Translation, rotation, and scale invariant image registration technique using angular and radial difference functions

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Received November 21, 2007

An algorithm is proposed for registering images related by translation, rotation, and scale based on angular and radial difference functions. In frequency domain, the spatial translation parameters are computed via phase correlation method. The magnitudes of images are represented in log-polar grid, and the angular and radial difference functions are given and applied to measure shifts in both angular and radial dimensions for rotation and scale estimation. Experimental results show that this method achieves the same accurate level as classic fast Fourier transform (FFT) based method with invariance to illumination change and lower computation costs.

OCIS codes: 100.0100, 100.2000.

doi: 10.3788/COL20080611.0827.

Image registration is a fundamental image processing task referring to matching images taken at different times or different viewpoints[1−13]. The frequency domain approaches have been much exploited in recent years[10−13]. Among them, most Fourier-based schemes use the shift property of Fourier transform to estimate translation, scale, and rotation[9]. As the representative one of these methods, the algorithm in Ref. [9] enables robust estimation with insensitivity to illumination change, however, its computation burden is heavy with three fast Fourier transforms (FFTs) plus three inverse fast Fourier transforms (IFFTs). Another translation estimation tool, difference function (DF), was introduced for roto-translation estimation in frequency domain[10,12,13]. Although DF performs excellent in translation and rotation estimation, it does not account for scaling due to its original definition[10].

To resolve this problem, we propose an algorithm for registration of images related by affine transform, where the linear components are restricted to scaling and rotation. The scheme consists of four steps. Firstly, phase correlation is used to compute spatial translation value. Secondly, the spectra of images are transformed to log-polar coordinate to decouple rotation and scaling into translation form. Thirdly, project each spectral image along every single dimension, the angular one, and the radial one. Finally, a new DF of two shifted signal is proposed with the property that the shift is only determined by one zero point of the DF. The angular difference function (ADF) and radial difference function (RDF) are defined and used for rotation and scaling estimation. Experimental results show its applicability with low computation complexity and feasibility to illumination change.

Considering that the most common relative motion between objects and camera can be taken as the camera position movements, rotation around its optical axis, and focus changing, the affine transformation model with translation parameter, rotation angle, and scale parameter is adopted in the following discussion.

Assume two images $f_1(x)$ and $f_2(x)$ related as

$$f_2(\mathbf{x}) = f_1(\mathbf{A}\mathbf{x} + \mathbf{T}), \tag{1}$$

where $\mathbf{A}$ is the linear component matrix of affine motion model, $\mathbf{T}$ is the two-dimensional (2D) translation vector.

Hereby we focus on the most commonly used affine motion model with $\mathbf{A}$ specified in the form

$$\mathbf{A} = \mathbf{s}\begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix}, \tag{2}$$

where $\mathbf{s}$ is the scale factor, $\Delta\theta$ is the rotation angle of images. By applying the Fourier transform on both sides of Eq. (1), we have

$$F_1(\boldsymbol{\omega}) = k\exp(-j(\boldsymbol{\omega}\cdot\mathbf{T}))F_2(\mathbf{A}'\boldsymbol{\omega}), \tag{3}$$

where $\boldsymbol{\omega}$ is the frequency, $k = \det(\mathbf{A}) = s^2$, and $\mathbf{A}'$ is the transpose of $\mathbf{A}$. Let M_1 and M_2 be the magnitudes of $F_1(\boldsymbol{\omega})$ and $F_2(\boldsymbol{\omega})$, from Eq. (3) we get

$$M_1(\boldsymbol{\omega}) = kM_2(\mathbf{A}'\boldsymbol{\omega}). \tag{4}$$

Equations (3) and (4) imply that the translation vector $\mathbf{T}$ affects only the phases of Fourier transforms in Eq. (3), while the linear component matrix $\mathbf{A}$ indicates the relation between the magnitudes of images.

Accordingly, the registration algorithm is composed of two stages. Firstly, use phase correlation technique to evaluate $\mathbf{T}$, which is a scheme proved to robustly estimate large spatial translations[9]. Secondly, recover $\mathbf{A}$ from the magnitudes of images, which is the main contribution of this letter.

