Filamentation with imperfect initial beam quality

Ying Zhang (张 颖)[1,2], Qihua Zhu (朱启华)[1], Xiaofeng Wei (魏晓峰)[1], Yidan Hao (郝轶聃)[1], and Chuntong Ying (应纯同)[2]

[1] *Research Center of Laser Fusion, China Academy of Engineering Physics, Mianyang 621900*

[2] *Department of Engineering Physics, Tsinghua University, Beijing 100084*

The problem of the intense femtosecond pulse with elliptical transverse distribution propagating in air is reduced to an analogous problem of a particle moving in a two-dimensional potential well using variational method. Different types of initial conditions are considered to provide direct physical insight of how the self-focusing and defocusing mechanisms can lead to long distance propagation and refocusing phenomenon. The difference between elliptical and circular transverse distribution beams is compared and discussed. The results are coincident with direct numerical simulations.

OCIS codes: 030.1640, 140.3300.

The observation of long-distance filamentation pulses in air has attracted much recent interest[1−7]. Many applications have been found such as supercontinuum generation[1], remote sensing[2] and lightning discharge control[3]. It has been demonstrated that multiple photons ionization is the dominant defocusing mechanism which combined with diffraction to counteract self-focusing effect[4]. Several models have been established to describe this phenomenon such as self-guided pulse propagation model[5], modified moving-focus model[6] and dynamic spatial replenishment model[7]. But the nonlinear Schrödiger equation is so complicated that cannot be numerical simulated easily[8,9]. Due to the fact, Aközebek *et al.*[10] used variational method to provide a semi-analytical result which is qualitatively in good agreement with experiment observations.

But all these previous discussions and simulations assumed that pulse has a circular symmetric transverse distribution. Actually laser focus spot is not always a circle due to the intensity fluctuation at the near field and imperfect focusing. If the shape of the initial beam spot is not circular symmetric distribution, the dynamic process of pulse propagation will be different. Our aim here is to provide a better understand of how an imperfect beam is propagating in air.

In this paper, we use elliptical focus spot as an example of the imperfect initial beam quality, The model has been described by Mlejnek[7] and the approximation has been discussed by Aközbek[10]. Here we ignored avalanche ionization, electron recombination, group velocity and absorption. The simplified equation of electric field is written as

$$\frac{\partial \varepsilon}{\partial z} = \frac{i}{2k}\left(\frac{\partial^2 \varepsilon}{\partial x^2} + \frac{\partial^2 \varepsilon}{\partial y^2}\right) + i\left[\alpha\left|\varepsilon\right|^2 - \gamma\left|\varepsilon\right|^{2n}\right]\varepsilon, \qquad (1)$$

where $\alpha = n_2 k$, $\gamma = \frac{\sigma\beta^{(n)}\tau}{2n\hbar}g(t)$. n_2 is the nonlinear refraction coefficient and k is wave vector of the laser beam. σ is the cross section for inverse bremsstrahlung, τ is the electron collision time, and $\beta^{(n)}$ is the n-photon ionization coefficient. $g(t)$=0.5($t_{\min} + t$), and $t_{\min}$ is a cutoff determined by the initial pulse.

Variational analysis method has been used by Aközbek[10] to describe a circle Gaussian beam propagation in air. We use the same method to describe the propagation of an elliptical Gaussian beam. Equation (1) can be derived from the Lagrangian as

$$\frac{\delta L_c}{\delta \varepsilon^*} = \frac{\partial}{\partial z}\left[\frac{\partial L_c}{\partial\left(\frac{\partial \varepsilon^*}{\partial z}\right)}\right] + \frac{\partial}{\partial x}\left[\frac{\partial L_c}{\partial\left(\frac{\partial \varepsilon^*}{\partial x}\right)}\right] + \frac{\partial}{\partial y}\left[\frac{\partial L_c}{\partial\left(\frac{\partial \varepsilon^*}{\partial y}\right)}\right] - \frac{\partial L_c}{\partial \varepsilon^*} = 0, \qquad (2)$$

where the Lagrangian function is

$$L_c = \frac{i}{2}\left(\varepsilon\frac{\partial \varepsilon^*}{\partial z} - \varepsilon^*\frac{\partial \varepsilon}{\partial z}\right) + \frac{1}{2k_0}\left(\left|\frac{\partial \varepsilon}{\partial x}\right|^2 + \left|\frac{\partial \varepsilon}{\partial y}\right|^2\right) - \frac{\alpha}{2}\left|\varepsilon\right|^4 + \frac{\gamma}{(n+1)}\left|\varepsilon\right|^{2n+2}. \qquad (3)$$

