Lidar signal de-noising based on discrete wavelet transform

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Lidar is an efficient tool for remote monitoring, but the effective range is often limited by signal-to-noise ratio (SNR). The reason is that noises or fluctuations always strongly affect the measured results. So the weak signal detection is a basic and important problem in the lidar systems. Through the power spectral estimation, we find that digital filters are not suitable for processing lidar signal buried in noise. We present a new method of the lidar signal acquisition based on discrete wavelet transform for the improvement of SNR to increase the effective range of lidar measurements. Performance of the method is investigated by detecting the simulating and real signals in white noise. The results of Butterworth filter, which is a kind of finite impulse response filter, are also demonstrated for comparison. The experiment results show that the approach is superior to the traditional methods.

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Lidar is most widely used in atmospheric research in environments. Lidar can be used to study the high cirrus clouds over equatorial regions[1], high-latitude polar stratospheric clouds[2], stratospheric ozone[3] and stratospheric aerosols[4]. Lidar is the unique detector, which can provide us remote or selective sensing of about 20 different gaseous compounds in the atmosphere.

Lidar transmits electromagnetic radiation and measures the radiation that is scattered back to receiver. The backscattered radiation detected by a lidar can be described by the lidar equation. For a simple backscattered lidar, the lidar equation can be written as

$$p_{\mathrm{r}}(\lambda_{\mathrm{L}}) = \frac{C}{R^2}\frac{h}{2}O(R)\frac{\beta(\lambda_{\mathrm{L},R})}{4\pi} \exp\left(-2\int_0^R k_{\mathrm{e}}(\lambda_{\mathrm{L}}, R')\mathrm{d}R'\right), \quad (1)$$

where $p_{\mathrm{r}}(\lambda_L)$ is the power returned to the lidar at the laser wavelength λ_1; C is the lidar constant; R is the range; $h = c \times t_{\mathrm{p}}$ with t_{p} being the pulse duration and c the speed of light. The term $O(R)$ describes the overlap between the laser beam and the receiver field of view and it is equal to 1 for ranges where there is complete overlap of the laser and the receiver's field of view. In addition, $\beta(\lambda_{\mathrm{L},R})$ stands for the combined aerosol and molecular backscatter coefficient and $k_{\mathrm{e}}(\lambda_{\mathrm{L}}, R)$ denotes extinction coefficients at the laser wavelength λ. For an elastic backscatter (one wavelength) lidar, this combined backscattering can be obtained by solving the lidar equation following the method suggested by Ref. [4].

The main limitation of the effective range of lidar system is caused by the fact that the signal-to-noise ratio (SNR) falls rapidly with an increase of the distance R, which involves all types of lidars[5]. Figure 1 shows a simulating lidar signal without noise and illustrates the characteristic of ideal lidar signal. Figure 2 shows a real lidar signal, which backscattered by molecular and aerosol, recorded someday by our own lidar system located in Anhui Institute of Optics and Fine Machine (AIOFM), Hefei, China.

In practice, several efficient procedures should be applied in order to improve the quality of lidar data. The signal, $p_{\mathrm{r}}(\lambda_{\mathrm{L}})$, has to be achieved by averaging several hundreds or thousands of lasers pulses. This averaging is necessary to reduce random noise and interferences as well as to increase the precision of digitalization. By moving average method[5], the signal is only smoothed over the distance. So it cannot eliminate nonsensical values (especially negative values) produced by noise.

One of the most important problems that have to be solved with the applications of digital filters is the correct choice of the filter type and the filter parameters. The most difficult choice is that of the cut-off frequency of the filter. It is often selected arbitrarily or adopted a certain theoretical model[6−8]. The wavelet transform (WT) has recently become a data analysis tool in many applications like estimation, classification, and compression. The wavelet expansions tend to concentrate the signal energy into a relatively small number of large coefficients. This energy concentrate property of the WT makes the wavelet domain appropriate for signal estimation. In this paper, we present a new method, which based on discrete WT (DWT), to process lidar signal de-noising and increase the effective range of lidar measurements. Smoothing removes high frequency and retains low ones, de-noising attempts to remove whatever noise is present and retain whatever signal

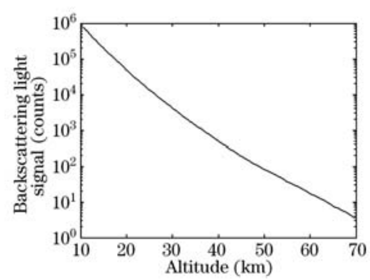

Fig. 1. Simulating lidar signal without noise.

