Effect of temperature on three-in-one composite compensator

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In order to reduce the error caused by the change of temperature, the relation between three-in-one composite compensator's phase retardation and temperature is calculated according to the characteristic of multiple-order quartz wave-plate. The results show that the influence on phase retardation increases with the increase of temperature and order of wave-plate. The phase retardation and the angle that the second wave-plate rotates are in a linear relationship in certain ranges. However, there is also some regularity in nonlinear areas which can be determined by theoretical calculation. The calculated result is well in agreement with experiments.

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Three-in-one composite compensator[1−3], an important optical component in polarized technology which can achieve the continuous modulation of phase retardation in certain ranges[4], is used widely in polarized light analysis, ellipse polarized light measurement, and optical modulation. Three-in-one composite compensator is usually affected by external fields, especially temperature field which is very essential to be studied. The three-in-one composite compensator of quartz is a perfect new-type compensator[1] with high precision and the adjustable phase retardation precision can be controlled within 1%. In the letter, the temperature effect of three-in-one composite quartz compensator is studied.

The phase retardation of the wave-plate where optical axis parallels its surface satisfies the following equation[5]:

$$\delta = \pm\frac{2\pi d(n_{\mathrm{o}} - n_{\mathrm{e}})}{\lambda} = 2\pi N, \tag{1}$$

where d is the thickness of the wave-plate, λ is the wavelength, N is the order of wave-plate, n_{o} and n_{e} are the refractive indexes of o light and e light, respectively.

When the temperature of the system is changed, the thickness of the wave-plate and the double refractive indexes change due to expansion. The thermal expansion coefficient of quartz crystal satisfies the following equation[6]:

$$\alpha = A \times 10^{-6} + B \times 10^{-8}T + C \times 10^{-11}T^2, \tag{2}$$

where A is a coefficient related to temperature, B and C are the dimensions with reciprocal of temperature's square and cube respectively. A is 13.37, B and C are 0 in the temperature range of $273-353$ K, so we can obtain the equation of the thickness of quartz that is perpendicular to optical axis:

$$d' = d[1 + 1.337 \times 10^{-5}(T' - T)]. \tag{3}$$

The relation between double refractive indexes and temperature, which is more complicated than the relation between thickness and temperature, is related to wavelength. The refractive index is variational with the variation of wavelength. At a wavelength of 632.8 nm, the quartz crystal satisfies the following equations[6]:

$$\frac{\mathrm{d}n_{\mathrm{o}}}{\mathrm{d}T} = -5.47 \times 10^{-6}/°\mathrm{C},$$

$$\frac{\mathrm{d}n_{\mathrm{e}}}{\mathrm{d}T} = -6.51 \times 10^{-6}/°\mathrm{C}. \tag{4}$$

From Eq. (4), the refractive index is decreasing with the increase of temperature, so the equation of double refractive indexes at temperature T' is

$$n'_{\mathrm{e}} - n'_{\mathrm{o}} = 0.00906 - (T' - 296) \times 1.04 \times 10^{-6}. \tag{5}$$

From Eqs. (1), (3), and (5), the phase retardation of 632.8 nm wave-plate at temperature T' can be expressed as

$$\delta' = 2\pi N[1 - 1.01420 \times 10^{-4}(T' - 296) - 1.53475 \times 10^{-9}(T' - 296)^2]. \tag{6}$$

It can be known from Eq. (6) that with the increase of N, the influence of phase retardation caused by the change of temperature is more and more obvious.

Assume δ_1, δ_2, and δ_3 represent the phase retardations of the first, the second, and the third wave-plates, respectively. When δ_1 equals δ_3, the first wave-plate's fast axis parallels to the third's, and the angle between the first wave-plate's fast axis and the second's is θ, the composite wave-plate's phase retardation δ satisfies the following equation[7]:

$$\cos(\delta/2) = \cos\delta_1 \cos(\delta_2/2) - \sin\delta_1 \sin(\delta_2/2)\cos 2\theta. \tag{7}$$

If the first and the third wave-plates are $\lambda/4$ plates and the second is a $\lambda/2$ one, the relation between δ and θ satisfies

$$\delta = 2\pi - 4\theta. \tag{8}$$

From Eq. (8), different values of δ can be obtained by changing the value of θ. With the variation of temperature, all of δ_1, δ_2, and δ_3 will change. Obviously, Eq. (8) is not satisfied. However, the relationship between δ and θ can still be deduced by Eq. (7). For the sake of calculation convenience, all of the three wave-plates are supposed to be of the same order.

