# Relationship between normalized light intensity and attenuated total reflection ratio

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Attenuated total reflection (ATR) ratio is usually utilized to study the properties of surface plasmon resonance (SPR) sensors. The relationship between normalized light intensity and ATR ratio is investigated, and a modification coefficient is put forward to describe the relationship. A mathematical expression is built up for the coefficient based on Fresnel principle. The result shows that the ATR ratio, which cannot be measured directly in experiments, can be determined with the coefficient and the normalized intensity of light. The characteristic of the coefficient is also discussed.

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Surface plasmon resonance (SPR) is one of the promising optical techniques with potential applications in different fields[1−4]. SPR sensors have the advantages such as flexibility, low cost, and small size[5,6]. The principle of SPR sensors is that surface plasmon wave (SPW), which can be excited at the interface between a metal film and an absorbing medium, is extremely sensitive to tiny change in the refractive index (RI) of absorbing medium. As the phenomenon of SPR is excited by a p-polarized light, the attenuated total reflection (ATR) occurs in sensor. However, the ratio of ATR could not be measured directly in experiments, because the incident and reflected light beams are all inside the sensor so that their intensities cannot be determined. The normalized intensity (the ratio of light intensity out of sensor to that launched into the sensor) is usually considered as the response of sensor for studying its characteristics[7−9]. But in fact, it is the ratio of ATR that is related to RI of absorbing medium. In this work, the relationship between normalized light intensity and ATR ratio is investigated.

SPR sensors have been developed into three types, prism-coupled, integrated optical waveguide-coupled, and optical fiber-coupled sensors[10−12]. In this research, a prism-coupled Krestschmann configuration is employed. The 50-nm-thick Ag film was deposited at the bottom of an equilateral triangle prism by radio frequency (RF) magnetron sputtering. The prism was made of K9 glass. A film thickness monitor (FTM) was used to control the thickness of the film during deposition.

The experimental scheme built for measuring normalized intensity of sensor is shown in Fig. 1. A 5-mW, 632.8-nm He-Ne laser was used as the light source. A polarizer with a high extinction ratio ($\geq 5 \times 10^4$) was installed to choose polarization of the incident light. The light beam was adjusted with an iris in front of the prism. The prism-coupled SPR sensor was positioned on a goniometer to change the angle of incidence in the ATR case. The light transmitting out of the sensor was detected with a digitized power meter.

Figure 2 illustrates the refraction and total internal reflection of light inside the prism. In order to excite SPW effectively, a p-polarized optical beam was launched into the prism. In the ATR case, the energy of the incident light is transferred to charge-density wave (surface plasmon) and the intensity of the light reflected from the bottom of the prism is reduced.

The ratio of ATR is given by

$$R_{\mathrm{SP}} = \left| \frac{\gamma_{12} + \gamma_{01} \mathrm{e}^{-2\alpha_1 d}}{1 + \gamma_{12}\gamma_{01}\mathrm{e}^{-2\alpha_1 d}} \right|^2 \tag{1}$$

with

$$\alpha_0 = \left(\beta^2 - \mathbf{k}_0^2 \varepsilon_{\mathrm{s}}\right)^{1/2}, \quad \alpha_1 = \left(\beta^2 - \mathbf{k}_0^2 \varepsilon_{\mathrm{m}}\right)^{1/2},$$

$$\alpha_2 = \left(\beta^2 - \mathbf{k}_0^2 \varepsilon_{\mathrm{g}}\right)^{1/2}, \quad \beta = \mathbf{k}_0 \sqrt{\varepsilon_{\mathrm{g}}} \sin\theta_0,$$

$$\gamma_{01} = \frac{\varepsilon_{\mathrm{s}}\alpha_1 - \varepsilon_{\mathrm{m}}\alpha_0}{\varepsilon_{\mathrm{s}}\alpha_1 + \varepsilon_{\mathrm{m}}\varepsilon_0}, \quad \gamma_{12} = \frac{\varepsilon_{\mathrm{m}}\alpha_2 - \varepsilon_{\mathrm{g}}\alpha_1}{\varepsilon_{\mathrm{m}}\alpha_2 + \varepsilon_{\mathrm{g}}\alpha_1},$$

where $\varepsilon_{\mathrm{g}}$, $\varepsilon_{\mathrm{m}}$ and $\varepsilon_{\mathrm{s}}$ are the dielectric constants of prism,

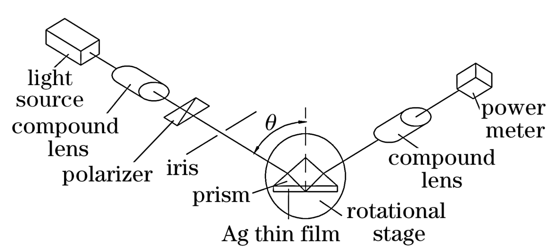


Fig. 1. Experimental scheme.

