Sub-wavelength atom localization in double-dark resonant systems

Dongchao Cheng (程东超), Chengpu Liu (刘呈普), Yueping Niu (钮月萍), Shiqi Jin (金石琦), Ruxin Li (李儒新), and Shangqing Gong (龚尚庆)

State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800

We propose two schemes of atom localization based on the interference of double-dark resonances in a tripod and a Λ-type four-level system. It is demonstrated that the localization is significantly improved owing to the interference of double-dark resonances. In the tripod scheme, the localization can be manipulated by the parameters of an additional control field. Via adjusting the Rabi frequency of the field, one can double the probability of detecting the atom within subwavelength domain. By decreasing the detuning of the field, higher spatial resolution can be achieved. In the Λ-type four-level system, via adjusting the probe field detuning, we can not only make the atom localized at the nodes of the standing-wave field with high precision, but also increase the detecting probability of the atom at a particular position by a factor of 2.

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In recent years, precision position measurement of an atom passing through a standing-wave field has attracted considerable attention. Many atom localization schemes based on atomic coherence and quantum interference effects have been proposed. Zubairy and co-workers proposed several schemes using the resonance fluorescence from a two-level system[1], the measurement of spontaneous emission in a multi-level system[2−6] or the measurement of the probe absorption[7,8]. Paspalakis *et al.*[9,10] put forward a scheme based on the formation of the dark state in a three-level Λ-type atom. Agarwal *et al.*[11] proposed a scheme based on the phenomena of coherent population trapping in a Λ-type atom.

On the other hand, double-dark resonances have been demonstrated in a variety of four-level systems[12−17], where the probe absorption spectrum is characterized by two electromagnetically induced transparency (EIT) windows, separated by a sharp absorption peak[18] resulting from interaction of double-dark resonances. In this paper, we present two localization schemes utilizing the interacting double-dark resonances in a tripod and a Λ-type four-level atomic system. It is shown that the property of atom localization can be significantly improved owing to the interference of double-dark resonances.

In the first place, we consider atom localization in a tripod system. The atomic system under consideration is shown in. Fig. 1. The atom is in a tripod configuration, with a lower level $|0\rangle$, two metastable levels $|1\rangle$ and $|2\rangle$, and a single upper level $|3\rangle$, which decays out of the system. The atom, moving in the z direction, passes through the classical standing-wave field aligned the x axis. The transition $|3\rangle \leftrightarrow |0\rangle$ is coupled by a weak probe field with Rabi frequency Ω_0. The transition $|3\rangle \leftrightarrow |1\rangle$ is coupled by a classical standing-wave field with position dependent Rabi frequency $\Omega_1(x) = \Omega_1 \sin(kx)$. Here Ω_1 is the constant part of $\Omega_1(x)$ and $k = 2\pi/\lambda$ is the wave number of the classical standing-wave field. The transition $|3\rangle \leftrightarrow |2\rangle$ is coupled by an additional control field with Rabi frequency Ω_2.

The interaction Hamiltonian of this system, in the dipole and rotating-wave approximations, is given by

$$H = \Omega_0 \exp(-i\delta_0 t)|3\rangle\langle 0| + \Omega_1(x)\exp(-i\delta_1 t)|3\rangle\langle 1| + \Omega_2 \exp(-i\delta_2 t)|3\rangle\langle 2|, \quad (1)$$

where δ_n represents the detuning of the laser field with the $|n\rangle \leftrightarrow |3\rangle$ transition.

The atom-field state-vector $|\Psi(t)\rangle$ can be written as

$$|\Psi(t)\rangle = \int \mathrm{d}x f(x)|x\rangle[a_0(x,t)|0\rangle + a_1(x,t)|1\rangle a_2(x,t)|2\rangle a_3(x,t)|3\rangle], \quad (2)$$

where $a_n(x,t)$ $(n = 1,2,3)$ represents the time and position dependent probability amplitude for the atom to be in level $|n\rangle$, $f(x)$ and is the center-of-mass wave function of the atom.