It is worth noting that in Eq. (4) the magnitudes not only depend on the linear component matrix $\mathbf{A}$ but on the coefficient k resulting from s. Namely, the task in this paper is to evaluate the parameters $(s, \Delta\theta)$ in $\mathbf{A}$ with the presence of k. To resolve the problem, there are three steps involved. Firstly, decouple the rotation and scaling into translation form in which we represent both the spectra in log-polar coordinate as

$$M_1(r,\theta) = kM_2(r-\Delta r, \theta-\Delta\theta), \tag{5}$$

where $r = \log\rho$, $\Delta r = \log s^{-1}$ for ease of notation.

Secondly, take Radon (projection) transform of Eq. (5) on two sides, denoted formally by $R(\cdot)$. According to the shifty property, we have

$$R_1(r) = kR_2(r-\Delta r), \quad R_1(\theta) = kR_2(\theta-\Delta\theta), \tag{6}$$

which implies that the 2D translation estimation can be reduced to evaluations of two one-dimensional (1D) translation problems, only by directly projecting the spectral image onto r and θ dimensions. This property renders respective computation of rotation and scaling possible with the presence of the magnitude change information k. Unlike phase correlation method, which uses not magnitude information but phase information aiming at eliminating the influence of k, the algorithm in this paper presents a different technique for translation, rotation, and scale-invariant image registration at low computation cost even under varying illumination condition.

Assume any non-negative 1D function $f_1(x)$, $f_2(x)$, $x \in [0, N]$ related by

$$f_2(x) = kf_1(x - \Delta x). \tag{7}$$

Define the extended DF as

$$\Delta f = f_1^{\mathrm{G}}(x) - f_2^{\mathrm{G}}(-x), \tag{8}$$

where

$$f_i^{\mathrm{G}} = \frac{f_i - \bar{f}_i}{\sigma(f_i)}, \quad i = 1, 2, \tag{9}$$

$\sigma(f_i)$ is the standard deviation (STD) of f_i, and $\bar{f}_i$ is the mean value of f_i. Replacing $f_2(x)$ in Eq. (9) with Eq. (7), then

$$f_2^{\mathrm{G}}(-x) = \frac{f_1(-x - \Delta x) - \overline{f_1(-x - \Delta x)}}{\sigma(f_1(-x - \Delta x))}$$
$$= f_1^{\mathrm{G}}(-x - \Delta x), \tag{10}$$

and then

$$\Delta f = f_1^{\mathrm{G}}(x) - f_1^{\mathrm{G}}(-x - \Delta x). \tag{11}$$

Let $\Delta f = 0$, necessarily $x_0 = -\Delta x/2$, which straightforwardly shows that Δx is determined only by the zero x_0, with nothing dealing with the difference between the magnitudes of $f_1(x)$ and $f_2(x)$, denoted by k in Eq. (7).

This result can be easily extended to our purpose of estimating $(s, \Delta\theta)$ by computing the DFs along the angular and radial dimensions respectively with the outcome of Eq. (6). Define the ADF as

$$\Delta \mathrm{Ang}(\theta) = R_1(\theta) - R_2(-\theta), \quad \theta \in [0, \pi], \tag{12}$$

and $\Delta\theta = -2\theta_0$; define the RDF as

$$\Delta \mathrm{Rad}(r) = R_1(r) - R_2(-r + N), \tag{13}$$

and $\Delta r = N - 2r_0$, N is the length of r. Once the translation Δr is evaluated, the scaling s can be computed as $s = \mathrm{e}^{-\Delta r}$.

The performance of the developed algorithm is experimentally tested on different sets of images. It should necessarily be noticed that the translation estimation part using phase correlation method has been proved most effective and robust in performance[9]. Moreover, Fourier transform shift theorem supports that, for affine model, translation parameter can be separately evaluated from the rotation and scaling parameters. More detailed demonstration of translation problem refers to Refs. [9,12,13]. Consequently, we lay the stress on the rotation and scaling estimation performance in the following experiments.

Figure 1 is an example of rotation estimation without illumination variations and scaling. Figure 1(a) is rotated randomly in the range of $[0, 2\pi]$. The estimation result is given in Fig. 1(b), where we present the angles that the image was originally rotated and the corresponding estimated angle. Clearly, the algorithm achieves effective tracking of the rotation angles.