From Eqs. (6) and (7), the relation between δ and θ can be drawn, as shown in Fig. 1. It can be seen that δ and θ satisfy the linear relationship as shown in Eq. (8) strictly at 296 K, but the relationship no longer exists when temperature changes. The adjustable ranges of δ will reduce whether the temperature increases or decreases. The variation of δ will increase with temperature departing from 296 K. The variation is the largest when $\theta = 0$ and $\theta = \pi/2$, because the sign of $\cos 2\theta$ changes at 0 and $\pi/2$. Figure 1(a) is the relationship between phase retardation δ and θ of a 23-order compensator at different temperatures and Fig. 1(b) shows a 10-order one. From Fig. 1, the variation is larger with larger order. However, δ and θ still approximately satisfy linear relationship. For example, in the range of $0.3-1.4$, δ satisfies $\delta = 6.1526 - 3.89120\theta$ when $T = 283$ K in Fig. 1(a) and the error is within 0.6%. In Fig. 1(b), the order of wave-plate is smaller, and the change of δ is also smaller than the former one. In summary, the adjustable scope is smaller with larger temperature change and larger order. So we should choose the wave-plates with small orders for three-in-one composite compensator. Meanwhile, when the temperature changes, δ can be modified by adjusting θ properly. For example, $\delta = \pi$ when $\theta = \pi/4$ at 296 K, while θ should be 0.776 to ensure $\delta = \pi$ at 283 K, which is about 0.5° smaller than $\pi/4$. The precision of step motor can achieve the second-order amplification to satisfy the request. It can be obtained by calculation for the δ in nonlinear areas.

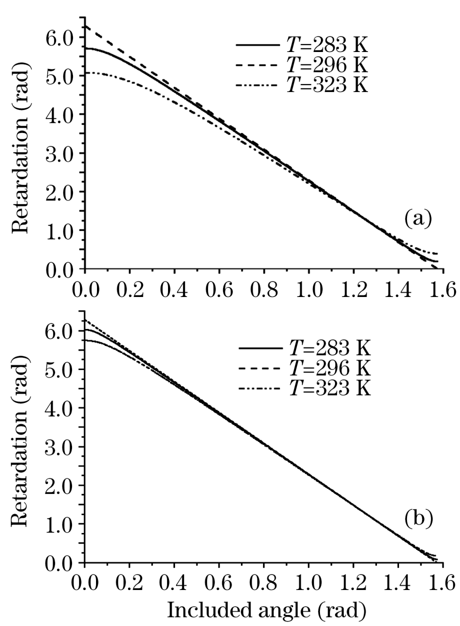

Fig. 1. Relation between δ and θ ($\lambda = 632.8$ nm). (a) For a 23-order wave-plate; (b) for a 10-order wave-plate.

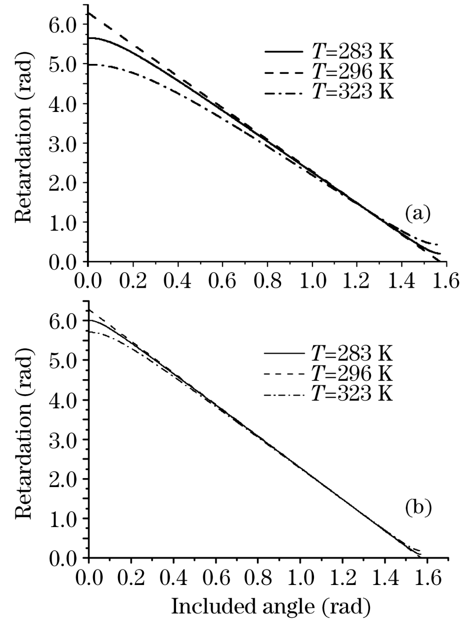

Fig. 2. Relation between δ and θ ($\lambda = 589.3$ nm). (a) For a 23-order wave-plate; (b) for a 10-order wave-plate.

For comparison, the relation between δ and θ at 589.3 nm is given in Fig. 2.

