Influence of dielectric microcavity on the spontaneous emission rate of atom: a perspective on the closed-orbit theory of photons

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The formulas of the quantum electrodynamics have been applied to calculate the spontaneous emission rate of excited atom in dielectric microcavity. The results exhibit damping oscillating patterns which depend sensitively on the scaling parameter and geometrical structure. Compared with the case that the emitting atom is immersed in dielectric, the spontaneous emission rate is depressed obviously and the center or the mean value of the oscillations is intimately related to the real refractive index of the local position where the atom is. In order to explain this phenomenon, we utilize the closed-orbit theory to deal with the classical trajectories of the emitted photon, and extract the corresponding frequencies of the oscillations by Fourier transform. It is found that the oscillations can be represented in terms of the closed-orbits of the photon motion constrained in dielectric microcavity, thus providing another perspective on the spontaneous emission of atom sandwiched by dielectric slabs.

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It has been perceived that the spontaneous emission rate of atom depends on the material environment since Purcell's pioneering work[1]. Modification of the spontaneous emission rate has been experimentally shown for free atoms[2−4], organic molecules[5] and Eu^{3+} complexes[6] in solution, single-mode Er^{3+}-doped tellurite fiber[7] and Er^{3+}-doped SiO_2[8]. This field has attracted much attention recently. With quantum electrodynamics (QED), Urbach and Rikken calculated the spontaneous emission rate for the nonabsorbent dielectric film system[9]. The lifetime distribution for an assembly of atoms in inhomogeneous electromagnetic environments has become the most popular topic in several investigations and played an important role in the atomic chip and optical crystal[10−17]. Wang *et al.* investigated the spontaneous emission rate of atoms near a dielectric interface and inside a dielectric slab with the closed-orbit theory[18,19]. Compared with the early work done in order to investigate the case for a thin quantum well sheet enclosed by a one-dimensional dielectric microcavity, the spontaneous emission rate was calculated by QED formulas as functions of a scaling variable measuring the overall size of the system. The results exhibited damping oscillations and rounded the real refractive index of dielectric microcavity if the local field and absorption effect had not been considered. We present a general physical mechanism for such oscillations[18,19] and give a visible physical image to interpret the spontaneous emission rate of atom in dielectric microcavity based on the closed-orbit theory which was created by Du and Delos[20,21]. The frequencies of the oscillations are extracted by Fourier transformation, which consist with the classical actions of the emitted photons along the closed-orbit trajectory. One closed-orbit of the emitted photon going out from and returning back to the emitting atom contributes an oscillatory peak in the spontaneous emission rate.

The model structure under consideration is shown in Fig. 1. A microcavity with real refractive index n_1 is sandwiched by two semi-infinity nonabsorbent dielectric slabs whose real refractive indices are n_2 and n_3, respectively. All the dielectric materials are assumed to be nonmagnetic. In a Cartesian coordinate system, the origin is in the middle of the microcavity while the z axis is perpendicular to the interfaces. The departure of an excited atom from the upper (lower) boundary is marked with d_1 (d_2). So the width of the dielectric microcavity is $d = d_1 + d_2$.

According to Fermi's golden rule, the QED formula for the spontaneous emission rate is

$$w = \frac{2\pi e^2}{\hbar^2 c} \left|\vec{D}_{12}\right|^2 F(z), \tag{1}$$

where e denotes the electron charge, c is the speed of light in vacuum, $\vec{D}_{12}$ is the dipole matrix element of the atomic transition. Since the orientation of the atomic dipole moment is random for the moment, then $F^j(z) = [F^x(z)+F^y(z)+F^z(z)]/3$, where F^j $(j = x, y, z)$

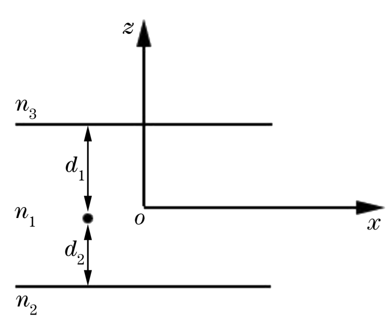

Fig. 1. Model structure of emitting atom in a microcavity system. n_j (j = 1, 2 and 3) are real refractive indices. The symmetric case $n_2 = n_3$ is investigated in this paper.