The estimation of both scaling and rotation was tested on 'plane' images in Fig. 2. Figure 2(b) is a rotated and scaled version of Fig. 2(a), with 45° rotation angle and 1.5 times dilation. Figures 2(c) and (d) are $M_1(r, \theta)$ and $M_2(r, \theta)$, the magnitudes of Figs. 2(a) and (b). It is easy to see the translations in horizontal dimension resulting from rotation and the shifts in vertical dimension resulting from scaling. Notably, the magnitude change generated by scaling is also visible. Figure 2(e) corresponds to the ADF of Figs. 2(c) and (d) in the rotation angle dimension, and Fig. 2(f) corresponds to the RDF of Figs. 2(c) and (d) in the $\log\rho$ dimension. In Fig. 2(e), where the zero θ_0 is clearly visible, there are also other zeros introduced by either the conjugate symmetry of

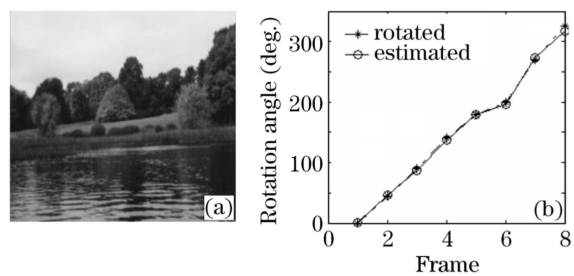

Fig. 1. Rotation estimation without illumination variation and scaling.

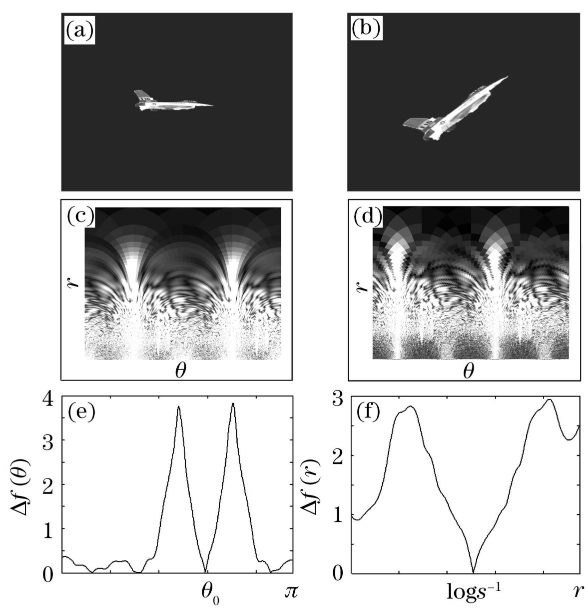

Fig. 2. (a) Original image; (b) the 45° rotated and 1.5 times scaled version of (a); (c),(d) magnitudes of (a) and (b), respectively; (e) ADF of (c) and (d); (f) RDF of (c) and (d).

Fig. 3. Two Lena images with artificially introduced luminance variations.

Table 1. Rotation and Scale Estimation with Different Illumination Variation and Scaling

Original Parameters				ADF and RDF		Phase Correlation	
α	ρ	k	$\Delta\theta$	ρ'	$\Delta\theta'$	ρ'	$\Delta\theta'$
1	1	1	5°	1.01	5.18°	1.00	5.18°
1	1.5	1.5	30°	1.52	28.77°	1.46	30.18°
1.5	1.5	2.25	135°	1.47	136.79°	1.40	135.09°
1.5	0.8	1.2	313°	0.87	312.83°	0.81	312.83°

spectra magnitudes or the discretization of the function, even by the noise disturbance. However, this can be resolved in the same manner described in Ref. [9]. The estimated result is $\theta = 44.15°$ and $s = 1.57$.

