## Transmission properties of one-dimensional photonic crystals containing double-negative and single-negative materials

Xia Li (李 侠)<sup>1,2</sup>, Kang Xie (谢 康)<sup>1</sup>, and Haiming Jiang (姜海明)<sup>1</sup>

<sup>1</sup>School of Opto-Electronic Information, University of Electronic Science and Technology of China, Chengdu 610054 <sup>2</sup>Department of Physics, Chengdu University of Technology, Chengdu 610059

## Received June 5, 2007

The transmission properties of one-dimensional photonic crystals containing double-negative and singlenegative materials are studied theoretically. A special kind of photonic band gap is found in this structure. This gap is invariant with scaling and insensitive to thickness fluctuation. But when changing the ratio of the thickness of two media, the width of the gap could be enlarged. The defect modes are analyzed by inducing a linear defect layer in the structure. It is found that the number of defect modes will increase when the thickness of the defect layer becomes larger.

OCIS codes: 260.1180, 120.7000, 230.4170.

Photonic crystal (PC) is an artificially fabricated structure with periodically modulated dielectric function. Because of the similarity between photonic band gap and traditional electronic band gap, PC has been studied intensively for their potentials for confining and guiding light in very small structures<sup>[1]</sup>. In existing discussions, the new type of gaps and the defect modes are the two most attracting things<sup>[2-4]</sup>.

Conventional photonic band gap (PBG) originates from the interference of Bragg scattering in a periodical dielectric structure. This Bragg gap is decided by lattice constant and working wavelength, but it is sensitive to disorder and may be deteriorated by randomness. So some new types of PBG are needed. Later the negative refractive index materials attracted much attention because of their special refraction characteristics [5-7]. In the stacking alternating layers of double-positive (DPS) and double-negative (DNG), a type of PBG corresponding to zero (volume) averaged refractive index was found<sup>[8,9]</sup>. This zero- $\bar{n}$  gap is invariant with scaling and insensitive to disorder, but its middle will shift noticeably to meet the zero- $\bar{n}$  condition and its width will change a little when the ratio of thickness of the two media changes. In another multilayer structure composed of the epsilon-negative (ENG) media with negative permittivity but positive permeability and the mu-negative (MNG) media with negative permeability but positive permittivity, a new type of photonic band gap called zero- $\Phi_{\rm eff}$  gap was found<sup>[10,11]</sup>. This gap opens at the zero effective phase delay point, its width could be enlarged by changing the ratio of thickness of the two media but its middle is fixed. These two findings are important, and at the same time they are suggestive. We can suppose that other new types of PBG could be found in different structures composed of negative refractive index materials.

On the other hand, the defect modes have been discussed frequently in all kinds of  $PCs^{[12-14]}$ . In those works about one-dimensional photonic crystals (1D PCs), there is a common conclusion that one defect usually induces one defect mode. Many researches have been focused on the analysis of the only defect mode, and the possibility that there are several defect modes induced by one defect has been missed.

In this paper we will discuss the transmission properties of 1D PCs containing DNG and ENG materials. By using transfer-matrix method (TMM), a new type of PBG is found and its special characteristic is discussed. Then we induce one linear defect layer in this structure and analyze the defect modes when changing the thickness of the defect layer.

Suppose the 1D PC containing 32 layers (16 periods) with alternating thicknesses  $d_A$  and  $d_B$ . The ENG material (A) and DNG material (B) are respectively described as:

$$\mu_{\rm A} = 3,$$
  $\varepsilon_{\rm A} = 1 - 100/\omega^2,$   
 $\mu_{\rm B} = 1 - 100/\omega^2,$   $\varepsilon_{\rm B} = 1.21 - 100/\omega^2,$ 

where  $\omega$  is the angular frequency measured in GHz and such dispersion has been used in other works<sup>[15]</sup>.