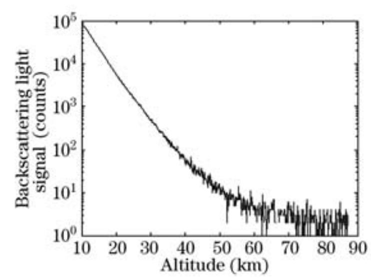

Fig. 2. Real lidar signal with noise.

is present regardless of the signal's frequency content. Wavelet de-noising does involve nonlinear soft threshold in the WT domain, and consists of three steps: a linear forward transform, a nonlinear threshold de-noising and a linear inverse WT. Furthermore, wavelet shrinking de-noising is considered as a nonparametric method. Thus, it is distinct from parametric method[9] in which we must estimate parameters for a particular model that must be assumed a priori.

In a real experiment there are many sources of noises and interferences, which can affect the lidar signal. 1) The random interferences and noises are produced mainly by the lidar system. 2) Interferences produced by high currents are switched in the laser circuits during the pulse. Interferences of this type are usually difficult to reduce by a simple shielding. 3) The background produced by light also comes from the sources other than the lidar laser. It can be eliminated in the similar way to the interferences occurring due to discharges in the laser circuits. 4) Light statistics is another important index for the lidar precision. It means that the parameter in Eq. (1) is determined by the detector characteristic. 5) Finally, there exists a digitalization noise, which is produced because of the reduced precision of devices converting the signal from the analogue form to the digital one.

Figure 3 shows the power spectral density (PSD) of simulating lidar signal illustrated in Fig. 1 and Fig. 4 shows the PSD of real lidar signal illustrated in Fig. 2, respectively. The goal of spectral estimation is to describe the distribution (over frequency) of the power contained in a signal, based on a finite set of data. According to spectral estimation, the noise of lidar signal is distributed in wide band and the noise and the signal are almost distributed in the same band interval. So it is impossible to eliminate the noise using digital filters by selecting a cut-off frequency simply.

The WT decomposes a signal into a set of basis functions, called a wavelet basis, which are from a single

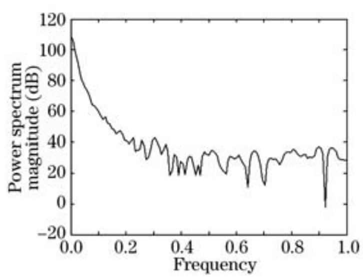

Fig. 3. PSD of simulating lidar signal.

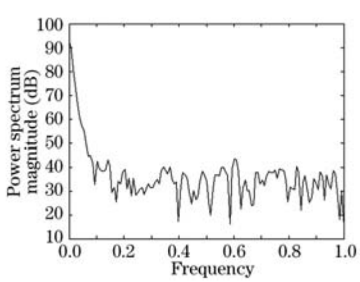

Fig. 4. PSD of real lidar signal with noise.

prototype wavelet by dilation and translation. By analogy with the Fourier transform (FT), WT maps the signal into another domain, in this case the time-scale (frequency) domain. However, the FT spreads the information of small or suddenly changing features over a wide frequency, while the WT is localized in time, which makes it more appropriate for applications to transient, non-periodic signals[10].

Wavelet functions $w(t)$ are defined as the waveforms that are locally positioned in both time and frequency domain, and satisfy following admissible condition

$$\int_{-\infty}^{+\infty} w(t)\mathrm{d}t = 0. \quad (2)$$

Equation (2) means that the wavelets do not have a dc component. One important property of wavelet basis function is its location in both time and frequency domains simultaneously.

