Carrier-envelope phase of ultrashort pulsed Laguerre-Gaussian beam

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The carrier-envelope phase of the ultrashort pulsed Laguerre-Gaussian beam is studied. The order of Laguerre function affects seriously the variations of the carrier-envelope phase with the propagation distance increasing. The beam waist also affects the carrier-envelope phase in a few Rayleigh distances. The variation of the carrier-envelope phase is larger on the axis than on the beam periphery in propagation.

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The rapid advances of laser technology in the past few decades have made the production of extremely short laser pulses, containing only a few, even only one, cycles of optical oscillations[1−3], which led to many new questions. The carrier-envelope (CE) phase is the phase of the carrier wave with respect to the pulse envelope. Figure 1 describes the relation of the pulse envelope with the electric field for a few-cycle optical pulse. The CE phase becomes a physically important parameter for ultrashort laser pulses because many physical processes are driven by it. Recently, significant progress was achieved in stabilizing the CE phase[4,5]. Nowadays much research focuses on the ultrashort laser pulses[6−13], especially the CE phase.

Lindner *et al.*[6] studied the Gouy phase shift for few cycle laser pulses. The results show that the CE phase undergoes a smooth variation over a few Rayleigh distances. Brabec *et al.*[7] demonstrated that the desription of the pulses in terms of an envelope and carrier oscillations remains to be useful and physically meaningful for few cycle pulses. Porras *et al.*[8,9] studied the CE phase of the pulsed Gaussian beam. They have gotten some useful relations between the CE phase and some parameters of the pulsed Gaussian beam. In this paper, the CE phase of the ultrashort pulsed Laguerre-Gaussian beam is studied. The results show that the order of Laguerre function affects seriously the variations of the CE phase with the propagation distance increasing. The beam waist also affects the CE phase in a few Rayleigh distances. The variation of the CE phase is larger on the axis than on the beam periphery.

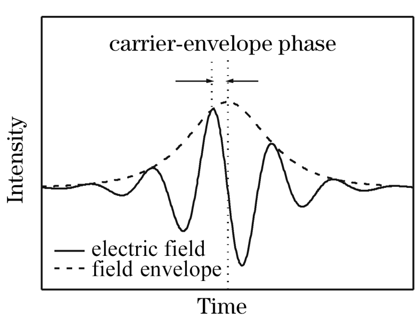

Fig. 1. Sketch of the CE phase.

We take a Laguerre-Gaussian beam form with a Gaussian pulse shape at the initial place, which is described as

$$E\left(r,0,t\right)=E_0L_n^m\left(-\frac{2r^2}{w_0^2}\right)\exp\left(-\frac{r^2}{w_0^2}\right)\times\exp\left(-\frac{t^2}{T^2}\right)\exp\left(-i\omega_0t\right),\qquad(1)$$

where E_0 is a constant related to the intensity of the laser beam, $r=\sqrt{x^2+y^2}$, w_0 is the beam waist, T is the pulse duration, ω_0 is the carrier frequency.

Suppose $\widetilde{E}\left(x,y,0,\omega\right)$ is the Fourier transform of the incident field at $z=0$, which is written as

$$\widetilde{E}\left(r,0,\omega\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}E\left(r,0,t\right)\exp\left(i\omega t\right)\mathrm{d}t.\qquad(2)$$

According to Eqs. (1) and (2), $\widetilde{E}\left(r,0,\omega\right)$ is easily obtained,

$$\widetilde{E}\left(r,0,\omega\right)=\frac{E_0T}{\sqrt{2}}L_n^m\left(-\frac{2r^2}{w_0^2}\right)\exp\left(-\frac{r^2}{w_0^2}\right)\times\exp\left(-\frac{\left(\omega-\omega_0\right)^2T^2}{4}\right).\qquad(3)$$

By using the beam formula, the expression of each sub-monochromatic wave along the z axis could be obtained as

$$\widetilde{E}\left(r,z,\omega\right)=\frac{E_0T}{\sqrt{2}w\left(z,\omega\right)}\left[\frac{\sqrt{2}r}{w\left(z,\omega\right)}\right]^m L_n^m\left(\frac{2r^2}{w^2\left(z,\omega\right)}\right)\times\exp\left(-\frac{r^2}{w^2\left(z,\omega\right)}\right)\cos\left(m\right)\exp\left(-\frac{\left(\omega-\omega_0\right)^2T^2}{4}\right)\times\exp\left[i\left(\frac{\omega}{c}z+\frac{\omega}{c}\frac{r^2}{2R^2\left(z,\omega\right)}-\left(m+2n+1\right)\arctan\left(\frac{z}{z_{\mathrm{R}}\left(\omega\right)}\right)\right)\right],\qquad(4)$$

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where

$$z_{\mathrm{R}}=\frac{\omega w_0^2}{2c},$$

$$R(z,\omega)=z+\frac{z_{\mathrm{R}}^2(\omega)}{z},$$

$$w(z,\omega)=w_0\sqrt{1+\left(\frac{z}{z_{\mathrm{R}}}\right)^2}, \tag{5}$$

are respectively the Rayleigh distance, the radius of curvature of the wave fronts, and the beam width at any propagation distance.

