Characterization for defect modes of one-dimensional photonic crystals containing metamaterials

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Transmission studies for one-dimensional photonic crystals (1DPCs) containing single-negative (SNG) materials inserted with multiple defects are presented. The numbers and positions of the defect modes inside zero-phase (zero-$\phi_{\rm eff}$) gap are found to be well characterized by effective medium theory.

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Recently, the defect modes inside the photonic band gap (PBG) have been widely studied due to their peculiar properties and potential applications[1−11]. Among them, the defects of the one-dimensional photonic crystals (1DPCs) containing metamaterial were studied most. In comparison with the defect modes in the conventional PBG, the defect appeared either in the zero-phase (zero-$\phi_{\rm eff}$) gap or in the zero-index (zero-n) gap is quite interesting[3,12]. It is almost independent of the incident angle, the scaling of the lattice constant, and the disorder.

In this paper, we investigate the characteristics of the defect modes appearing in the zero-$\phi_{\rm eff}$ gap from the 1DPCs combined with two types of single-negative (SNG) materials. The SNG materials include the mu-negative (MNG) media with negative permeability (μ) but positive permittivity (ε) and the epsilon-negative (ENG) with negative ε but positive μ. By adjusting the numbers of defect layers, we realize the defect modes with various numbers in the gap. To our interest, we obtain the frequency equation for the defect modes based on the effective medium theory. Numerical results are in good agreement with those based on the transfer matrix method.

We consider the system that 1DPCs composed of alternating layers of MNG material and ENG material are doped by a defect layer with positive refractive-index material, as shown in Fig. 1(a). The physical properties of MNG and ENG are supposed to be

$$\varepsilon_1 = \varepsilon_{\rm a}, \quad \mu_1 = \mu_{\rm a} - \frac{\alpha}{\omega^2 + i\omega\Gamma_{\rm m}} \tag{1}$$

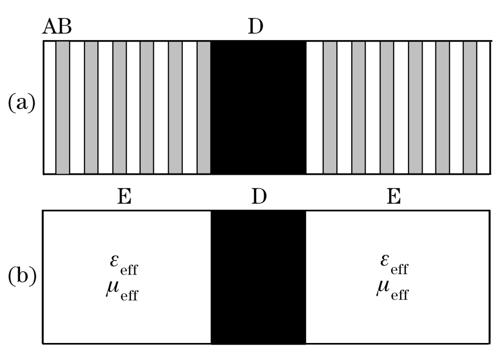

Fig. 1. Schematic of the two structures with a defect. (a) The light gray and white regions denote the MNG and ENG layers, respectively; (b) the white regions denote the effective material. Both the dark gray regions denote the defect layer.

in MNG materials and

$$\varepsilon_2 = \varepsilon_{\rm b} - \frac{\beta}{\omega^2 + i\omega\Gamma_{\rm e}}, \quad \mu_2 = \mu_{\rm b} \tag{2}$$

in ENG materials. These kinds of dispersion for ε and μ may be realized in special microstrips[13]. In Eqs. (1) and (2), $\Gamma_{\rm m}$ and $\Gamma_{\rm e}$ are the damping constants, and ω is the frequency measured in GHz. Without loss of generality, we choose $\mu_{\rm a} = \varepsilon_{\rm b} = 0.4$, $\varepsilon_{\rm a} = 20.5$, $\mu_{\rm b} = 1.6$, $\alpha = 360$, $\beta = 160$, $\Gamma_{\rm m} = \Gamma_{\rm e} = 2\pi \times 3 \times 10^6$ s^{-1}[8]. The thicknesses of MNG, ENG, and defect layer are assumed to be $d_1 = d_2 = 8$ mm and $d_3 = 64$ mm. The refractive index of the defect layer is $n_3 = 3.4$.