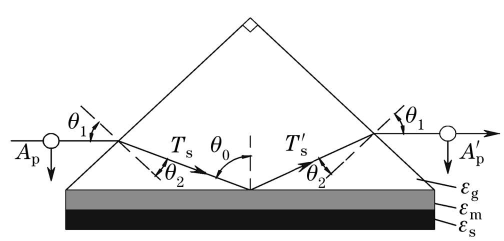


Fig. 2. Refraction and total internal reflection of light inside the prism.

 

metal film and medium, respectively, $\mathbf{k}_0$ is the wave vector of incident light in vacuum. Equation (1) shows that the ratio of ATR is a direct response to the permittivity of ambient medium, $\varepsilon_s$. The incident light and reflected light of ATR are all inside the sensor, so it is impossible to detect their intensity. The normalized intensity, i.e., the relative transmitted power of light, has been used to study the characteristics of sensors[13−15]. In the following, we will study relationship between normalized intensity and ATR ratio.

Before ATR occurs, refraction of the incident light has existed. Based on Fresnel formula, the amplitude of refraction light is given by

$$T_S = \frac{2\sin\theta_2 \sin\theta_1}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)} A_P, \quad (2)$$

where $A_P$ denotes the amplitude of light launched into sensor. The ratio of ATR can be rewritten as

$$R_{SP} = \left(\frac{T'_S}{T_S}\right)^2, \quad (3)$$

where $T'_S$ is the amplitude of reflected light in ATR. On the other side of prism, the amplitude of the second refraction light is expressed as

$$A'_P = \frac{2\sin\theta_1 \cos\theta_2}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)} T'_S. \quad (4)$$

The normalized intensity is

$$R_0 = \left(\frac{A'_P}{A_p}\right)^2 = \left(\frac{A'_P}{T'_S}\right)^2 \times \left(\frac{T'_S}{T_S}\right)^2 \times \left(\frac{T_S}{A_P}\right)^2. \quad (5)$$

Defining a parameter $K$ with $R_{SP} = R_0 \times K$, we can get

$$K = \left(\frac{T'_S}{A'_P}\right)^2 \times \left(\frac{A_P}{T_S}\right)^2 = \frac{\sin^4(\theta_1+\theta_2)\cos^4(\theta_1-\theta_2)}{16\sin^2\theta_1\cos^2\theta_1\sin^2\theta_2\cos^2\theta_2}. \quad (6)$$

Apparently, it is not accurate to replace $R_{SP}$ with $R_0$. The parameter $K$ discussed above could be called a modification coefficient of ATR. Several values of $K$ are given in Fig. 3. It decreases slightly with the increase of resonant angle $\theta_0$ ($\theta_0 = 45° + \theta_2$).

The analysis above is in such condition that the SPR system is based on angle interrogation technique. If the SPR system is based on wavelength interrogation method, the incident angle $\theta_1$ is fixed, the coefficient $K$ will also change due to the existence of dispersion in prism. The relationship between the coefficient $K$ and wavelength $\lambda$ is discussed in the following.

According to the experiential formula, the refractive index of prism is given by

$$n = n_0 + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots, \quad (7)$$

where $n_0$ is a constant, the coefficients $B$ and $C$ have certain values, and $\lambda$ denotes the wavelength (in nanometers). If the high order items are neglected, Eq. (7) can be rewritten as

$$n = n_0 + \frac{B}{\lambda^2}. \quad (8)$$

The reflection angle $\theta_2$ is wavelength-dependent. Because $\sin\theta_2 = \frac{1}{n}\sin\theta_1$, by taking differentiation of $K$ with respect to wavelength $\lambda$, we can get

$$\frac{dK}{d\lambda} = \frac{dK}{d\theta_2} \cdot \frac{d\theta_2}{d\lambda}. \quad (9)$$

From Eq. (6), the differentiation of $K$ by $\theta_2$ is

$$\frac{dK}{d\theta_2} = -\frac{4\sin^3(\theta_1+\theta_2)\sin(\theta_1-\theta_2)}{\sin^2 2\theta_1 \sin^2 2\theta_2}.$$

Based on Eq. (8) and $\sin\theta_2 = \frac{1}{n}\sin\theta_1$, the differentiation of $\theta_2$ by $\lambda$ is

$$\frac{d\theta_2}{d\lambda} = \frac{2\sin^2\theta_2}{\cos\theta_2} \cdot \frac{B}{\lambda^3 \sin\theta_1}.$$

So,

$$\frac{dK}{d\lambda} = -\frac{8B\sin^2(\theta_1+\theta_2)\sin(\theta_1-\theta_2)\sin^2\theta_2}{\sin^2 2\theta_1 \sin^3 2\theta_2 \cos\theta_2 \sin\theta_1} < 0. \quad (10)$$

Equation (10) demonstrates that the coefficient $K$ decreases with the increase of resonant wavelength $\lambda$.