The conditional position probability distribution, i.e., the probability of finding the atom at position x in the standing-wave field provided that the atom is found in its internal state $|3\rangle$, is

$$P(x,t|3\rangle) = |N|^2|f(x)|^2|a_3(x,t)|^2, \quad (3)$$

here N is a normalization factor. The dependence of a_3 on x makes it possible to obtain information about the x

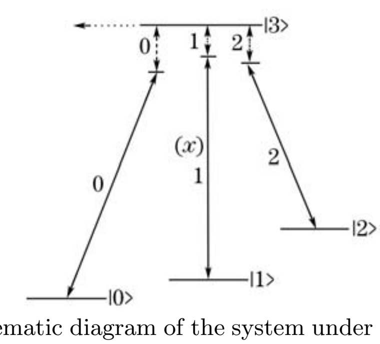

Fig. 1. Schematic diagram of the system under consideration. The atom interacts with a standing-wave field that couples the $|3\rangle \leftrightarrow |1\rangle$ transition, a probe laser field that couples the $|3\rangle \leftrightarrow |0\rangle$ transition, and a control laser field that couples the $|3\rangle \leftrightarrow |2\rangle$ transition.

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position of the atom as it passes through the standing-wave field via measuring the population in upper state.

Making use of the assumption that the atom is initially in its ground state $|0\rangle$, and that the probe field is very weak, we can determine $a_3(x,t)$ analytically by means of time-dependent perturbation theory. Then we obtain the conditional position probability distribution

$$\begin{aligned} &P(x,t\to\infty|3\rangle) = \\ &\quad |N|^2|f(x)|^2\frac{\Omega_0^2}{\left(\delta_0+\frac{\Omega_2^2}{\delta_2-\delta_0}++\frac{\Omega_1(x)^2}{\delta_0-\delta_1}\right)^2+\frac{\gamma^2}{4}} \\ &\quad = |N|^2|f(x)|^2F(x), \end{aligned} \tag{4}$$

where γ is the decay outside the system, and $F(x)$ is the filter function. As $f(x)$ is assumed to be nearly constant over many wavelengths of the standing-wave field, the conditional position probability distribution is determined by the filter function.

The filter function in Eq. (4) is complex, therefore, we have to limit ourselves to specific cases where the form of the filter function is simplified. Paspalakis et al. have demonstrated that the absorption spectrum of a tripod system may be symmetric or asymmetric and its shape depends critically on the system parameters[16,17]. Goren *et al.*[18] symmetrically detuned two pumps with equal Rabi frequency, and found that the central peak appears exactly at the line center. Inspired by these results, we restrict our discussion under the limitation that $\delta_1=-\delta_2$. Moreover, we apply an exactly resonant probe field. Upon substituting $\delta_1=-\delta_2$ and $\delta_0=0$ into Eq. (4), we simplify the filter function as

$$F(x)=\frac{4\Omega_0^2\delta_2^2}{4(\Omega_2^2-\Omega_1^2\sin^2(kx))^2+\gamma^2}. \tag{5}$$

It is easy to find that the maxima of the peaks are located at

$$kx=\pm\arcsin\frac{\Omega_2}{\Omega_1}+m\pi, \tag{6}$$

where m is an integer.

The full width at half of the maximum height of all the peaks is given by

$$w=\left|\arcsin\frac{\sqrt{\Omega_2^2+\gamma\delta_2/2}}{\Omega_1}-\arcsin\frac{\sqrt{\Omega_2^2-\gamma\delta_2/2}}{\Omega_1}\right|. \tag{7}$$

Equation (6) shows that the position of the localization peak may be manipulated via changing Ω_2. According to Eq. (7), for given Ω_1 and Ω_2, the peak-position is fixed, but the peak-width can still be narrowed by decreasing δ_2. When δ_2 is decreased exactly to be zero, i.e., the two transparency windows coincide at $\delta_0=0$, then perfect EIT occurs at $\delta_0=0$ instead of absorption. In such a case, the atom localization cannot be realized. This can also be seen from Eq. (5), $F(x)\equiv 0$ for $\delta_2=0$. From the above analysis, we can see that the interacting double-dark resonances provide us the opportunity to control the localization results by the parameters of the additional control field.