are the jth components of the zero-point fluctuation of electromagnetic field. In this model, the formula could be given by

$$F^j(z) = \sum_{\lambda=s,p}\sum_{\mu=2}^{3}\int_0^{2\pi}\int_0^{k_0 n_\mu}\frac{\hbar\omega_0}{2\varepsilon_0}\left|E^j_{k\lambda\mu}(\vec{r})\right|^2\beta\mathrm{d}\beta\mathrm{d}\varphi, \quad (2)$$

where $k_0=\omega_0/c$ is the wave number of the emitted light in vacuum, ε_0 is the vacuum permittivity, $E(\vec{r})$ is the electric field evaluated at the position of emitting atom for various modes, and s (p) indicates the TE (TM) polarization states whose electric (magnetic) field vector is parallel to the (x,y) plane, and $k_{jz}=\sqrt{k_0^2n_j^2-\beta^2}$ ($j=1,2,3$, respectively for the three different dielectric materials). Further details can be found in Ref. [9].

In the following, we use F to represent the relative spontaneous emission rate $F(z)/F_{\mathrm{v}}$, where F_{v} is the zero-point fluctuation of the electromagnetic field in vacuum at the position of the excited atom and can be expressed as

$$F_{\mathrm{v}} = \frac{\hbar\omega_0^3}{6\pi^2\varepsilon_0 c^2}. \quad (3)$$

In order to conveniently describe the damping oscillations in the spontaneous emission rate, we introduce $R=(d_2-d_1)/(d_1+d_2)$ to denote the emitting atom's position relative to the upper and lower interfaces and introduce a scaling variable α to measure the overall size of the system. For any R, we calculate the spontaneous emission rate $F^i(\alpha)$ ($i=x,y,z$) by Eqs. (1) and (2) in which $d_1=\alpha d_1^0$, $d_2=\alpha d_2^0$, $d=\alpha d^0$, where $d^0=\lambda_0=510$ nm is a typical size of the system. It is easy to show $d_1^0=\frac{(1-R)d^0}{2}$, $d_2^0=\frac{(1+R)d^0}{2}$, and the atomic coordination z is given by $z=\alpha R d^0/2$.

We set $n_1=1.00$ for vacuum and $n_2=n_3=1.49$ for the dielectric media. In Fig. 2, the calculated spontaneous emission rate F is plotted as a function α with a step size $\Delta\alpha=0.0035$ for the three situations $R=0$, $\frac{1}{3}$, and $\frac{3}{4}$, respectively. In the figure, dotted line denotes $F^x(\alpha)$ and dashed line represents $F^z(\alpha)$, while the solid line stands for the average of the three directions, $(2F^x(\alpha)+F^z(\alpha))/3$. In Fig. 2(a), the emitting atom is placed at the origin, and the emission rate presents a simple sinusoidal-damping oscillations; in Fig. 2(b), the distance from the emitting atom to the upper interface is one third of the dielectric microcavity width, and some oscillations with higher frequencies appear; in Fig. 2(c), the emitting atom is further moved up, whose distance to the upper interface is one eighth of the microcavity width, and the damping oscillations take on much more complex and consist of more higher frequencies. From Figs. 2(a)—(c), it is obvious that the center or the mean value of the damping oscillations is related to the real refractive index of the dielectric microcavity.