If the refractive index of absorbing medium is 1.3330, and the sensor is considered with angle interrogation technique, the relationship between normalized intensity and ATR ratio is shown in Fig. 4. The curve of normalized intensity is very similar to that of ATR ratio, so it is reasonable to use the normalized intensity and the modification coefficient to study properties of SPR sensors.

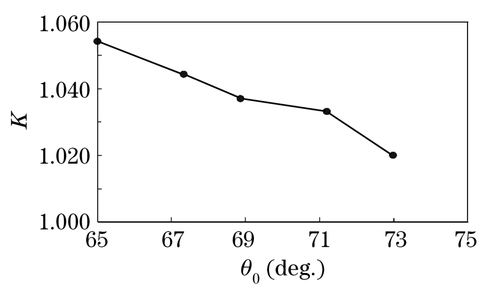


Fig. 3. Modification coefficient $K$ varying with resonant angle $\theta_0$.

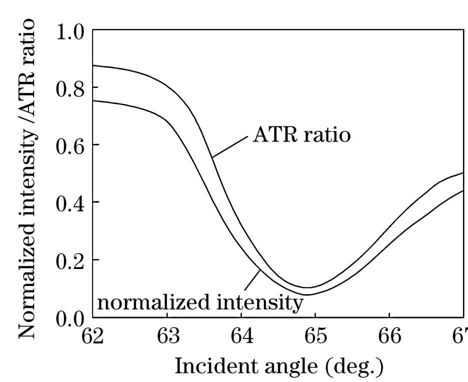


Fig. 4. Normalized intensity and ATR ratio show similar dependence on incident angle.

In summary, the relationship between normalized intensity and ratio of ATR has been investigated. The ATR ratio can be determined based on the normalized intensity and a modification coefficient. With the ATR ratio, the resonant angle or wavelength can be detected precisely. This ratio also illustrates that the energy of light is transferred to charge-density wave (surface plasmon).

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## References

1. F. S. Damos, R. C. S. Luz, and L. T. Kubota, J. Electroanal. Chem. **581,** 231 (2005).
2. Y. Chen, H. Huang, X. Yu, and L. Qi, Carbohydr. Res. **340,** 2024 (2005).
3. Y. Wang and W. Knoll, Anal. Chim. Acta **558,** 150 (2006).
4. Y.-B. Shin, J.-M. Lee, M.-R. Park, M.-G. Kim, B. H. Chung, H.-B. Pyo, and S. Maeng, Biosens. Bioelectron. **22,** 2301 (2007).
5. M. Lotierzo, O. Y. F. Henry, S. Piletsky, I. Tothill, D. Cullen, M. Kania, B. Hock, and A. P. F. Turner, Biosens. Bioelectron. **20,** 145 (2004).
6. B. D. Gupta and A. K. Sharma, Sensor. Actuator. B **107,** 40 (2005).
7. R. Naraoka and K. Kajikawa, Sensor. Actuator. B **107,** 952 (2005).
8. W. W. Lam, L. H. Chu, C. L. Wong, and Y. T. Zhang, Sensor. Actuator B **105,** 138 (2005).
9. B. A. Morris and A. Sadana, Sensor. Actuator. B **106,** 498 (2005).
10. J. Dostálek, J. Čtyroký, J. Homola, E. Brynda, M. Skalsky, P. Nekvindová, J. Špirková, J. Škvor, and J. Schröfel, Sensor. and Actuator. B **76,** 8 (2001).
11. H. P. Ho, S. Y. Wu, M. Yang, and A. C. Cheung, Sensor. Actuator. B **80,** 89 (2001).
12. Z.Cao, L. Wu, and D. Li, Chin. Opt. Lett. **4,** 160 (2006).
13. Y. Wu and Y. Yuan, Acta Opt. Sin. (in Chinese) **25,** 199 (2005).
14. Y. Wu, Z. Gu, and Y. Yuan, Chin. Opt. Lett. **4,** 91 (2006).
15. D. J. Gentleman, L. A. Obando, J.-F. Masson, J. R. Holloway, and K. S. BooKsh, Anal. Chim. Acta **515,** 291 (2004).