First of all, we consider the probability of finding the atom within a wavelength domain of the standing-wave field. Setting all parameters to be dimensionless, we plot the filter function $F(x)$ versus the normalized position kx for two cases: $\Omega_2\in(0,\Omega_1)$ and $\Omega_2=\Omega_1$ in Figs. 2 and 3, respectively. The dashed curve is a sine-squared function to illustrate the position-dependent Rabi frequency of the standing-wave field. Evidently, subwavelength localization can be realized in such a system. In Fig. 2, there are four peaks within a unit wavelength of the standing-wave field. In other words, there are four different probable positions of the atom in a unit wavelength domain when the population in upper level is detected. Thus, the probability of finding the atom at a particular position is 1/4. In fact, from Eq. (6), we can find that for $\Omega_2\in(0,\Omega_1)$, there are four probable peak-positions within the interval $[\pi,\pi]$. Therefore, when Ω_2 is adjusted within the interval $(0,\Omega_1)$, the probability of finding the atom at a particular position is always 1/4. However, when $\Omega_2=\Omega_1$, as Fig. 3 shows, the number of peaks is reduced to two, and the probability of finding the atom at a particular position is 1/2. Therefore, when the Rabi frequency of the control field is equal to the maximum Rabi frequency of the standing-wave field, the probability of detecting the atom within one wavelength domain of the standing-wave field is doubled.

Now, we turn to the localization precision problem. In Figs. 4 and 5, we still plot the filter function $F(x)$ versus normalized position kx for the two different cases: $\Omega_2\in(0,\Omega_1)$ and $\Omega_2=\Omega_1$. When plotting Fig. 4 (or Fig. 5), we use the same parameters as in Fig. 2 (or Fig. 3), but with smaller detuning δ_2 than in Fig. 2 (Fig. 3). Comparing Fig. 4 with Fig. 2, we can find that the position and the number of peaks in Fig. 4 exactly correspond to

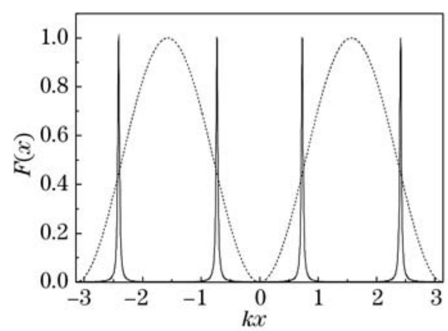

Fig. 2. Filter function $F(x)$ as a function of kx when $\Omega_2=2.00$. The dashed curve is a sine-squared function. The parameters are $\delta_0=0.00$, $\delta_2=-\delta_1=1.50$, $\Omega_1=3.00$, and $\gamma=0.20$. All parameters are measured in arbitrary units.

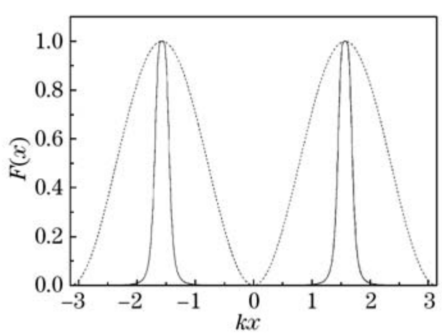

Fig. 3. Filter function $F(x)$ as a function of kx when $\Omega_2=3.00$. The parameters are the same as Fig. 2.

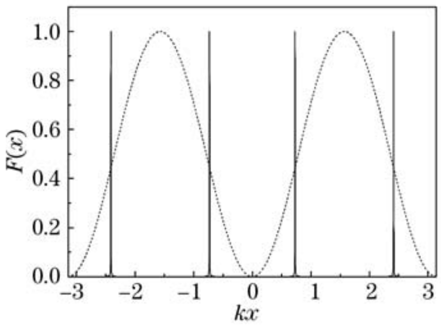

Fig. 4. Filter function $F(x)$ as a function of kx when $\Omega_2 = 2.00$. The parameters are the same as Fig. 2, but with $\delta_2 = -\delta_1 = 0.15$.