The more popular orthogonal wavelet bases have several interesting properties that make them suitable as tool in signal analysis. In particular it has been possible to construct fast and efficient algorithms that enable WT to be practical tools in signal processing. Continuous wavelet decomposition can be written as

$$W(s,b) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} w_{(s,b)}(t) f(t)\mathrm{d}t, \quad (3)$$

where $W(s,b)$ is the WT coefficient; $\frac{1}{\sqrt{s}}$ is a normalization factor for conservation of energy; s is called scale parameter, b is translation factor, $w_{(s,b)}(t) = w((t-b)/s)$ is the wavelet function at a particular scale s, i.e. the same wavelet function is dilated or contracted according to the scale; and $f(t)$ is the function to be analyzed. The scale s can be interpreted as a measure of frequency. A short scale contains high-frequency components whereas a long scale contains low-frequency components. Equation (3) can also be interpreted as a convolution of the signal with the wavelet function in the time domain. Since the wavelets are locally positioned and have no DC component, the WT has sensitivity to the transient signals. And it carries out cross correlation procedure between wavelets and signals in the time domain, and de-noising procedure in the frequency domain. The WT also has a linear property

$$W_{\mathrm{s+n}}(s,b) = W_{\mathrm{s}}(s,b) + W_{\mathrm{n}}(s,b), \quad (4)$$

where $W_{\mathrm{s}}(s,b)$ is the WT coefficient of the signal without noise, $W_{\mathrm{n}}(s,b)$ is the WT coefficient of noise and $W_{\mathrm{s+n}}(s,b)$ is the WT coefficient of the signal with noise.

Functionally, WT can be applied to a finite group of data much like the discrete FT (DFT). The DWT uses two kinds of basis function consisting of a set of scaling functions, $\phi(t)$, called the "father" wavelet (interpreted as a low-pass filter), and wavelet functions, $w(t)$, called the "mother" wavelet (interpreted as a high-pass filter). A common algorithm for calculating discrete wavelet coefficients is the so-called Mallat algorithm[11]. The aim of the DWT is to decompose any signal $f(t)$ into a summation of all possible wavelet bases at different scales. The original input signal, $f(t)$, passes through these two complementary filters and emerges as two signals.

The wavelet function $w(t)$ can be written as a linear combination of the scaling function. The scaling function has the property that it can be written in terms of scaled versions itself,

$$\phi\left(\frac{t}{2}\right) = \sqrt{2}\sum_n h_n \phi(t-n), \quad (5)$$

$$w\left(\frac{t}{2}\right) = \sqrt{2}\sum_n g_n \phi(t-n). \quad (6)$$

There are two sequences, h_n and g_n, of coefficients. h_n are related to the low-pass filtering and g_n to the high-pass filter in the DWT algorithm. At each scale high and low pass filters are applied to the input signal. The actual shapes of these filters are determined by the kind of wavelet function used. The output from the high-pass filter at each scale is recorded as the wavelet coefficients. The low-pass filter extracts the low frequency components for the next scale where another set of high and low-pass filters is employed. At each successive scale the length of the vector upon which the filters operate is halved; this is referred to as decimation. Thus, the total number of available scale is $\log_2(N)$, where N is the length of the input data vector.

The coefficients of DWT are related not only to the input signal $f(t)$, but also to the types of mother wavelets $w(t)$ and its scaling function $\phi(t)$. When a mother wavelet, whose features are most matched the signal is chosen, the DWT coefficients of signal are bigger than that of noise in each scale. The success of wavelet de-noising based on threshold lies in that usually the signal will be decomposed into a few large coefficients whereas the noise component will give rise to small coefficients only. It is the features that we can use to remove the noise in signal.

Through wavelet decomposition, a similarity index $W(s,b)$ between the signal and the wavelet is calculated. The index called wavelet coefficient represents how closely the wavelet $w(t)$ correlates with the original signal. If the index $W(s,b)$ is large, the similarity is strong; otherwise, it is weak. As a result, we use the following formula to reconstruct

$$f(t) = W(s,b) \otimes w(t), \quad (7)$$

where $\otimes$ denotes inverse DWT.

