# Photonic band gap of one-dimensional periodic structure containing dispersive left-handed metamaterials

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Band structures of one-dimensional (1D) photonic crystals (PCs) containing dispersive left-handed metamaterials are studied theoretically. The results show that the structure possesses a type of photonic band gap originating from total internal reflection (TIR). In contrast to photonic band gaps corresponding to zero average refractive index and zero phase, the TIR gap exhibits sharp angular effect and has no polarization effect. It should also be noted that band structures of transverse electric (TE) and transverse magnetic (TM) mode waves are exactly the same in the PCs we studied.

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Photonic crystals (PCs) have received considerable attention in recent years because of their intrinsic physical properties as well as important potential applications in devices[1−5]. In conventional one-dimensional (1D) PCs, which are made of dielectric media with positive refractive indices, there may exist photonic band gap (PBG) originating from the interference of Bragg scattering[6]. It is well known that, as the incident light changes from normal to oblique incidence, the effective optical lengths of all the medium layers reduce for two independent electromagnetic modes: transverse electric (TE) mode and transverse magnetic (TM) mode. This strongly influences the interference process within the PCs. At increasingly oblique angle, the gap of the TE mode increases, whereas that of the TM mode decreases. In addition, the center of the gap shifts to higher frequencies[7]. For this reason, the conventional PBG has been used for mirrors with its incident light being restricted to a narrow frequency range or a particular angular range. In order to overcome the angular effect, some researchers attempted to realize PBG related to the mechanism beyond Bragg scattering in the left-handed materials (LHMs)[8−10]. Such materials possess negative refractive indices, which is due to negative permeability (or permittivity)[9,11]. It has been demonstrated that stacking alternating layers of positive and negative (or only negative) media leads to new type of PBGs, such as zero average refractive index (zero-$\tilde{n}$), zero phase (zero-$\phi_{\text{eff}}$), and complete PBGs.

In this letter, we show that stacking alternating layers of dispersive left-handed metamaterials can lead to a new type of photonic gap with properties different from that of a Bragg gap. This new type of gap arises naturally when $\varepsilon\mu < \sin^2\theta$, where $\varepsilon$, $\mu$, and $\theta$ are the permittivity, permeability, and the incident angle, respectively. Stop band of this type can be achieved in composites containing dispersive left-handed metamaterials as long as the $\varepsilon\mu < \sin^2\theta$, $\mu_1 = \varepsilon_2$, $\mu_2 = \varepsilon_1$, and $d_1 = d_2$ conditions are satisfied, where $d_i$ ($i = 1, 2$) are the thicknesses of the corresponding left-handed metamaterial.

We start by showing that in a 1D layered stack, $\varepsilon\mu < \sin^2\theta$ implies the existence of a total internal reflection (TIR) gap. Numerical calculations are performed to demonstrate the unusual properties of this new type of photonic gap, assuming dispersions that are most frequently adopted for negative $\mu$ and $\varepsilon$. The results demonstrate that the TIR-gap is sensitive to angular change and insensitive to length scale change, and band structures of TE and TM mode waves are exactly the same in the PCs we studied.

Let a wave be incident from air at an angle $\theta$ onto 1D-PC containing two different dispersive left-handed metamaterials with dielectric permittivities $\varepsilon_{1,2}$ and magnetic permeabilities $\mu_{1,2}$, respectively, as shown in Fig. 1. Suppose wave vectors $\boldsymbol{k}(\omega)$ lie in the $x$-$z$ plane. For TE-polarized waves, the electric field component can be described by the linear Helmholtz-type equation. By solving the equation, the dependence of $K_\text{b}$, which is the dimensionless Bloch wave number, on wave vector component along the layers can be explicitly obtained for two-layered periodic structures[8,10]

$$2\cos(K_\text{b}) = \text{Tr}(\mathbf{M}) = 2\cos(k_{1x}d_1)\cos(k_{2x}d_2) - \left(\frac{k_{2x}\mu_1}{k_{1x}\mu_2} + \frac{k_{1x}\mu_2}{k_{2x}\mu_1}\right)\sin(k_{1x}d_1)\sin(k_{2x}d_2), \quad (1)$$

where $\mathbf{M}$ is a $2\times2$ transfer matrix for the two-layered structure, $k_{jx}$ are the $x$ components of the wave vector in the first ($j = 1$) and second ($j = 2$) media. For the

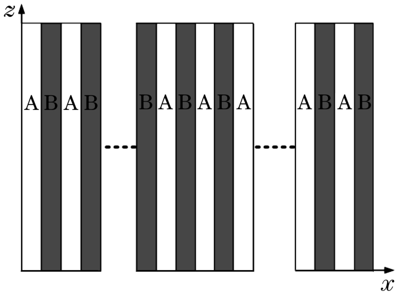

Fig. 1. Schematic of a 1D PC with dispersive left-handed metamaterials.