The trial solution of Eq. (1), which is different from Ref. [10], is written as

$$\varepsilon(z,x,y,t) = A(z,t)\exp[-\frac{x^2}{a^2(z,t)} - \frac{y^2}{b^2(z,t)} + ic(z,t)x^2 + id(z,t)y^2 + i\phi(z,t)], \qquad (4)$$

where variational parameters of A, a, b, c, d, and ϕ are dependent both on z and t. There parameters are sufficient to describe an elliptical distribution laser beam, including the effect of self-phase modulation through c and d. The initial field is assumed to be

$$\varepsilon(0,x,y,t) = \varepsilon\exp\left(-\frac{x^2}{a_0^2} - \frac{y^2}{b_0^2} - \frac{t^2}{t_0^2} + ic_0x^2 + id_0y^2\right). \qquad (5)$$

Insert Eq. (4) into Eq. (3) and integrating over the transverse coordinate to get the reduced Lagrangian $< L_c > (z,t) = \int L_c \mathrm{d}x\mathrm{d}y$. Using the equation of motion in the presence of multiple-photon ionization

$$\frac{\partial}{\partial z}\left[\frac{\partial\langle L_c\rangle}{\partial\left(\frac{\partial \mu_i}{\partial z}\right)}\right] - \frac{\partial\langle L_c\rangle}{\partial \mu_i} = 0, \qquad (6)$$

where $\mu_i = A, a, b, c, d, \phi$ $(i = 1, \cdots, 6)$, we obtain the following sets of coupled equations

$$\left\{\begin{array}{l} \frac{\partial a}{\partial z} = \frac{2}{k}ac \\ \frac{\partial c}{\partial z} = -\frac{2}{k}c^2 + \frac{2}{k}\frac{1}{a^4} - \frac{\alpha}{2}\frac{A^2}{a^2} + \frac{2n\gamma}{(n+1)^2}\frac{A^{2n}}{a^2} \end{array}\right., \qquad (7)$$

$$\left\{\begin{array}{l} \frac{\partial b}{\partial z} = \frac{2}{k}bd \\ \frac{\partial d}{\partial z} = -\frac{2}{k}d^2 + \frac{2}{k}\frac{1}{b^4} - \frac{\alpha}{2}\frac{A^2}{b^2} + \frac{2n\gamma}{(n+1)^2}\frac{A^{2n}}{b^2} \end{array}\right., \qquad (8)$$

1671-7694/2007/S1S246-04 © 2007 Chinese Optics Letters

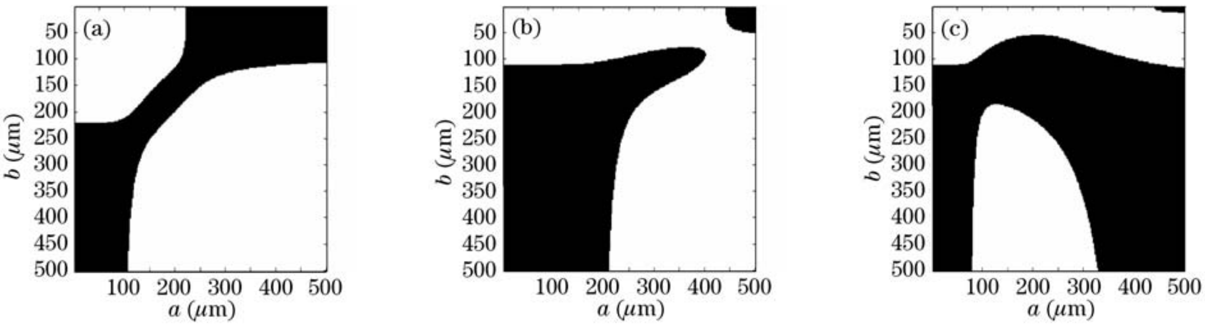

Fig. 1. Two-dimensional potential well with different initial parameters. The white areas denote $U(a,b) \gg 0$ and the black areas denote $U(a,b) \ll 0$. (a) $P = 5P_{\rm cr}$, $a_0 = b_0 = 200$ μm; (b) $P = 5P_{\rm cr}$, $a_0 = 100$ μm, $b_0 = 400$ μm; (c) $P = 2P_{\rm cr}$, $a_0 = 100$ μm, $b_0 = 400$ μm.