We consider a transverse electric (TE) wave normal incident to the 1D PC structure from vacuum along zdirection and the electric field is polarized along x direction. The transfer matrix is induced as<sup>[15]</sup>

$$M_{j} = \begin{pmatrix} \cos\beta & \frac{\sqrt{\mu_{j}}}{\sqrt{\varepsilon_{j}}}\sin\beta \\ -\frac{\sqrt{\varepsilon_{j}}}{\sqrt{\mu_{j}}}\sin\beta & \cos\beta \end{pmatrix},$$
$$\beta = \frac{\omega}{c}n_{j}(\omega)d_{j}, \qquad n_{j} = \sqrt{\varepsilon_{j}}\sqrt{\mu_{j}}.$$

The transfer matrix of the whole structure can be written as  $X = \prod M_j(d_j)$ , and then we get the transmission coefficient  $t(\omega) = \frac{2}{[x_{11}+x_{22}+i(x_{12}-x_{21})]}$ . The power transmittance is determined by  $T = tt^*$ .

Figure 1 shows that a special kind of band gap is found in the 1D PC containing DNG and ENG materials. This gap is not a gap with common physical sense. It is formed



Fig. 1. Special PBG in 1D PC containing ENG and DNG with thicknesses  $d_{\rm A} = 12$  mm and  $d_{\rm B} = 6$  mm.

simply by two limits: the lower limit expressing that the transmission coefficient in 1D PC composed of ENG and DNG becomes to zero from this point; the upper limit representing the frequency from which the two media (ENG and DNG) both become normal materials (DPS) and the corresponding transmission condition changes. So in the following, we will see that the upper limit of this band gap always keeps invariant.

We also find that the gap is robust against disorder and variant scaling. In Fig. 2, the solid line is the transmittance through 16 periods and the ratio of  $d_A/d_B$  is 2; the dotted line is the transmittance through the same structure but the lattice constant is scaled to 1/2 of its original value; the dashed line is the transmittance through the structure with thickness fluctuation of  $\pm 3$ mm averaged over 32 layers under the condition that the average ratio of  $d_A/d_B$  remains 2. From the figure we can see that this gap is invariant with any disorder or scaling change.

This result could be explained by the fact that the lower limit of the gap is determined by the dispersion of the metamaterials. Therefore, like the zero- $\Phi_{\rm eff}$  gap and zero- $\bar{n}$  gap, the lower limit of the new gap is robust against disorder and variant scaling. The traditional Bragg gap, however, relying on interference mechanisms, is influenced by disorder and scaling variance.

Now we discuss that the width of the gap may be enlarged freely by varying the ratio of  $d_A/d_B$ . Figure 3 shows the transmittance spectra of the PBG with



Fig. 2. Transmittance spectra showing that the PBG is invariant with disorder or scaling chage. Solid line: 16 periods,  $d_{\rm A} = 12$  mm,  $d_{\rm B} = 6$  mm; dotted line: the lattice constant is scaled to 1/2 of the original value; dashed line: thickness fluctuation of  $\pm 3$  mm averaged over 32 layers (+ and – are equally probable).



131

Fig. 3. Variance of band gap with the ratio of thicknesses of two media. Solid line:  $d_{\rm A} = 12$  mm,  $d_{\rm B} = 6$  mm; dotted line:  $d_{\rm A} = 12$  mm,  $d_{\rm B} = 12$  mm; dashed line:  $d_{\rm A} = 6$  mm,  $d_{\rm B} = 12$  mm.

different ratios of two media. From these results we can conclude that the width of the new band gap can be enlarged by increasing the ratio of  $d_{\rm A}/d_{\rm B}$ , and this characteristic is similar with the unique feature of zero- $\Phi_{eff}$  $gap^{[11]}$ . At the same time we also see that the middle of the gap shifts noticeably when the  $d_A/d_B$  varies, and this characteristic is shared by the zero- $\bar{n}$  gap. So this special band gap has the peculiarities of both zero- $\Phi_{\rm eff}$  gap and zero- $\bar{n}$  gap, but undoubtedly it is different from both zero- $\Phi_{\rm eff}$  gap and zero- $\bar{n}$  gap: zero- $\Phi_{\rm eff}$  gap originates from the interaction of evanescent waves, and the zero- $\bar{n}$ gap comes from the interaction of propagating waves. In the PC discussed in this paper, material A whose refractive index is a pure imaginary number, supports only evanescent electromagnetic fields; material B with a negative refractive index, supports propagating waves. The transmission properties of this PC are determined by the interaction of propagating waves and evanescent waves, and this mechanism may lead to the result that the width of the gap could be enlarged freely by decreasing  $d_{\rm A}/d_{\rm B}$ . In addition, we can see that the transmission coefficient becomes larger when  $d_A/d_B$  becomes smaller.