The phase of the monochromatic Laguerre-Gaussian beam in Eq. (4) is

$$\varphi(r,z,\omega)=\frac{\omega_0}{c}z+\frac{\omega_0 r^2}{2cR(z,\omega_0)} - (m+2n+1)\arctan\left(\frac{z}{z_{\mathrm{R}}(\omega_0)}\right). \tag{6}$$

The phase fronts of the pulsed Laguerre-Gaussian beam satisfies $\omega_0 t-\varphi(x,y,z,\omega)=\text{const.}$, namely,

$$\omega_0 t-\frac{\omega_0}{c}z-\frac{\omega_0 r^2}{2cR(z,\omega_0)} +(m+2n+1)\arctan\left(\frac{z}{z_{\mathrm{R}}(\omega_0)}\right)=\text{const.}. \tag{7}$$

Thus the time of the phase fronts from the initial place to the position $q(x,y,z)=\sqrt{x^2+y^2+z^2}$ is

$$\tau'=\frac{z}{c}+\frac{r^2}{2cR(z,\omega_0)}-\frac{(m+2n+1)}{\omega_0}\arctan\left(\frac{z}{z_{\mathrm{R}}(\omega_0)}\right). \tag{8}$$

The time of the pulse peak from the initial place to the same position $q(x,y,z)$ is the derivative of the phase $\varphi(x,y,z,\omega)$ with respect to frequency, evaluated at the carrier frequency, i.e,

$$\tau''=\frac{z}{c}+\frac{r^2}{2cR(z,\omega_0)}-\frac{r^2 z_{\mathrm{R}}^2(\omega_0)}{2czR^2(z,\omega_0)} +\frac{(m+2n+1)z_{\mathrm{R}}(\omega_0)z}{\omega_0\left(z_{\mathrm{R}}^2(\omega_0)+z^2\right)}. \tag{9}$$

Then expression of the CE phase at any propagation distance could be obtained as $\varphi_{\mathrm{CE}}(x,y,z)=\omega_0\tau'-\omega_0\tau''$, i.e,

$$\varphi_{\mathrm{CE}}(x,y,z)=\frac{\omega_0 z_{\mathrm{R}}^2(\omega_0)r^2}{czR^2(z,\omega_0)}-\frac{(m+2n+1)z_{\mathrm{R}}(\omega_0)z}{z_{\mathrm{R}}^2(\omega_0)+z^2} -(m+2n+1)\arctan\left(\frac{z}{z_{\mathrm{R}}(\omega_0)}\right). \tag{10}$$

Now we give some numerical results. In the numerical simulation, supposing $m=0$ for convenience, we consider firstly the effect of the order of Laguerre function on the CE phase on the axis, which is shown in Fig. 2, with $\omega_0=3.2$ fs^{-1}, $w_0=20$ μm, $z_{\mathrm{R}}=2.13$ mm. It is evident that the order of Laguerre function affects the CE phase seriously. For comparison, the Gouy shift of pulsed Gaussian beam is given in Fig. 2 with square line. The pulsed Gaussian beam undergoes a half π-phase shift from 0 to infinity, while the CE phase of the pulsed Laguerre-Gaussian beam is $(m+2n)$ times as large as that of the pulsed Gaussian beam from 0 to infinity. Thus the slope of the CE phase of the pulsed Laguerre-Gaussian beam is sharper than that of the pulsed Gaussian beam, especially in the first Rayleigh distance.

Figure 3 shows the CE phase of the pulsed Laguerre-Gaussian beam with different w_0. We find that they all undergo a same phase shift from 0 to infinity independent of beam waist, i.e. w_0, which is consistent with Fig. 2. The CE phase with a larger w_0 is smaller than that with a smaller w_0. The difference of the CE phase with different beam periphery is given in Fig. 4, where it has the same parameters as in Fig. 3. The maximal difference of the CE phases between $w_0=25$ μm and $w_0=20$ μm appears at $z=1.53$ mm and the neighborhood is 0.456π. The maximal difference of the CE phases between $w_0=15$ μm and $w_0=20$ μm appears at $z=0.91$ mm and the neighborhood is 0.583π.

The variations of the CE phase at different transversal positions with the propagation distance increasing are shown in Fig. 5. We find that the CE phase outside the axis is smaller than that on the axis, and the CE phase

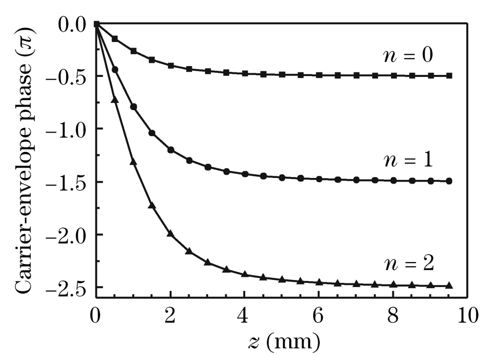

Fig. 2. Effect of the different order of Laguerre function on the CE phase on the axis. The CE phase of the pulsed Laguerre-Gaussian beams undergoes a $(1/2+n)\pi$ phase shift from 0 to infinity, which leads to a sharper slope. Parameters: $\omega_0=3.2$ fs^{-1}, $w_0=20$ μm, $z_{\mathrm{R}}=2.13$ mm, $m=0$.