Table 1. Characteristic Parameters of Air[10,11]

n_b	n_2 (cm^{2}W^{-1})	n	$\beta^{(n)}$ (m^{2n-3}W^{-n+1})	τ (s)
1.0	5.57×10^{-19}	7	6.5×10^{-104}	3.5×10^{-13}

$$\frac{\partial A}{\partial z} = -\frac{1}{k} A\left(c+d\right), \tag{9}$$

$$\frac{\partial\left(A^2 ab\right)}{\partial z} = 0. \tag{10}$$

It is surprisingly to see that parameters (a, c) and (b, d) are separately described by two sets of equations. Equations (7) and (8) seem the same as Eqs. (14) and (16) in Ref. [10]. The total power is defined as $P{=}\pi A^2 ab/2$, where P is a constant inferred from Eq. (10). Equations (7)$-$(9) can be combined to give

$$\frac{1}{2}\left(\frac{\partial a}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial b}{\partial z}\right)^2 + U\left(a,b\right) = 0, \tag{11}$$

where

$$\begin{aligned} U\left(a,b\right) = {} & \frac{1}{a^2} + \frac{1}{b^2} - p\left[\frac{2}{ab} - \left(\frac{1}{ab_0} + \frac{1}{a_0 b}\right)\right] \\ & + q\left[\frac{2}{a^n b^n} - \left(\frac{1}{a^n b_0^n} + \frac{1}{a_0^n b^n}\right)\right] \\ & - \left(\frac{1}{a_0^2} + \frac{1}{b_0^2}\right) - 2\frac{a_0^2 c_0^2}{k^2} - 2\frac{b_0^2 d_0^2}{k^2}, \end{aligned} \tag{12}$$

$p = \alpha k P/\pi$, and $q = 2^{n+1} k\gamma P^n/\pi^n (n+1)^2$ are constants decided by material.

Equation (10) is similar to Eq. (17) in Ref. [10]. Here a classical particle is moving in a two-dimensional potential well $U(a,b)$, but in Ref. [10], a particle is just moving in a one-dimensional potential well. a and b describe the position of the particle, and z acts as time. Figure 1 shows some potential well under different initial condition when a collimated beam propagates in air with the characteristic parameters shown in Table 1. The well has a sharp edge (where $U(a,b) = 0$) so it is better to be described by black and white colors. $P_{\rm cr} = \lambda_0^2/2\pi n_0 n_2$ is the critical power for self focusing of a continuous wave (CW) laser beam. Note that the well is not symmetrical when $a_0 \neq b_0$.

Figures 2 $-$ 4 describe the detail evolvement of beam shape and peak intensity under the initial condition in Fig. 1. As shown in Fig. 2(a), $a_0 = b_0$, so the potential well is symmetrical. The particle will oscillate in the well which means that the beam radius will focus, diffract and then refocus, diffract. Peak intensity of the beam oscillates with beam radius (shown in Fig. 2(b)). If $a_0 \neq b_0$, things will be different. As shown in Fig. 3, long axis and short axis will undergo independent oscillation. $a < b$, $a = b$, $a > b$, and $a = b$ appear repeatedly. That means the beam will change from ellipse to circle, then ellipse with long and short axes exchanged, and then circle, after, ellipse, repeatedly. It is interesting to see that peak intensity will also oscillate with propagation distance, which reach maximum when the beam shape is circular, as shown in Fig. 3(b). That means that refocus phenomenon is also expected to appear with elliptical initial laser beams theoretically.

