Absolute small-angle measurement based on optical feedback interferometry

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We present a simple but effective method for small-angle measurement based on optical feedback interferometry (or laser self-mixing interferometry). The absolute zero angle can be defined at the biggest fringe amplitude point, so this method can also achieve absolute angle measurement. In order to verify the method, we construct an angle measurement system. The Fourier-transform method is used to analysis the interference signal. Rotation angles are experimentally measured with a resolution of 10^{-6} rad and a measurement range of approximately from -0.0007 to $+0.0007$ rad.

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Small-angle measurement has important applications in many fields such as optical collimation, micro-electro-mechanical system (MEMS), atomic-force microscope imaging, and precision machining. Many methods of measuring angle have been presented[1−6]. High-precision measurement of angles is mainly performed with optical interferometers. However, traditional interference system is usually bulky instruments unsuitable for space-critical applications and is easily affected by external vibration.

The physical basis of optical feedback interferometry that is also known as laser self-mixing interferometry, is the interference of the back-reflected wave with the standing wave inside the laser resonant cavity. Optical feedback interferometer demonstrates unique features including a simple, single-axis optical arrangement that requires minimal optical components and high sensitivity at low light level, and results in it has the advantages of very small size, low cost, and easy construction. These characteristics are attractive for many applications[7−9]. A method for the angle measurement of a remote flat surface based on optical feedback in a laser diode (LD) has been proposed, for which the optical feedback interference was used as an amplitude modulation of LD output signal[10].

We present a simple but effective method for small-angle measurement based on optical feedback interferometry in this letter. This method can also achieve absolute angle measurement through defining the absolute zero of rotation angle, whereas traditional interferometry is difficult to perform absolute angle measurement because of the interference fringe ambiguity. To determine the absolute angle is clearly of a great practical benefit. Optical feedback interferometer is used to measure changes in the optical path length with an accuracy of $\lambda/2$ by counting interference signal peaks. In order to obtain the accuracy beyond $\lambda/2$, the phase analysis method of interference signal is used recently. The Fourier-transform phase analysis method[11] is used to analyze the feedback interference signal in this letter.

In a low-feedback regime, the effect of multiple reflections in the external cavity is very small. The optical feedback interference signal of a He-Ne laser can be expressed as[12]

$$I = I_0(1 + m\cos\phi), \tag{1}$$

where I_0 is the power of the laser output without optical feedback, m is the interference fringe visibility, $\phi = 4\pi L/\lambda$ is the phase of interference fringe, L is the length of the external cavity, and λ is the wavelength of the laser.

The principle of angle measurement is presented in Fig. 1. A reflecting mirror M is placed at the surface of measured object. The length of the external cavity is L_0. The rotation center of object is at the pint O, and the distance between M and O is d. The laser beam deviates from the rotation center O with the deviation angle α and deviation distance h. The reflected beam from the mirror is back into the laser resonant cavity, so that the laser operation is affected, causing a feedback interference signal.

When the measured object rotates an angle θ, the length of the external cavity changes ΔL is given by

$$\Delta L = d(\tan\alpha - \tan\frac{\theta}{2})\tan\theta. \tag{2}$$

If θ is a small angle (less than 1×10^{-3} rad), we have $\tan\theta \approx \theta$, $\tan\frac{\theta}{2}\tan\theta \approx \frac{\theta^2}{2} \approx 0$. Equation (2) is rewritten as

$$\Delta L \simeq h\theta. \tag{3}$$

Strictly speaking, if the reflecting mirror is placed in

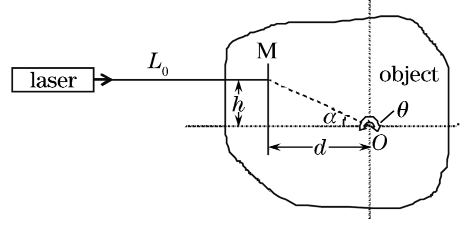

Fig. 1. Schematic of angle measurement principle.

the rotation center, i.e., $d = 0$ and $\alpha = \pi/2$, we have $\Delta L = h\theta$.