Equation (11) also guarantees the algorithm's invariance to intensity change and it is tested on Lena images in Fig. 3. The images are rotated and scaled with different luminance variations. Table 1 reports the estimated results, with the presentation of estimation using phase correlation method[9] for comparison. Notice that the accuracy of the proposed algorithm is comparable to that of the phase correlation method. This result is quite encouraging concerning with comparison of complexity of the two methods. To estimate 2D translation, the proposed algorithm requires computation of one FFT and one IFFT. Furthermore, the scheme involves two times of computation of ADF and RDF for scale and rotation using the spectra magnitudes, which are the products of FFT for no extra computation. Considering an image with the site of $N \times N$, when the number of zeros of ADF or RDF is m, the complexity of ADF and RDF scheme is

$$\begin{aligned} &1 \times (N(\lg(N) - 3) + 2N^2 + 4) \times M_c \\ &+1 \times (3N(\lg(N) - 1) + 4) \times A_c \\ &+2 \times (4N^2 + 14N + 4mN) \times A \\ &+2 \times (12 + 4N) \times M. \end{aligned} \quad (14)$$

In comparison, the complexity of phase correlation method is

$$\begin{aligned} &6 \times (N(\lg(N) - 3) + 2N^2 + 4) \times M_c \\ &+6 \times (3N(\lg(N) - 1) + 4) \times A_c. \end{aligned} \quad (15)$$

In Eqs. (14) and (15), M_c is complex multiplication, A_c is complex addition, M is real multiplication, A is real addition. The computation of flip, absolute value etc. is ignored. Subtraction is taken as addition and division is taken as multiplication. The computation of log-polar transform is also ignored. Evidently, compared with the complexity of the phase correlation method which is up to three FFTs and three IFFTs, ADF and RDF is far less than two FFTs plus two IFFTs, even considering the search procedure for m zeros. The reason is that FFT or IFFT consumes $O(N^2)$ complex multiplications plus $O(N \lg N)$ complex additions, which are enormous in the order compared with $O(N)$ real multiplications plus $O(N^2)$ real additions.

In conclusion, an algorithm for translation, rotation, and scale invariant image registration is investigated. The algorithm is operated in the frequency domain, where phase correlation is used for translation estimation and magnitude information is employed for rotation and scale estimation. By transforming the spectral images into log-polar form, rotation, and scale are reduced to translations in both angular and radial dimensions. Then, a new DF is proposed, which determines the translation value of two shifted functions only by its zero without concerning of any magnitude change. Consequently, ADF and RDF are defined and used for rotation and scale estimation. Experimental results demonstrate its applicability, which is characterized by the same accuracy level with classic correlation estimation, the lower computation complexity, and the invariance to illumination change.

This work was supported by the Astronautics Technique Creation Project "vision-based spacecraft high accuracy realtime position and attitude measurement method research". The authors' e-mail addresses are lilihit@163.com (L. Li), zqs2000@yahoo.com.cn (Q. Zeng), and mengfanfeng@sohu.com (F. Meng).

References

1. L. G. Brown, ACM Computing Surveys **24**, 325 (1992).
2. B. Zitová and J. Flusser, Image and Vision Computing **21,** 977 (2003).
3. J. B. A. Maintz and M. A. Viergever, Medical Image Analysis **2,** 1 (1998).
4. G. Liu, D. Liu, F. Liu, and Y. Zhou, Acta Opt. Sin. (in Chinese) **28,** 454 (2008).
5. J. P. W. Pluim, J. B. A. Maintz, and M. A. Viergever. IEEE Trans. Medical Imaging **22,** 986 (2003).
6. F. Ye, L. Su, and S. Li, Chin. Opt. Lett. **4,** 386 (2006).
7. X. Yang and J. Pei, Chin. Opt. Lett. **3,** 510 (2005).
8. L. Song, Y. Zhou, and J. Zhou, Chin. Opt. Lett. **3,** 392 (2005).
9. B. S. Reddy and B. N. Chatterji, IEEE Trans. Image Processing **5,** 1266 (1996).
10. L. Lucchese and G. M. Cortelazzo, IEEE Trans. Signal Processing **48,** 1769 (2000).
11. J. Jin, Q. Wang, and Y. Shen, Chin. Opt. Lett. **5,** 389 (2007).
12. Y. Keller, Y. Shkolnisky, and A. Averbuch, IEEE Trans. Pattern Anal. Machine Intell. **27,** 969 (2005).
13. Y. Keller, A. Averbuch, and M. Israeli, IEEE Trans. Image Processing **14,** 12 (2005).