special case of $\mu_1 = \varepsilon_2 = \mu$, $\mu_2 = \varepsilon_1 = \varepsilon$, and $d_1 = d_2 = d$. Equation (1) becomes

$$2\cos(K_\mathrm{b}) = 2 - (\frac{\mu}{\varepsilon} + \frac{\varepsilon}{\mu} + 2)\sin^2(\frac{\omega}{c}d\sqrt{\varepsilon\mu - \sin^2\theta}), \tag{2}$$

if

$$\left|2 - (\frac{\mu}{\varepsilon} + \frac{\varepsilon}{\mu} + 2)\sin^2(\frac{\omega}{c}d\sqrt{\varepsilon\mu - \sin^2\theta})\right| \geq 2. \tag{3}$$

Except for being an integral multiple of $\pi$, Eq. (3) implies that Eq. (2) has no real solution for $K_\mathrm{b}$, indicating a spectral gap. This is the familiar Bragg condition for conventional PBG materials. However, if we stack alternating layers containing dispersive left-handed metamaterials, there is an extra possibility

$$\varepsilon\mu - \sin^2\theta < 0. \tag{4}$$

Inequation (4) will also lead to Eq. (2) an imaginary solution for $K_\mathrm{b}$ and thus a spectral gap appears. Inequation (4) is virtually the TIR-condition for the interface of air and the given left-handed metamaterial[11]. We call this gap the TIR-gap. In comparison with the Bragg gap, the TIR-gap has unique properties. In particular, the TIR-gap is angular sensitive, but has no polarization effect.

Based on Ref. [8], we give a theoretical model to expatiate the TIR-gap, the values of $\mu_{1,2}(f)$ and $\varepsilon_{1,2}(f)$ are given by[10,12]

$$\varepsilon_1(f) = \mu_2(f) = 0.5 + \frac{5^2}{0.9^2 - f^2} + \frac{10^2}{11.5^2 - f^2}, \tag{5}$$

$$\mu_1(f) = \varepsilon_2(f) = 0.5 + \frac{3^2}{0.902^2 - f^2}, \tag{6}$$

where $f$ is the frequency measured in GHz. It is noted these kinds of dispersion for $\mu_{1,2}(f)$ and $\varepsilon_{1,2}(f)$ may be realized in special microstrips[13]. The numerical values of $\mu_{1,2}(f)$ and $\varepsilon_{1,2}(f)$ are given in Fig. 2. In the following calculation, $\theta = \pi/6$, $d_1 = d_2 = 5$ mm are used.

The numerical values of $\mu_{1,2}(f) \times \varepsilon_{1,2}(f)$ and $\sin\theta$ are shown in Fig. 3(a). From 3.49 to 6.30 GHz, inequation (4) is satisfied and a gap opens at the corresponding frequency range. Figure 3(b) shows the band structure and Fig. 3(c) gives the transmissivity of a stack of 8 unit cells. The band structure and the transmissivity clearly show a band gap.

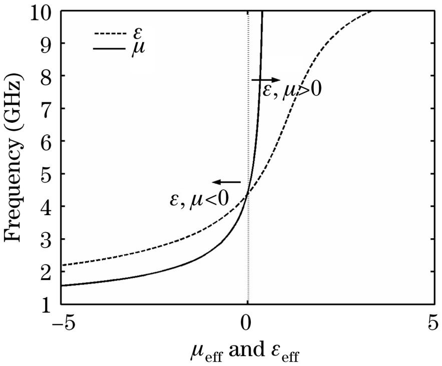

Fig. 2. Effective $\varepsilon$ and $\mu$ of the left handed metamaterials, as given by Eqs. (5) and (6).

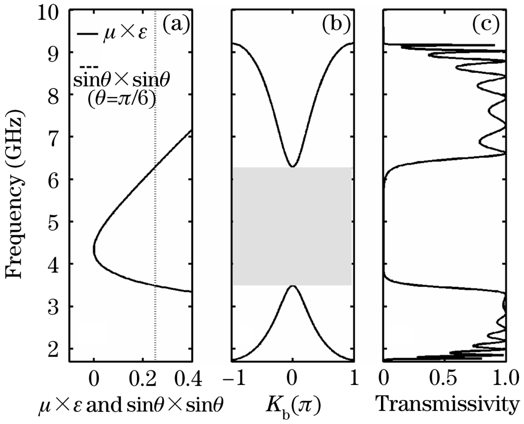

Fig. 3. (a) Numerical values of $\mu_{1,2}(f) \times \varepsilon_{1,2}(f)$ and $\sin\theta$, (b) dispersion relationship of a photonic crystal with alternate layers of dispersive left-handed metamaterials, grey areas are TIR-gap, $\theta = \pi/6$, $d_1 = d_2 = 5$ mm, (c) transmittance through 8 unit cells, corresponding to the band structure in (b).

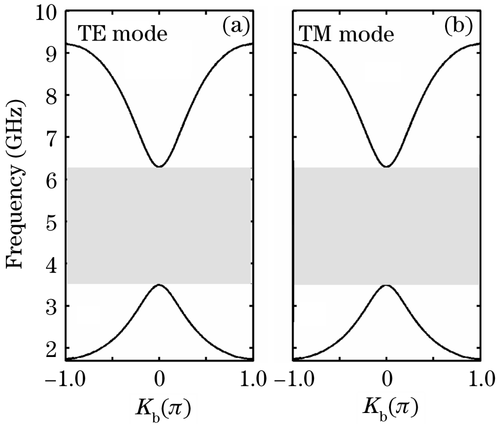

Fig. 4. Band structures of (a) TE-polarized waves (b) TM-polarized waves for the same structure with parameters given by Eqs. (5) and (6).