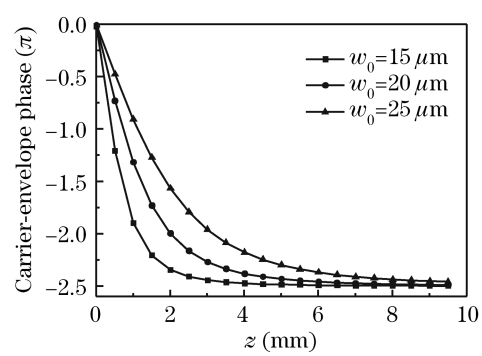

Fig. 3. Effect of the different beam waist on the CE phase on the axis. Parameters: $\omega_0=3.2$ fs^{-1}, $w_0=20$ μm, $z_{\mathrm{R}}=2.13$ mm, $m=0$, $n=2$.

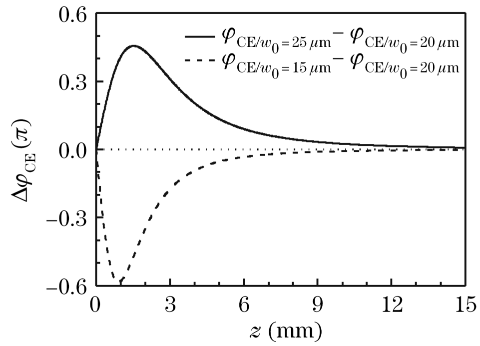

Fig. 4. Difference of the CE phase with different beam waists. The solid curve denotes the difference of the CE phase between $w_0 = 25$ μm and $w_0 = 20$ μm; the dashed curve denotes the difference of the CE phase between $w_0 = 15$ μm and $w_0 = 20$ μm. The parameters are the same as those in Fig. 3.

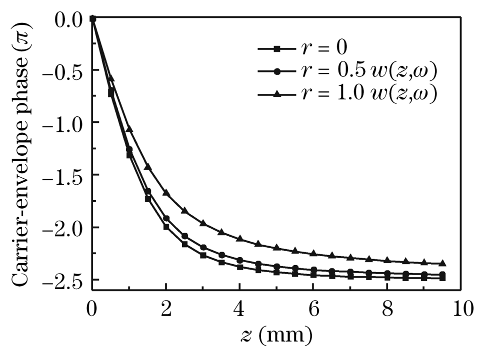

Fig. 5. Variation of the CE phase at different transversal positions with the propagation distance. Parameters: $\omega_0 = 3.2$ fs^{-1}, $w_0 = 20$ μm, $z_R = 2.13$ mm, $m = 0$, $n = 2$.

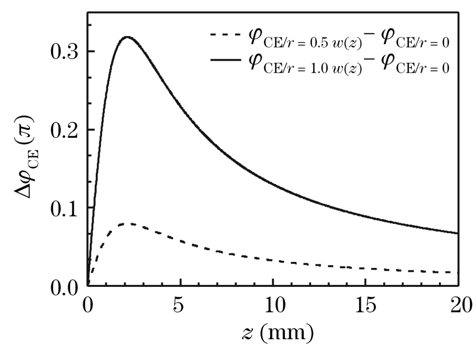

Fig. 6. Difference of the CE phase at different beam peripheries. The dashed curve denotes the difference of the CE phase between $r = 0$ and $r = 0.5w\,(z,\omega)$; the solid curve denotes the difference of the CE phase between $r = 0$ and $r = 1.0w\,(z,\omega)$. The parameters are the same as those in Fig. 5.

shift becomes smaller when the transversal position is nearer the axis. The difference of the CE phase with different beam waist is given in Fig. 6, where it has the same parameters as in Fig. 5. The maximal difference of the CE phases between $r = 0.5w\,(z,\omega)$ and $r = 0$ appears at $z = 2.1$ mm and the neighborhood is 0.08π. The maximal difference of the CE phases between $r = 1.0w\,(z,\omega)$ and $r = 0$ also appears at the same distance, i.e. $z = 2.1$ mm, and the neighborhood is 0.318π.

In conclusion, the CE phase of the ultrashort pulsed Laguerre-Gaussian beam is studied. We find that the order of Laguerre function affects seriously the variations of the CE phase. The beam waist also affects the CE phase in a few Rayleigh distances. When the beam waist is shorter, the pulsed beam undergoes a larger CE phase shift in a few Rayleigh distance. The variations of the CE phase on and outside the axis along the propagation distance is also studied. The result shows that the CE phase on the axis is larger than that on the beam periphery at any propagation distance.

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