Finally we discuss the defect modes in the special band gap. Supposing that the defect is induced by using a linear defect layer C (a kind of DPS material with refractive index  $n_{\rm C} = 3$ ) to replace a layer B in the periodical structure, thus the structure has the form of  $(BA)^{8}C(AB)^{8}$ . When the thickness of layer C becomes larger, we will get more and more defect modes in the gap, as shown in Fig. 4. The band gap we discussed here corresponds to the structure with  $d_{\rm A} = 12$  mm and  $d_{\rm B} = 6$  mm. Figure 4(a) shows there is one defect mode in the structure with  $d_{\rm C} = 80$  mm, Fig. 4(b) shows there are two defect modes in the structure with  $d_{\rm C} = 110$  mm, Fig. 4(c) shows three defect modes appearing in the gap when the thickness of the defect layer is 170 mm, and Fig. 4(d) shows there is no defect mode when the thickness of the defect layer is 60 mm. So we obtained more and more defect modes by increasing the thickness of the defect layer, and similar phenomena also have been found in other  $PCs^{[16,17]}$ .

In this paper we show that 1D PC containing ENG and DNG materials possess a new type of band gap. This gap is different from usual Bragg gap, zero- $\Phi_{\rm eff}$  gap and zero- $\bar{n}$  gap. It may be very useful in practical applications because of its special characteristics. We also discuss the linear defect modes in the special band gap, and the



Fig. 4. Defect modes in the PBG obtained by introducing a linear defect layer C in the periodical structure. The thicknesses of the defect layer are (a) 80, (b) 110, (c) 170, and (d) 60 mm, respectively.

interesting result is that the number of defect modes can be increased by augmenting the thickness of the linear defect layer.

This work was supported by the National Natural Science Foundation of China (No.60607005 and 60588502) and the Science and Technology Bureau of Sichuan Province (2006z02-010-3). X. Li's e-mail address is lixiamail@tom.com.

## References

 S. Fan, Z. Wang, D. A. B. Miller, P. R. Villeneuve, H. A. Haus, and J. D. Joannopoulos, Proc. SPIE 4870, 298 (2002).

- 2. Y. Zhang and J.-J. Shi, Chin. Phys. Lett. 23, 639 (2006).
- E. Yablonovitch, T. J. Gmitter, R. D. Meade, A. M. Rappe, K. D. Brommer, and J. D. Joannopoulos, Phys. Rev. Lett. 67, 3380 (1991).
- F.-F. Ren, R. Li, C. Cheng, J. Chen, Y.-X. Fan, J. Ding, and H.-T. Wang, Phys. Rev. B 73, 033104 (2006).
- D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. 84, 4184 (2000).
- S. Foteinopoulou, E. N. Economou, and C. M. Soukoulis, Phys. Rev. Lett. 90, 107402 (2003).
- D. R. Fredkin and A. Ron, Appl. Phys. Lett. 81, 1753 (2002).
- J. Li, L. Zhou, C. T. Chan, and P. Sheng, Phys. Rev. Lett. 90, 083901 (2003).
- H. Jiang, H. Chen, H. Li, Y. Zhang, and S. Zhu, Appl. Phys. Lett. 83, 5386 (2003).
- L.-G. Wang, H. Chen, and S.-Y. Zhu, Phys. Rev. B 70, 245102 (2004).
- H. Jiang, H. Chen, H. Li, Y. Zhang, J. Zi, and S. Zhu, Phys. Rev. E 69, 066607 (2004).
- S. M. Wang, C. J. Tang, T. Pan, and L. Gao, Phys. Lett. A 348, 424 (2006).
- X.-Q. Huang and Y.-P. Cui, Chin. Phys. Lett. 20, 1721 (2003).
- R. D. Meade, K. D. Brommer, A. M. Rappe, and J. D. Joannopoulos, Phys. Rev. B 44, 13772 (1991).
- K.-S. Tang, Y.-J. Xiang, and S.-C. Wen, Proc. SPIE 6020, 60200S (2005).
- J. Dong, Y. Chen, and H. Wang, Acta Phys. Sin. (in Chinese) 56, 268 (2007).
- Y. Chen, Y. Zhang, and S. Liu, Opt. Commun. 265, 542 (2006).