If the input energy is low, the beam radius will not oscillate. As shown in Fig. 4, $P = 2P_{\rm cr}$, which is not enough for a elliptical beam to oscillate. The long axis direction first focuses and then diffracts, not focuses again.

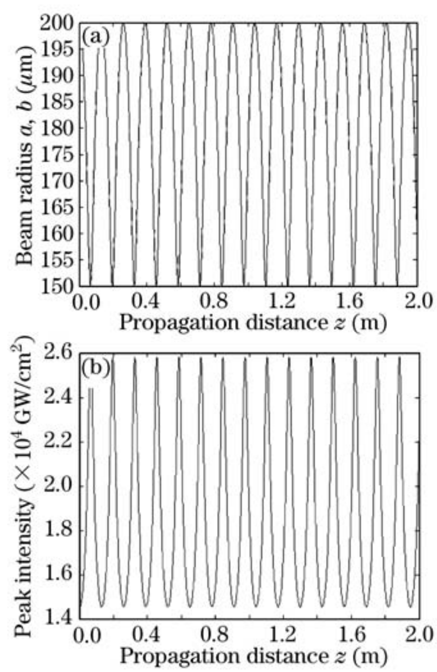

Fig. 2. Evolvement of beam parameters when $P = 5P_{\rm cr}$, $a_0 = b_0 = 200$ μm. (a) Beam radius a and b, here $a = b$; (b) peak intensity A^2.

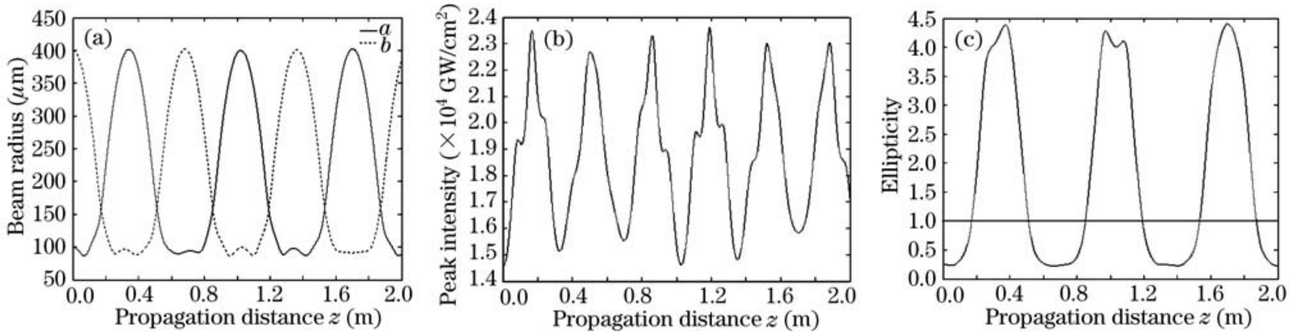

Fig. 3. Evolvement of beam parameters when $P = 5P_{\rm cr}$, $a_0 = 100$ μm, $b_0 = 400$ μm. (a) Beam radius a and b; (b) peak intensity A^2; (c) ellipticity a/b.

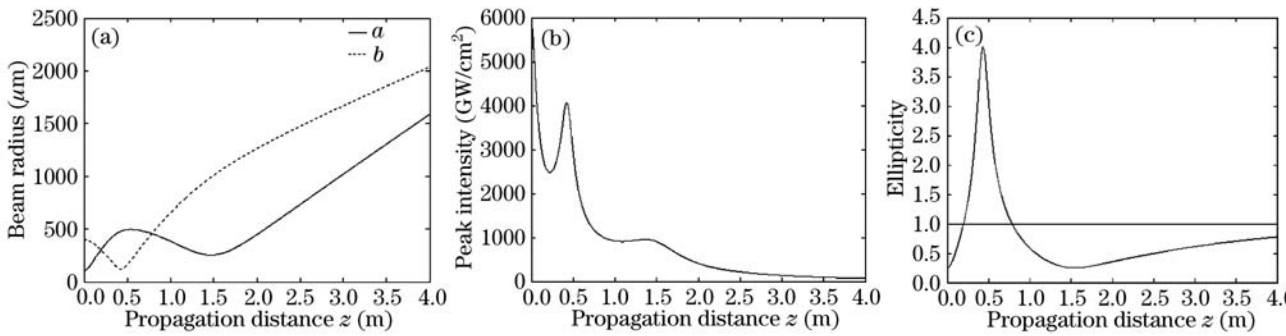

Fig. 4. Evolvement of beam parameters when $P = 5P_{\rm cr}$, $a_0 = 100$ μm, $b_0 = 400$ μm. (a) Beam radius a and b; (b) peak intensity A^2; (c) ellipticity a/b.