In this section, we present two experimental results on the proposed algorithm for signal de-noising. First, the simulating signal is generated from the function $y = \exp(-128 \times ((x-0.3)^2) - 3 \times (|x-0.7|^{0.4}))$. The input x is generated from the uniform distribution on [0, 1.024], and the corresponding functional output y is artificially contaminated by stochastic noises generated from normal Gaussian white noise with zero-means. Figure 5 shows the contaminated signal with the SNR of 2 dB. The SNR is here defined as

$$\mathrm{SNR} = 10\log\left\{(\sum_{i=1}^{n} I_k^2)/\left[\sum_{i=1}^{n}(I_k - \hat{I}_k)^2\right]\right\} \quad (\text{unit}: \text{dB}), \quad (8)$$

where I_k denotes the original signal and $\hat{I}_k$ is contaminated signal. In this experiment, the contaminated simulating signal is decomposed in 5 scale and the wavelet function is symlet wavelet with five order. Figure 6 shows the de-noised simulating signal with the SNR of 25 dB. For contrast, the original simulating signal was also illustrated. It suggested that sharp features of the original signal remain sharp in the reconstruction. Clearly, the method of wavelet coefficient de-noising based on DWT has better performance in terms of visual quality. In fact, the gain of SNR is more then ten times.

Assume that we are given N samples from real lidar signal observed with noise: $y_i = f(t)_i + e_i$, $i = 1, 2, \cdots, N$, where e_i is regarded as zero mean and variance Gaussian white noise.

In practice, the Gaussian wavelet or Mexican hat wavelet is often used as mother wavelet. The Mexican hat wavelet is compactly supported in the time domain rather than the frequency domain, and is often used in the case where high resolution is required in the time domain. For lidar signal is a transient signal in the time domain, Mexican hat wavelet is selected. The Mexican hat wavelet is given by

$$w(t) = \frac{2}{\sqrt{3}}\pi^{-1/4}(1-t^2)\mathrm{e}^{-t^2/2}. \quad (9)$$

Figure 7 shows the waveform of Mexican hat wavelet. And a scaling function used in this experiment is $\phi(t) = 1$, if $0 \le t \le 1$, otherwise $\phi(t) = 0$.

In fact, the lidar signal tends to dominate low-frequency components. It is expected that the majority of high-frequency components above a certain level are

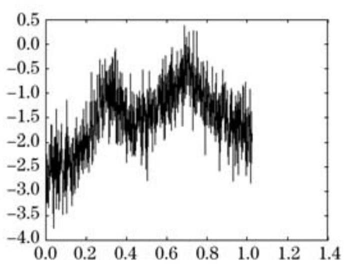

Fig. 5. Contaminated signal.

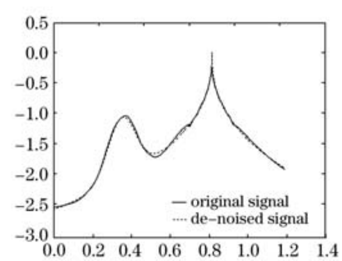

Fig. 6. Reconstructed signal.

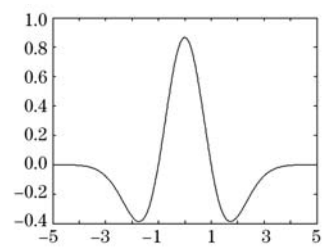

Fig. 7. Mexican hat wavelet.

noise components, which have very small wavelet coefficient values at short scales. To remove those elements regarded as noise among the whole wavelet coefficient, $W(s, b)$, there are many criteria giving threshold value. In this paper, we use the following universal threshold, $th = \sigma\sqrt{(2\log_2(N))}$ where N is the length of the input data vector and σ is the standard deviation of the noise. The latter is estimated from the median of the detail DWT coefficients at the first level (D^1) of signal decomposition.