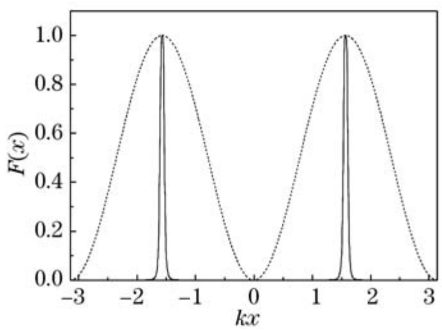

Fig. 5. Filter function $F(x)$ as a function of kx when $\Omega_2 = 3.00$. The parameters are the same as Fig. 3, but with $\delta_2 = -\delta_1 = 0.15$.

Fig. 2, but the peaks in Fig. 4 are much narrower. This shows that if only δ_2 is decreased, the peaks can become much narrower, namely, the precision of localization can be greatly enhanced. At the same time, the number of peaks does not change, i.e., the probability of finding the atom within a unit wavelength domain of the standing-wave field keeps unchanged. Similar conclusion can also be drawn for the case $\Omega_2 = \Omega_1$ by comparing Fig. 5 with Fig. 3. Therefore, via decreasing δ_2, the interacting double-dark resonances lead to enhanced localization precision with unchanged probability of finding the atom within a unit wavelength domain. The fact is easy to understand that decreasing δ_2 leads to narrower peaks. The decrease of the detuning δ_2 actually shortens the distance between the two EIT points, and naturally makes the width of the central absorption peak become narrower, which agrees with that of Ref. [18].

In the second place, we consider atom localization in a Λ-type four-level system.

The atomic system is show in Fig. 6. The transition $|a\rangle \leftrightarrow |c\rangle$ is coupled with a driving field aligned along the x direction with Rabi frequency $\Omega = \Omega_0 \sin(kx)$. A weak probe field with Rabi frequency ε couples the transition $|a\rangle \leftrightarrow |b\rangle$. An additional coherent perturbation field with Rabi frequency Ω_c couples the transition $|c\rangle \leftrightarrow |d\rangle$. The application of the perturbation field leads to a splitting of dark states and a "double-dark resonances" structure appears[12].

Assuming that the atom is initially in state $|b\rangle$, and that the probe field is very weak, then the conditional position probability distribution, i.e., the probability of finding the atom at position x in the standing-wave field

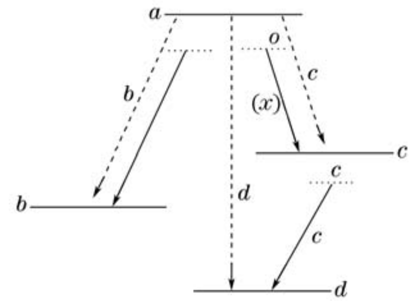

Fig. 6. Atomic system displaying double-dark resonances. γ_b, γ_c and γ_d represent the spontaneous decay rates from the upper level $|a\rangle$ to the three metastable states $|b\rangle$, $|c\rangle$ and $|d\rangle$. A driving field Ω, a weak probe field ε and an additional coherent perturbation field Ω_c couple their corresponding transitions.

provided that the atom is found in its internal state $|a\rangle$, is given by

$$P(x, t \to \infty|a\rangle) = |N|^2 |f(x)|^2 \varepsilon^2 A^2 / \left[(B - \Delta A)^2 + \gamma_{ab}^2 A^2\right], \quad (8)$$

here $A = \Omega_c^2 - (\Delta_0 - \Delta)(\Delta_0 + \Delta_c - \Delta)$, $B = \Omega^2(\Delta_0 + \Delta_c - \Delta)$, $\gamma_{ab} = (\gamma_b + \gamma_c + \gamma_d)/2$, N and $f(x)$ have the same meanings as in the above section. Obviously, the conditional position probability distribution is determined by the filter function

$$F(x) = \varepsilon^2 A^2 / \left[(B - \Delta A)^2 + \gamma_{ab}^2 A^2\right]. \quad (9)$$

As can be seen from Eq. (9), the localization depends not only on the parameters of standing-wave field and additional coherent perturbation field, but also on the probe field detuning Δ. Here we focus on the effects of probe field detuning on atom localization.