According to the closed-orbit theory, the formula of the spontaneous emission rate reads

$$F(\alpha)=F_0+\sum_i A_i\sin[S_i(\alpha)+\varphi_i], \quad (4)$$

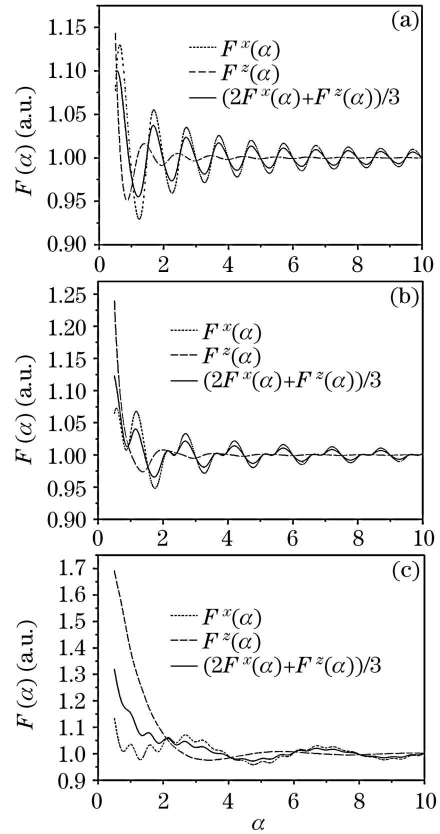

Fig. 2. Calculated spontaneous emission rate F (relative to the value in vacuum) for the microcavity with $n_1=1.00$, $n_2=n_3=1.49$. $d=\alpha d^0$ is the microcavity width, where $d^0=\lambda_0=510$ nm. (a) $R=0$, (b) $R=\frac{1}{3}$, (c) $R=\frac{3}{4}$.

where F_0 represents a background term and its value is equal to n_1, which can be attributed to the free emission process implicating that when the emitted photon leaves the emitting atom and never returns. The sum is over all classical orbits of emitted photon going out from and returning back to the emitting atom; the emitted photon obeys the laws of reflection and refraction when it travels in classical trajectories; $S_i(\alpha)=kL_i$ is the action of the photon along the ith closed-orbit, where k is the wave number, and L_i is the geometric length of the closed orbit; the amplitude A_i varies slowly and is a measure of the intensity of the returning group of photon wave. The phase accumulations, including Maslov corrections, are denoted in φ_i.

In order to extract the frequencies in the damping oscillations of the spontaneous emission rate, we follow the standard approach in the closed-orbit theory. Firstly we remove the background term F_0 and then multiply the result by α before Fourier transform. We define

$$F'(\alpha)=[\frac{F(\alpha)}{F_0}-1]\alpha, \quad (5)$$

and perform Fourier transform on F':

$$\tilde{F}(\gamma)=\sum_i F'(\alpha_i)\mathrm{e}^{-i\gamma\alpha_i}\Delta\alpha, \quad (6)$$

where the step is $\Delta\alpha=0.0035$ in our calculations.

Fourier transforms of the spontaneous emission rate $F(\alpha)$ are displayed in Fig. 3. In Fig. 3(a), two peaks appear at $\gamma=6.26$ and $\gamma=12.53$; there are three species

Table 1. Frequencies of Oscillations Compared with Actions of Photons' Closed Orbits

R	0		1/3			3/4		
Closed Orbit i	1st & 2nd	3rd & 4th	1st	2nd	3rd & 4th	1st	2nd	3rd & 4th
QED γ	6.26	12.53	4.17	8.34	12.56	1.49	11.00	12.54
S_i^0	6.28	12.56	4.16	8.37	12.56	1.57	10.99	12.56