Clearly, small-angle measurement is transformed to the measurement of external cavity length. Equation (1) is thus rewritten as

$$I = I_0[1 + m\cos(4\pi L_0/\lambda + 4\pi h\theta/\lambda)]. \quad (4)$$

Setting $\phi_0 = 4\pi L_0/\lambda$, $\phi_\theta = 4\pi h\theta/\lambda$, we have

$$I = I_0[1 + m\cos(\phi_0 + \phi_\theta)]. \quad (5)$$

When reflection beam is fed back into the laser resonant cavity with a changing angle, the laser output power will be affected in two ways. Firstly, from Eq. (4), we know that the laser output power has a fringe periodic change along with the rotation angle θ. As long as we obtain the phase value ϕ_θ in Eq. (5), the rotation angle measurements could be achieved. Secondly, the feedback level changes with the rotation angle. When the light is reflected back to the laser cavity along original path, the feedback level and the interference fringe amplitude are the largest. The bigger the deviation of feedback light from original path is, the smaller the feedback level and the interference fringe amplitude are. Using this feature, we can define the absolute zero angle at the biggest fringe amplitude point, so that the absolute measurement of the rotation angle can be achieved.

In order to obtain the phase value of interference signal, a Fourier-transform method is used to analyze the phase of interference signal. The interference fringes can be expressed by a Fourier series expansion as

$$I(t) = \sum_{n=-\infty}^{\infty} A_n \exp\{j[n\phi(t)]\}. \quad (6)$$

By using a fast Fourier transform (FFT) algorithm, we compute the one-dimensional (1D) Fourier transform by

$$F(f) = \int_{-\infty}^{\infty} I(t)\exp(-j2\pi ft)\mathrm{d}t. \quad (7)$$

We select only one spectrum $F(f_0)$ by filtering operation and compute its inverse Fourier transform to obtain a complex signal:

$$\hat{I}_1(t) = A_1 \exp\{j[\phi(t)]\}. \quad (8)$$

Finally, we obtain the phase of fringe signal from the real and imaginary parts of the complex signal:

$$\phi(t) = \arctan\frac{\mathrm{Im}\hat{I}_1(t)}{\mathrm{Re}\hat{I}_1(t)}. \quad (9)$$

In order to verify our new method, we constructed a rotation angle measurement system. The experimental setup is shown in Fig. 2. A He-Ne laser (632.8 nm, TEM_{00}) with the cavity length of 250 mm and the output power of 3 mW was used. The external reflecting mirror was fixed on a rotation stage (Beijing Optical Instrument Factory, MRS102) driven by a step motor with an accuracy of 0.0001562°. The reflectivity of the external reflection mirror was 0.04. The interference signal was detected by a silicon photovoltaic photoelectric detector (PD). The voltage signal from the PD was amplified and then digitized with a 200-kHz, 12-bit analog-to-digital (A/D) board (National Instrument, PCI-6025E, USA) on a computer bus. This A/D board was also used to send voltage to the step motor driver. The rotation angle of each step was set to 0.0003125° or 5.454×10^{-6} rad.

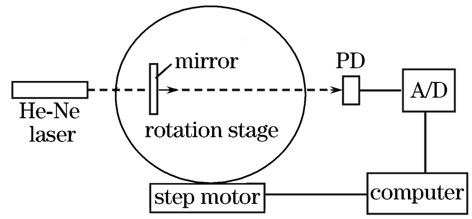

Fig. 2. Rotation angle measurement system.

Figure 3 shows the laser output signal obtained by the PD when the rotation stage rotates with step motor driving and the interference signal occurs in the step number range from 220 to 480. In order to obtain the biggest fringe amplitude point, curve fitting is carry out by the least square method for the interference signal. The dashed line in Fig. 3 is the fitting curve of interference signal, the lowest point of which is the biggest fringe amplitude point. At this point, the feedback level is the biggest and the feedback light is reflected back to the laser cavity along original path, so that the lowest point of curve can be confirmed as absolute zero angle for absolute angle measurements.

Figure 4 shows the phase distribution of interference signal obtained by Fourier-transform method and the

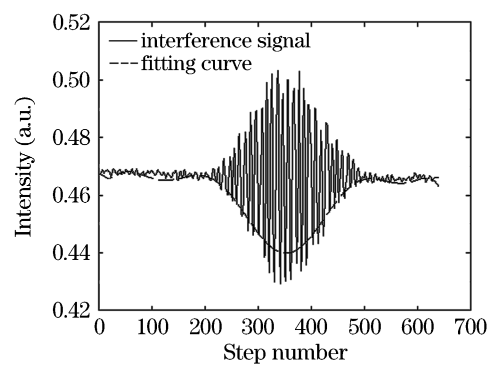

Fig. 3. Laser output signal obtained by PD.