It should be pointed out that for the case $\Delta_0 + \Delta_c - \Delta = 0$, i.e., satisfying the three-photon resonance, the filter function becomes

$$F(x) = \varepsilon^2 / \gamma_{ab}^2. \quad (10)$$

The filter function is a constant for fixed ε and γ_{ab}, and the atom cannot be localization. Therefore, $\Delta_0 + \Delta_c - \Delta \neq 0$ is a fundamental condition for realizing atom localization in such a scheme.

In Fig. 7, we present the filter function versus kx for four different probe field detunings. The figure shows that the probe field detuning has a significant effect not only on the numbers of atom localization peaks, but also on the degree of atom localization. On the one hand, when Δ is small (Figs. 7(a) and (b)), there are three peaks in the subwavelength domain, which lie at the nodes of the standing-wave field. This means that the detecting probability of the atom in the subwavelength domain is 1/2. With a further increase of Δ, as Figs. 7(c) and (d) show, there appear four peaks of the atom localization, and the detecting probability is reduced to 1/4. So, we can say that adjusting the probe field detuning can realize the quantum control of localization and reduce the uncertainty in measuring a particular position of the atom by a factor of 2. On the other hand, the peaks in Fig. 7(b) are narrower than those in Fig. 7(a). This indicates that a suitable increment in the probe field detuning

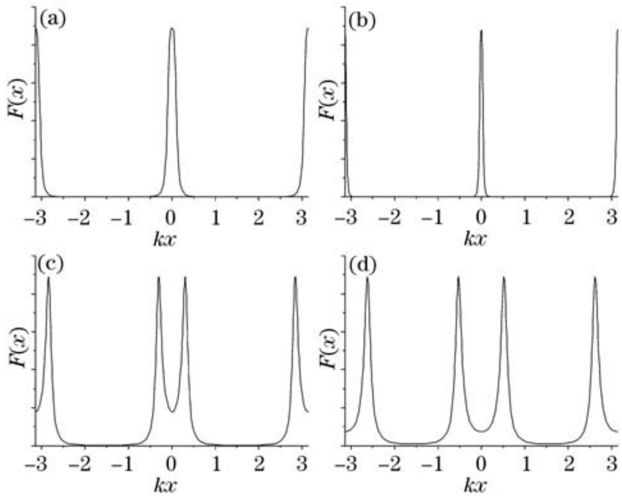

Fig. 7. Filter function $F(x)$ (in arbitrary units) as a function of kx for different probe field detunings. (a) $\Delta = 0.05$, (b) $\Delta = 0.15$, (c) $\Delta = 3.00$, (d) $\Delta = 5.00$.

can improve the degree of atom localization. However, when the probe field detuning is large, as Figs. 7(c) and (d) show, the degree of atom localization becomes worse than those in Figs. 7(a) and (b).

In summary, we presented two localization schemes utilizing interacting double-dark resonances. We demonstrated that atom localization via interacting double-dark resonances can show new features which are hardly found in the Λ-type system, where only one dark state exists. In the tripod system, the probability of detecting the atom within a unit wavelength domain is increased to 1/2 instead of usual 1/4 when the Rabi frequency of the control field is adjusted to the maximum Rabi frequency of the standing-wave field. The resolution for atom localization can be increased by reducing the detuning value to nonzero value. In the Λ-type four-level system, both the probability and the precision can be controlled by the detuning of the weak probe field. Adjusting the probe field detuning can not only make the atom localized at the nodes of the standing-wave field with high precision, but also increase the detecting probability of the atom at a particular position by a factor of 2.

D. Cheng's e-mail address is dchcheng@siom.ac.cn.

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