We study the defect-induced transmission of the hetero-structure $({\rm AB})_n{\rm D}_m({\rm AB})_n$, as shown in Fig. 1(a), where n, m are the period numbers of AB and D. Figure 2 shows the transmittance through the structure for different numbers of defects for normal incidence. It is shown that the numbers of defect modes inside the band

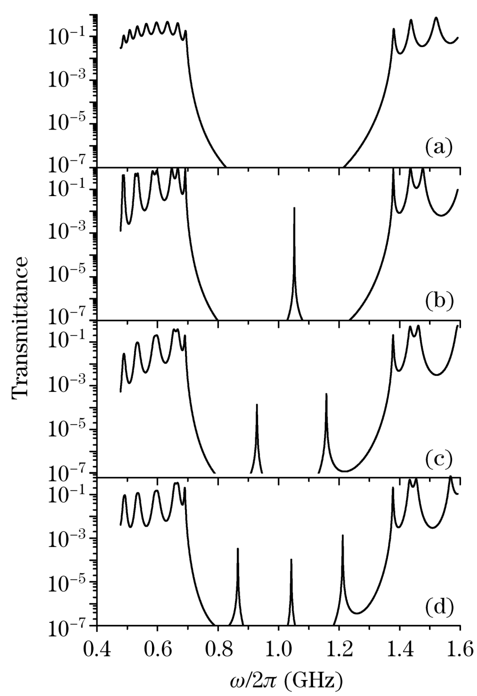

Fig. 2. Transmittance through periodic structure with different numbers of defects. (a) $m = 0$, (b) $m = 1$, (c) $m = 2$, and (d) $m = 3$.

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gap are 0, 1, 2, and 3 respectively. The conclusion is very useful, as the frequency, frequency interval, and the number of transmission channels can be tuned by adjusting the number of defect layers.

When the zero-ϕ_{eff} gap appears, the system may be treated within effective medium theory[14,15]. The effective permittivity ε_{e} and the effective permeability μ_{e} of the layered structure are given by[14]

$$\mu_{\text{e}} = \frac{d_1}{d}\mu_1 + \frac{d_2}{d}\mu_2, \tag{3}$$

$$\varepsilon_{\text{e}} = \frac{d_1}{d}\varepsilon_1 + \frac{d_2}{d}\varepsilon_2 - \sin^2\theta\left(\frac{d_1}{d\mu_1} + \frac{d_2}{d\mu_2}\right) + \sin^2\theta/\left(\frac{d_1}{d}\mu_1 + \frac{d_2}{d}\mu_2\right) \tag{4}$$

for TE polarization. Here θ is the incident angle and $d = d_1 + d_2$ is the lattice constant. Equations (3) and (4) indicate that the period structure containing two kinds of materials can be effectively simplified as one material which has the equivalent property of electromagnetic wave propagating, as shown in Fig. 1(b). Similar as Fig. 1(a), we denote the structure as ED$_m$E, where E is the equivalent material for (AB). The transmittance of the ED$_m$E structure is also calculated by the transfer-matrix method, as shown in Fig. 3. In comparison with Fig. 2, we find that the two structures almost yield the same transmission spectra.

Following, we would like to derive the frequencies of the defect modes. We suppose a TE wave normally incident on the EDE structure. The transfer matrix of E is written as

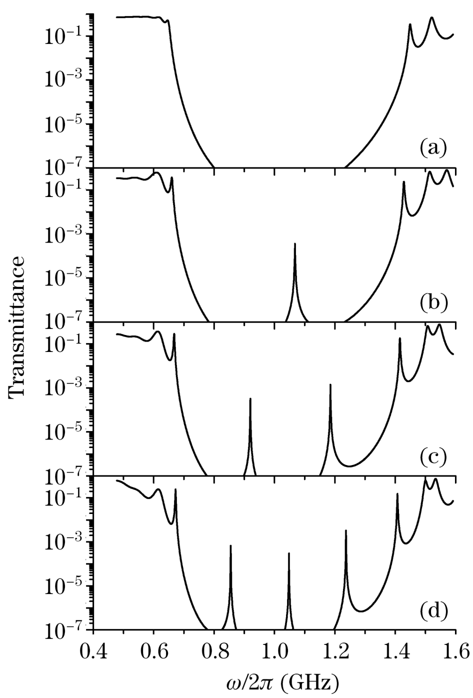

Fig. 3. Transmittance through effective material structure under different numbers of defects. (a) $m = 0$, (b) $m = 1$, (c) $m = 2$, and (d) $m = 3$.