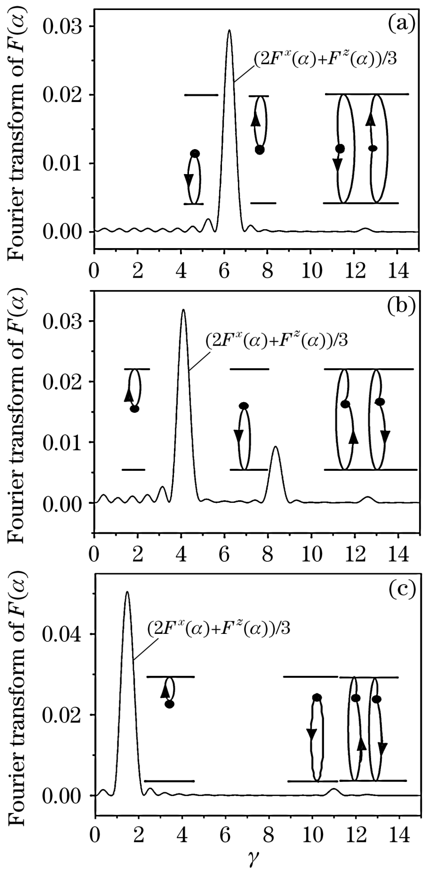

Fig. 3. Fourier transform of the spontaneous emission rate $F(\alpha)$ in Fig. 2. The responsible photon closed orbits are shown schematically near the peaks. (a) $R = 0$, (b) $R = \frac{1}{3}$, (c) $R = \frac{3}{4}$.

peaks at $\gamma = 4.17$, 8.34, and $\gamma = 12.58$, respectively in Fig. 3(b); and in Fig. 3(c), the first and second peaks move to $\gamma = 1.49$, 11.00 while the last one is located at $\gamma = 12.54$.

The closed-orbit theory predicts the ith peak at S_i^0 corresponding to $S_i = \alpha S_i^0 = k_0 L_i$. The actions of emitted photon around closed orbits can be specified by peaks' positions. In Table 1, we show the actions of the photon's closed orbits compared with the frequencies of the damping oscillations by Fourier transform when $R = 0$, $\frac{1}{3}$, and $\frac{3}{4}$, respectively.

The physical image can be interpreted as follows. The emitted photon is created near the atom when the atom decays from the excited states to lower states. This photon leaves away from the atom to a large distance and returns to the atom when it collides with the interface between different media, thus contributes a visible oscillation in the spontaneous emission rate. The trajectory of the photon forms one closed orbit. The damping oscillations in the spontaneous emission rate in Fig. 2 can

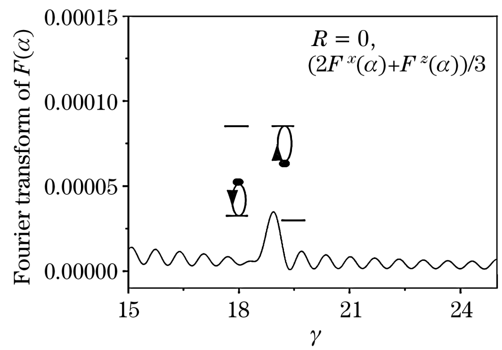

Fig. 4. Average of the spontaneous emission rate in Fig. 3(a) which is magnified at $\gamma = 18.90$. The responsible photon closed orbits are shown schematically near the peaks.

be regarded as quantum interference between the emitted photon wave and the photon wave returning back to the emitting atom. Each closed orbit produces an oscillatory contribution to the rate with a characteristic frequency determined by its action.

We actually predict peaks present again as γ increases. Figure 4 is the magnified part of the average of the spontaneous emission rate in Fig. 3(a) near $\gamma = 18.90$. Peak appears at $\gamma = 18.93$ while the action of the photon is $S_{1,2}^1 = 6\pi n_1 = 18.84$. This accords with our prediction. The emitted photon collides with the interfaces more than two times. Longer closed orbits usually correspond to higher frequencies with smaller amplitude oscillations. Their frequencies are much higher while amplitudes of the damping oscillations are much smaller. So these peaks are too weak to be seen in Fig. 3. It seems in line with the physical intuition.

In conclusion, the spontaneous emission rate in dielectric environments is calculated with QED. As the scale parameter of the system is varied, the result displays damping oscillations around the real refractive index of the dielectric microcavity. From the closed-orbit theory, each closed orbit produces a visible oscillation. The frequencies are extracted by Fourier transform, which is induced by the outgoing and incoming electromagnetic waves propagating along a closed orbit. The positions where the peaks appear agree well with the actions of the emitted photon. This study gives a visible physical insight into the spontaneous emission of the excited atom. It is a successful sample by using the closed-orbit theory to represent the oscillations in the spontaneous emission rate of atom in dielectric microcavity.

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