, and $A(x, y, z)$ is the complex envelope of the electrical field. Taking into account Gaussian optical beam from laser devices, here we take $q(x, y, 0) = \exp(-x^2/2 - y^2/2)$. I_0 is the input intensity, $x = X/r_0$, $y = Y/r_0$, r_0 is the input beam width unit, $z = Z/L_{\rm dif}$, $L_{\rm dif} = n_0\omega r_0^2/c$, $L_{\rm nl} = 2c/\omega n_2 I_0$, ω is the frequency, and n_2 is of the order of the nonlinear correction to the refractive index because of the Kerr effect. $p = L_{\rm dif}/L_{\rm ref}$ represents the guiding parameter, $L_{\rm ref} = c/(\delta n\omega)$, and δn denotes the refractive index modulation unit, which is small compared with the unperturbed index n_0. The weak modulation of linear refractive index in the transverse direction is defined by the real function $R(x, y)$. Here we consider the continuous, separable modulation[22]

$$R(x, y) = \cos^2(\pi x/d) + \cos^2(\pi y/d), \quad (2)$$

where the grid constant $d = 2$. $g(x, y)$ describes the spatial modulation of the nonlinearity. In spite of many possibilities, we discuss only three examples of interest for the applications.

1) $g(x, y) = 1$. In this case, nonlinear optical media is of spatially homogeneous nonlinearity. We consider the case in order to compare it with the inhomogeneous case. Figure 1 presents the propagation of light for different values of parameter p. It is obvious that the value of the guiding parameter p has a crucial influence on our results. In fact, the guiding parameter p denotes modulation depth of the refractive index. For different p, there exists different lattices due to different modulation depths. We do not have to vary the grid constant

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d, since the propagation Eq. (1) can be scaled in such a way that this is equivalent to changing p. For small p (corresponding to lower modulation depth), we can see that the input light can propagate in the stationary mode in this nonlinear system as shown in Fig. 1(b). We also find that the beam width decreases compared with the input beam, however, it could not influence the main character of the propagation. This indicates a complete formation of stable, slightly oscillating two-dimensional (2D) soliton mode. If the depth of the lattice is higher (for greater values of p), simulations show that this is indeed the case that the propagation of input beam should become unstable. In this sense, the stable 2D soliton depends on modulation depth relative to the value of p.

2) $g(x,y) = -\exp(-3x^2 - 3y^2)$. In this case, $g(x,y)$ describes a Gaussian nonlinearity such as the one generated by controlling the Feschbach resonances optically using a Gaussian beam[23−25]. The propagation of light is depicted in Fig.2. From the plot, we find that the propagation of light is various for different values of parameter p. If setting $p = 4.2$, one can clearly see that beam split occurs regularly, as shown in Fig. 2(b). Compared to Fig. 1(b), this is a new type of optical mode. In order to investigate this in detail, we consider the case 3).

3) $g(x,y) = -g_0\{1 - 3\alpha[\cos(wx) + \cos(\omega y)]\}$. In this case, the nonlinearity is harmonic. Beam split also occurs regularly when $p = 4.2$ as shown in Fig. 3(b). However, split law is different compared with Fig. 2(b).

Thus, in (2+1)-dimensional nonlinear optical media with inhomogeneous nonlinearities, we could adjust the parameter p to obtain beam regular split. As a matter of fact, the 2D regular split patterns are caused by lower modulation depth (for small p) for an effective periodic potential with inhomogeneous nonlinearities. And the growth of modulation depth (for increasing p) could strongly affect the regular split patterns. This leads to the conclusion that, lower lattice depth is necessary in order to stabilize beam regular split. We should expect dynamics similar to that of nonlinear development of the modulation instability (MI) of a continuous-wave (CW) state, seeded by an infinitesimal periodic perturbation. In MI, the instability growth results in formation of a periodic chain of solitons on a residual background. For applications, one needs soliton trains without the CW background. In contrast to the MI case, beams splited here are without the CW background. Moreover, weak modulation depth plays an important role for beam regular split. This might imply that the linear guided modes are excited for weak modulation depth. For large number of waveguides, propogation constant of the beam is close to that of the linear guided mode. That means, beam regular split also drastically depends on whether the linear guided mode has been excited. Again, when beam regular split is excited, a nonlinear transfer of beam energies from the initially fixed beams usually takes place.

Such an interesting nonlinear effect, actually, is the result of perfect balance among the diffraction, the refraction, spatially inhomogeneous nonlinearities, and linear continuous modulation. With the unique property,

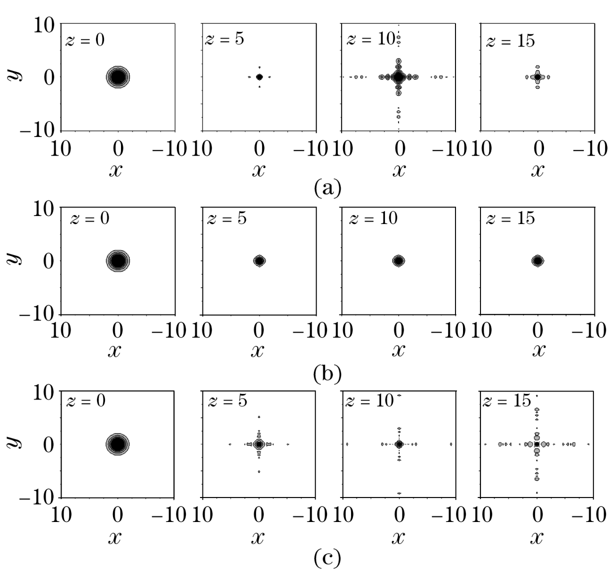

Fig. 1. Contour plots of light wave propagation. (a) $p = 13.6$; (b) $p = 4.2$; (c) $p = 27.2$.