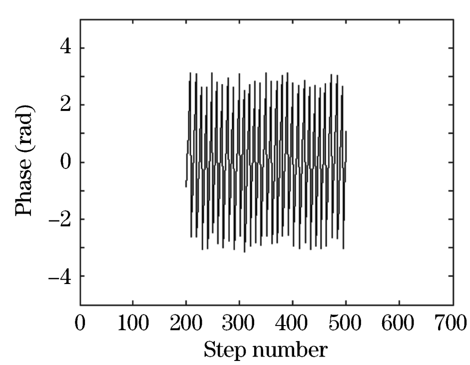

Fig. 4. Phase distribution of interference signal obtained by Fourier-transform method.

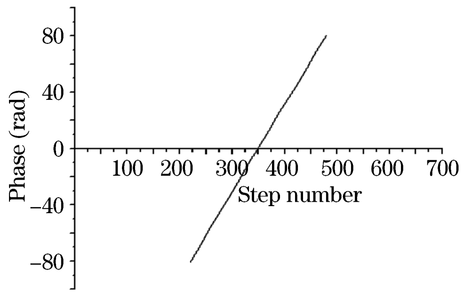

Fig. 5. Continuous phase distribution obtained by an unwrapping process.

phase value is wrapped within the region of $-\pi$ and $+\pi$. A phase unwrapping process must be carried out to obtain true phase with physical meaning. Unwrapping is processed according to the phase difference between two neighboring points[11] and the phase value at the point of absolute zero angle is set to zero. After an unwrapping process, the continuous phase distribution is obtained, as shown in Fig. 5.

At the point of absolute zero angle, we have $\theta = 0$ and $\phi_0 = 4\pi h\theta/\lambda = 0$, so that the absolute phase corresponding to absolute rotation angle is

$$\phi_{\mathrm{a}\theta} = \phi - \phi_0 = 4\pi h\theta/\lambda. \quad (10)$$

Through pre-calibration, the phase value corresponding to the rotation angle of each step is 0.6246 rad. According to this relationship between the angle and phase, we achieve the absolute rotation angle measurement, as shown in Fig. 6. The angle measurement range is approximately from -0.0007 to $+0.0007$ rad. Figure 7 shows the error distribution between experimental value and calculated value according to step number. Experimentally, rotation angles were measured with a resolution of 10^{-6} rad.

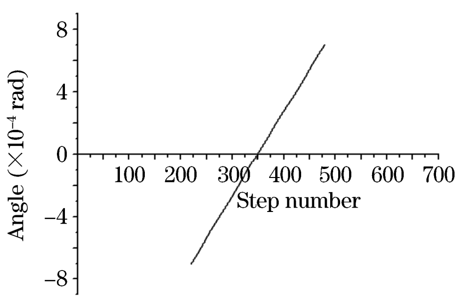

Fig. 6. Absolute rotation angles obtained from interference signal.

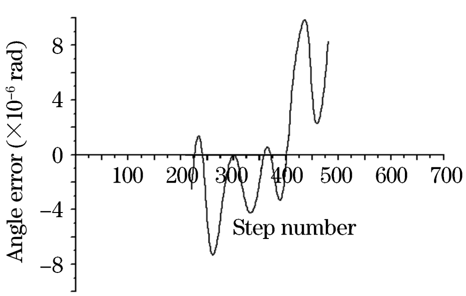

Fig. 7. Error distribution between experimental value and calculated value.

According to Eq. (2), we used a 1st order approximation of θ for the optical path ΔL if θ is a small angle (less than 1×10^{-3} rad).

The absolute error $\delta(\Delta L)$ produced by this approximation can be expressed as

$$\delta(\Delta L) = \left| d(\tan\alpha - \tan\frac{\theta}{2})\tan\theta - h\theta \right|. \quad (11)$$

And according to Eq. (2), the angle error $\delta\theta$ is given as

$$\delta\theta = \frac{\delta(\Delta L)}{d(\tan\alpha\sec^2\theta - \tan\frac{\theta}{2}\sec^2\theta - \frac{1}{2}\tan\theta\sec^2\frac{\theta}{2})}. \quad (12)$$

If $h = 5.5$ mm, $d = 7.5$ mm, and $\theta = 1 \times 10^{-3}$ rad, the angle error $\delta\theta$ is estimated to be 1.36×10^{-6} rad.

In conclusion, we demonstrated a new method for the measurement of the rotation angle based on optical feedback interferometry in the low-feedback regime. We constructed a rotation angle measurement system for the precision rotation stage. Experimental results show that our method is very effective. Compared with traditional interferometry, this angle measurement method is simple and absolute angle measurement can be achieved. This method can only be used in small-angle measurements by the restriction of feedback light. Experimentally, rotation angles were measured with a resolution of 10^{-6} rad and an angle measurement range of approximately from -0.0007 to $+0.0007$ rad.

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