$$\sigma = |\mathrm{median}(D^1)|/0.674. \quad (10)$$

Once the threshold value has been calculated one can apply a soft or hard modeling policy. For the lidar signal tends to dominate low-frequency components, the soft threshold, which is best in reducing noise but worst in preserving edges, was selected. In soft threshold, for each DWT coefficient $W(s, b)$ and threshold th, the soft threshold value is calculated as $W_{ij}^{\mathrm{st}} = \mathrm{sgn}(|W_{ij}| - th)$, if $|W_{ij}| \geq th$; $W_{ij}^{\mathrm{st}} = 0$, if $|W_{ij}| \leq th$. W_{ij} is the jth DWT coefficient at scale i of the decomposition. The threshold was applied only to detail coefficients.

The procedure of de-noising with soft thresholding is summarized as follows. 1) Remove meaningless data intervals in the original lidar signal, which are undetectable ranges of the lidar instrument. And the whole length of saved signal data set is extended to $2n$, by linear padding, which is the nearest length to the original signal data set. Apply DWT using Mexican hat wavelet based on Mallat algorithm to the original signal and obtain the matrix of wavelet coefficients. 2) Remove the elements regarded as noise among wavelet coefficients. A soft threshold approach is used to define the threshold value. The wavelet coefficient columns larger than the threshold value are selected and these columns indexes are saved, except for column indexes involved in the linear padding section. The selected indexes generate a new compressed lidar signal data set in the wavelet domain. The indexes are also used to reconstruct the original signal. 3) Apply inverse DWT using the saved indexes of selected wavelet coefficients to obtain de-noised and compressed lidar signal.

Figure 8 shows the de-noised lidar signal processed by DWT described in this paper. The effective range is great than 80 km. Figure 9(a) shows the PSD of lidar signal processed by DWT. Compared with the PSD of simulating lidar signal illustrated in Fig. 3, most components of the real lidar signal is retained regardless of the frequency content. Figure 9(b) shows the PSD of lidar signal processed by Butterworth filter. Obviously, the high components of lidar signal were lost.

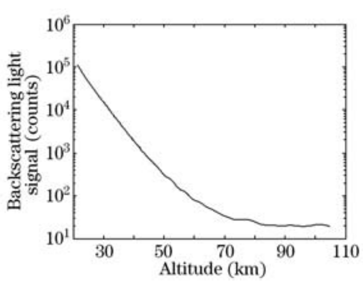

Fig. 8. De-noised lidar signal processed by DWT.

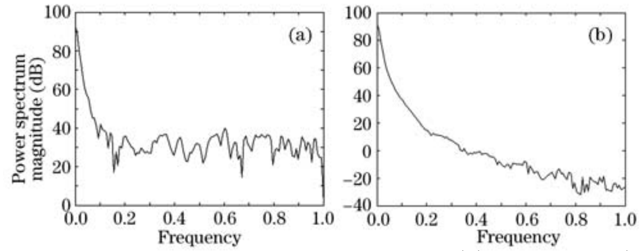

Fig. 9. PSD of lidar signal processed by (a) DWT and (b) Butterworth filter.

According to power spectral estimation, the noise is distributed in wide band, especially the noise and the signal, which came from long distance (greater then 40 km) with low SNR, are almost distributed in the same band interval. So it is impossible to eliminate the noise using conventional digital filter by selected a cut-off frequency simply. The wavelet coefficient de-noising algorithm by using nonlinear soft threshold method can remove the noise and retain the signal components regardless of the signal's frequency content. The reconstructing lidar signal algorithm from noise environment based on DWT is present in this paper. The experimental results about both simulating and real data demonstrate the effectiveness and efficiency of our proposed approach. In particular, the experimental results on real-world lidar signal show that the proposed approach is still efficient at greater distance (in the example above 80 km) where a poor SNR occurs. The reconstruction process of lidar signal by the wavelet coefficient de-noising shows that the noise in lidar signal almost was entirely suppressed and the effective range of lidar instrument increased greatly. Compared with the conversional digital filters, the PSD of processed lidar signal suggested that most of the lidar signal components are retained regardless of the signal frequency content. Future research work will include how to apply this method to solving more real problems.

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