In order to show the unique properties of the TIR-gap, we study the dispersion relationship of TM-polarized waves by replacing $\varepsilon \leftrightarrow \mu$ in Eq. (1). We get the same dispersion relationship as that of TE-polarized waves owing to $\mu_1 = \varepsilon_2$, $\mu_2 = \varepsilon_1$, and $d_1 = d_2$. That is to say TM- and TE-polarized waves have the same band structure, which is shown in Fig. 4. Consequently, filters without polarization effect can be designed by using this kind of structure. Thus, in the following study, we only demonstrate the properties of TE-polarized waves.

For the given dispersive materials by Eqs. (5) and (6), the larger of the incident angle, the wider frequency band will satisfy inequation (4), so TIR-gap is angular dependence. The angular effect of TIR-gap is explicitly demonstrated in Fig. 5. The dotted line in Fig. 5(a) is the transmissivity through 8 unit-cells at an angle of $\pi/6$ and $d_1 = d_2 = 5$ mm. The solid line corresponds to the transmittance of the same structure but the incident angle is $\pi/3$. At increasingly oblique angle, the gap edges of the TIR-gap shift contrarily, one direction is higher frequency and another is lower frequency, which is shown in Fig. 5(a), and the band width is enlarged, which is

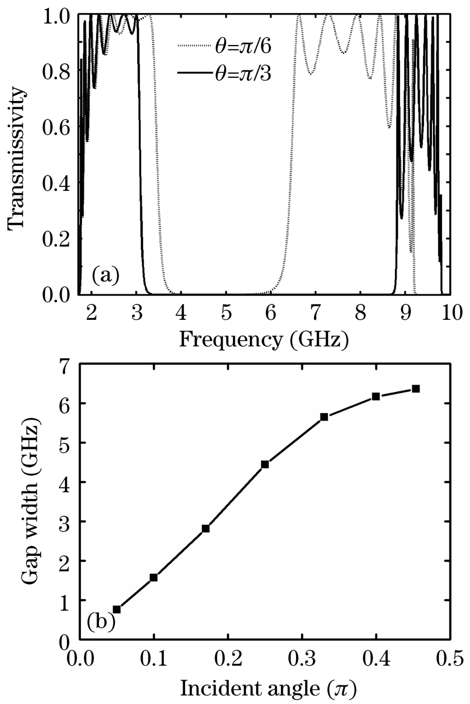

Fig. 5. Angular effect of TIR-gaps for 8 unit-cells, $d_1 = d_2 =$ 5 mm. (a) Dotted line is the transmission spectrum with incident angle of $\pi/6$, while solid line is for incident angle of $\pi/3$; (b) with the increase of the incident angle, the TIR-gap width is enlarged.

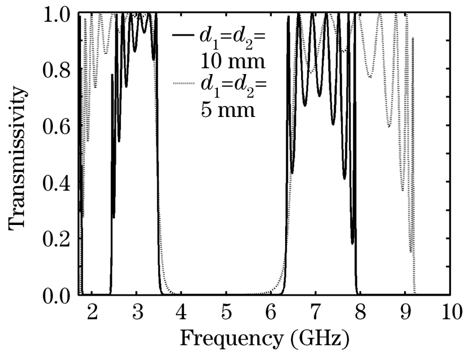

Fig. 6. Transmission spectrum for the PCs with the same parameters except for the layer thickness, dotted line: $d_1 = d_2 = 5$ mm, solid line: $d_1 = d_2 = 10$ mm.

shown in Fig. 5(b). The dependence of TIR-gaps on the incident angle means that photonic devices based on such PBGs can reflect different frequency without polarization effect by changing the incident angles.

Early researches have shown that both zero average refractive index and zero phase gap are all insensitive to unit-cell size change[8,9], which distinguishes them from Bragg gap. In contrast, we also study the corresponding characteristic of TIR-gap by changing the unit-cell size, which is shown in Fig. 6. The solid line in Fig. 6 is the transmittance through 8 unit-cells at an angle of $\pi/6$, and $d_1 = d_2 = 10$ mm, while the dotted line corresponds to the transmittance of the same incident angle but $d_1 = d_2 = 5$ mm. We can see from Fig. 6 that TIR-gap is also insensitive to the unit-cell size change, which corresponds to the essence of TIR, i.e..

In conclusion, we have showed that 1D PCs containing dispersive left-handed metamaterials possess a type of photonic band gap originating from TIR, which is distinct from Bragg gap, zero average refractive index and zero phase gaps. The TIR-gaps possess particular properties with angular effect and without polarization. It should also be noted that the band structures of TE and TM mode waves are exactly the same in the PCs, which can be used to design filters without polarization effect.

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