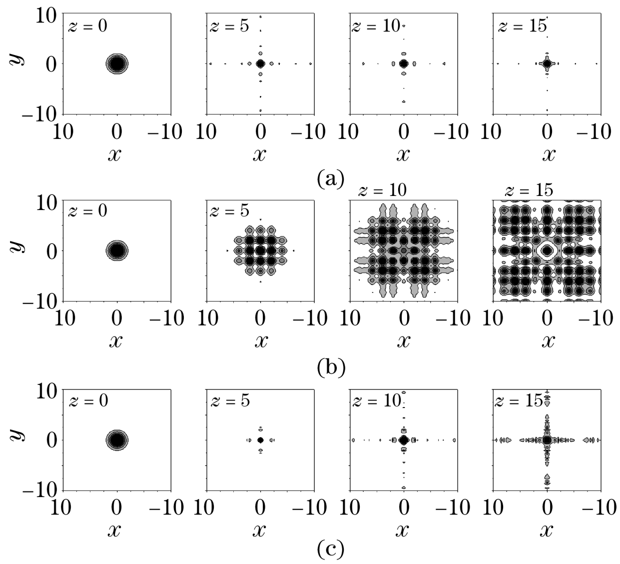

Fig. 2. Contour plots of light wave propagation. (a) $p = 13.6$; (b) $p = 4.2$; (c) $p = 27.2$.

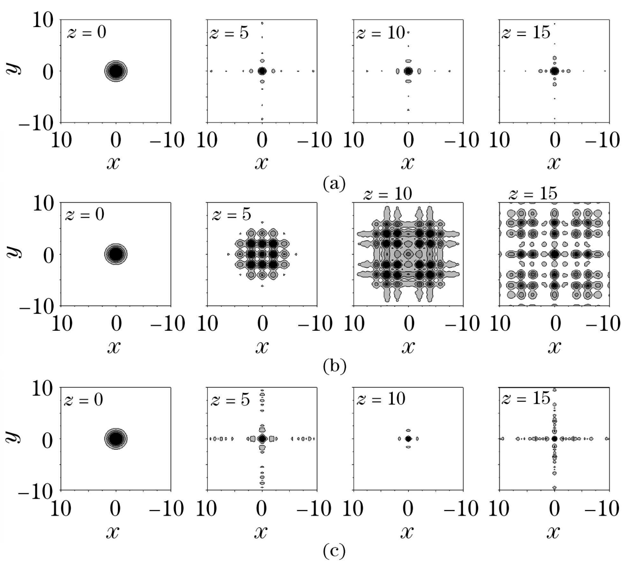

Fig. 3. Contour plots of light wave propagation for $g_0 = 1$, $\omega = 1$, and $\alpha = 0.05$. (a) $p = 13.6$; (b) $p = 4.2$; (c) $p = 27.2$.

some complicated devices, to which a lot of attention has been currently devoted, may be further developed in the nonlinear optical media. In fact, Ref. [26] has reported on the first experimental observation of 2D multicolored transverse arrays in a quadratic nonlinear medium under the pump of two crossly overlapped femtosecond beams. Certainly, optical mode presented in our work is symmetric mode. At this point, the effect could also support the development of the theory about similar problem mentioned above in other physical fields, even other scientific fields, such as biology.

We have also made numerical simulations to determine the stability of the beam regular split in the presence of perturbations and the violation of parameter profiles. The results reveal that the beam regular split is not very sensitive to the initial perturbations, such as small amounts of amplitude perturbations, indicating the stability of the beam regular split.

On the other hand, Figs. 1(a), 1(c), 2(a), 2(c), 3(a) and 3(c) show more complicated physical phenomena, which will be accompanied by more physical discussion.

In summary, we have considered (2+1)-dimensional nonlinear Schrödinger (NLS) equation with spatially inhomogeneous nonlinearities, which describes propagation of light in (2+1)-dimensional nonlinear optical media with inhomogeneous nonlinearities, and new types of optical modes and nonlinear effects in optical media have been presented numerically. The results reveal that the beam regular split can be obtained in (2+1)-dimensional nonlinear optical media with inhomogeneous nonlinearities, by adjusting the guiding parameter. It is worth noting that these results are useful not only for dealing with physical problems, but also for dealing with other problems, such as biological problems. Finally, the numerical simulation shows that the beam regular split is still stable in the presence of finite perturbations and the violation of parameter profiles. We emphasize that all the results obtained in this work provide a totally new insight into the problems. Furthermore, we think that the results could be obtained by other methods, such as equivalent-particle approximation method[27].

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