It can be known from Figs. 1 and 2 that for the compensators with the same order, the phase retardation of long wavelength changes less than the one of the shorter wavelength with the same temperature variation. This is because for the single wave-plates with the same order, the influence of phase retardation caused by temperature is smaller with longer wavelength[8,9]. In addition, there exists a θ value of 1.22, with which δ is always 1.4 at any temperature and order.

Figure 3 is the schematic diagram of the experimental equipment. In Fig. 3, S is the light source, P_1 and P_2 are polarization prisms with the same specification, P_c is three-in-one composite compensator which is set in temperature equipment T, G is detector, B is step motor, P is computer control system.

The principal axis of P_1 is determined as x axis, the travel direction of light is determined as z axis, the included angle between P_1 and P_2 is 45°, so the phase retardation is[10,11]

$$\delta = 2\arcsin\sqrt{(I_{\max} - I_{\min})/(I_{\max} + I_{\min})}, \tag{9}$$

where $I_{\max}$ and $I_{\min}$ are the largest and smallest intensities of light measured, respectively.

In Fig. 4, the theoretical curve and the experimental

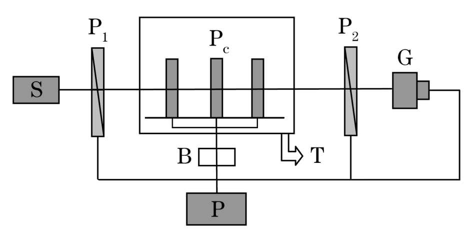

Fig. 3. Schematic diagram of the experimental equipment.

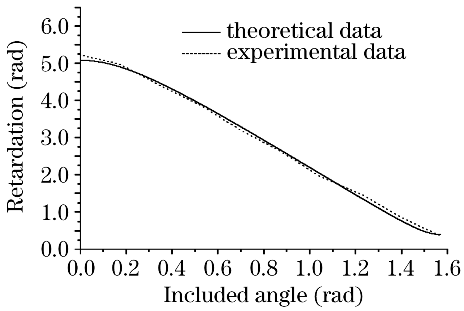

Fig. 4. Relation between δ and θ at 323 K (632.8 nm, 23 orders).

Table 1. Phase Retardations under Different Temperatures and Angles (rad)

Required	1.000	2.000	3.000	4.000
θ_1/δ_1	1.321/1.006	1.071/1.988	0.821/3.022	0.571/3.993
θ_2/δ_2	1.333/0.995	1.054/2.007	0.775/3.016	0.496/4.027

one are shown between δ and θ at 323 K of the 23-order 632.8-nm compensator. It can be seen that the theoretical data is well consistent with the experimental one.

The relation between θ_2 and δ_2 at 323 K is $\delta_2 = 5.7796 - 3.5851\theta_2$ and the range of θ_2 is $0.4 - 1.4$. Phase retardations at different temperatures are obtained by changing θ and the results are shown in Table 1. The first row of the data is the required phase retardation, θ_1 is the calculated value of the included angle from Eqs. (6) and (7) at 23 °C (296 K), and δ_1 is the measured data of phase retardation corresponding to θ_1, while θ_2 and δ_2 are at 50 °C (323 K).

From Table 1, we can see that the phase retardation can be modified by the proper adjustment of θ_2 which is determined by theoretical calculation when the temperature changes. The experimental data are well consistent with the theoretical one. So the phase retardation can be modified by this way under any temperature.

There are some factors causing the experimental error: 1) the angle between P_1 and P_2 is not 45°; 2) the extinction ratio of polarizer is not perfect; 3) the incident light is not perpendicular to the surface of wave-plate.

Though there exists error, the experimental curve tends to be consistent with the theoretical one, which indicates the value of our results.

In summary, the change of three-in-one composite compensator's phase retardation is larger with larger temperature change and larger order. The phase retardation and the angle that the second wave-plate rotates are in a linear relationship in certain ranges and we can change the angle to improve the precision of phase retardation by calculation. The adjustable range is different for the compensator of different wavelengths and different orders which should be confirmed by calculation. If the adjustable range is so large that the compensator can not satisfy it with the variation of temperature, repeated compensation can be adopted. For example, an electro-optic crystal, which can change the emergent light's phase by adjusting the voltage, can be set following a compensator and different phase retardations can be obtained by different voltages.

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