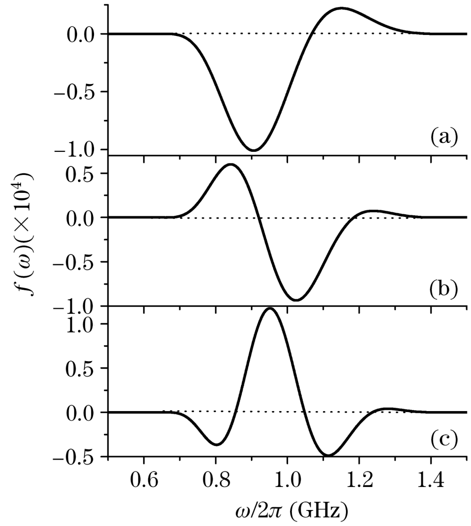

Fig. 4. $f(\omega)$ as a function of ω with different thicknesses of defect layer. (a) d_3 (64 mm), (b) $2d_3$, (c) $3d_3$.

$$\vec{M}_{\text{E}} = \begin{pmatrix} \cos(k_{\text{e}}d_{\text{e}}) & i\frac{\sqrt{\mu_{\text{e}}}}{\sqrt{\varepsilon_{\text{e}}}}\sin(k_{\text{e}}d_{\text{e}}) \\ i\frac{\sqrt{\varepsilon_{\text{e}}}}{\sqrt{\mu_{\text{e}}}}\sin(k_{\text{e}}d_{\text{e}}) & \cos(k_{\text{e}}d_{\text{e}}) \end{pmatrix}, \tag{5}$$

where $k_{\text{e}} = \frac{\omega}{c}\sqrt{\varepsilon_{\text{e}}}\sqrt{\mu_{\text{e}}}$ is the effective wave vector, c is the light speed in vacuum, and d_{e} is the thickness of E. In addition, the transfer matrix of D is

$$\vec{M}_{\text{D}} = \begin{pmatrix} \cos(k_3d_3) & in_3^{-1}\sin(k_{\text{e}}d_{\text{e}}) \\ in_3\sin(k_3d_3) & \cos(k_3d_3) \end{pmatrix}, \tag{6}$$

where $k_3 = \frac{\omega}{c}n_3$ is the wave vector in the defect layer. As a result, the total matrix of the whole EDE structure is

$$\vec{M} = \vec{M}_{\text{E}}\vec{M}_{\text{D}}\vec{M}_{\text{E}}. \tag{7}$$

The dispersion relation can then be expressed as

$$f(\omega) = \cos(2k_{\text{e}}d_{\text{e}})\cos(k_3d_3) - \frac{1}{2}\left(\frac{\sqrt{\mu_{\text{e}}}}{\sqrt{\varepsilon_{\text{e}}}}n_3 + \frac{\sqrt{\varepsilon_{\text{e}}}}{\sqrt{\mu_{\text{e}}}}\frac{1}{n_3}\right)\sin(2k_{\text{e}}d_{\text{e}})\sin(k_3d_3). \tag{8}$$

In Eq. (8), $|f(\omega)| > 1$ dictates the forbidden gap, while $|f(\omega)| \leq 1$ indicates the pass bands. We plot $f(\omega)$ as a function of ω in Fig. 4. Figures 4(a)—(c) correspond to d_3, $2d_3$, and $3d_3$, respectively. It is shown that when the gap is opened, $|f(\omega)|$ becomes very large. And when the defect mode inside the gap appears, $|f(\omega)| \to 0$. For instance, when the thickness of defect layer is d_3 (64 mm), we obtain the frequency $\omega/(2\pi) = 1.06783-1.06786$ GHz under the condition $|f(\omega)| < 1$. This is just the defect mode frequency (1.067 GHz) of the structure. The cases of $2d_3$ and $3d_3$ can also be discussed. Therefore, one can use Eq. (8) to determine the frequency of the defect modes.

In conclusion, the defect modes inside the zero-ϕ_{eff} gap were investigated. By modulating the number of defect layers, the defect modes can be tuned simultaneously. This phenomena can be well described within effective medium theory, and the frequency of the defect modes is derived within effective medium theory accordingly.

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