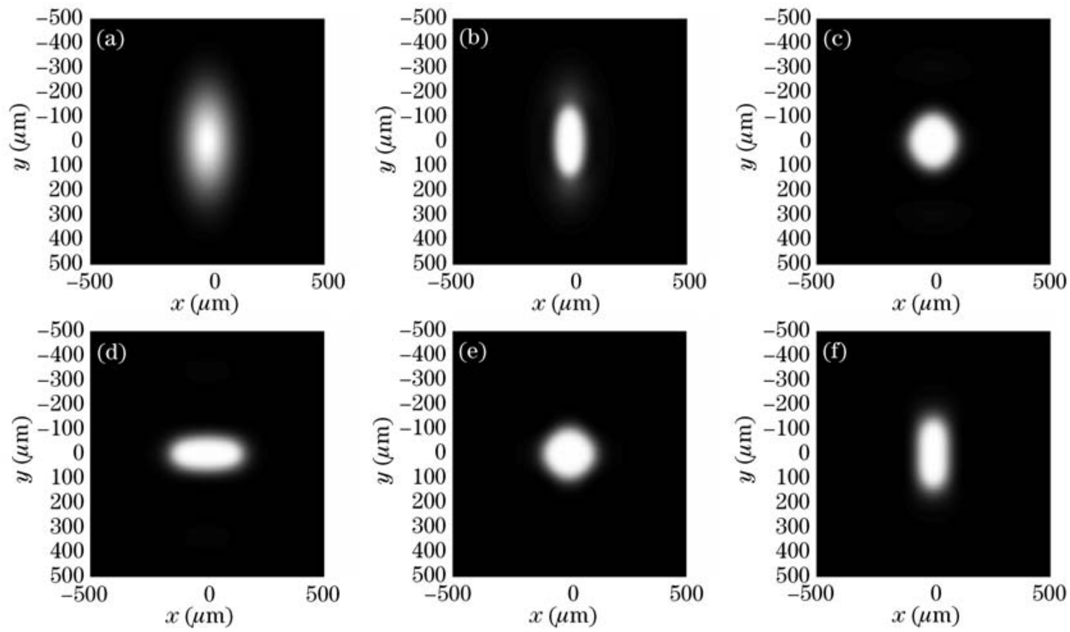

Fig. 5. Direct simulation of beam spot intensity distributions at different propagation distances of (a) 0, (b) 4, (c) 10.5, (d) 15, (e) 30, and (f) 36 cm. $P_0 = 5P_{\rm cr}$.

The short axis direction refocuses only one time. Peak intensity of the beam decreases continually except reach local maximum when $z \approx 0.4$ m.

The evolvement of beam shape can be predicted from Fig. 1. Figure 1(a) is symmetrical, which means a and b will follow the same path. Figures 1(b) and (c) are not symmetrical, which means a and b will move independently. In Fig. 1(c), the black area disperses into the region that $a \gg 0$ and $b \gg 0$, so the particle will possibly move to the place $a \gg 0$ and $b \gg 0$, which equals to the beam size will infinitely increase. If absorption is considered, potential well will change with the total power, which means laser beam will ultimately diverge.

Direct simulation is made to get a better comprehension. Here absorption is considered for better convergence. As shown in Fig. 5, the initial beam spot has an elliptical Gaussion distribution. When it propagates about 10 cm, it changes into a circular spot, after that it turns to an ellipse with its long axis and short axis exchange. Then, the beam changes into a circle again, finally, an ellipse, which is the same as the initial form. These processes will carry through repeatedly until energy is consumed by absorption. We found that direct simulation results are qualitatively coincident with variational analysis results. That is to say, trial solution in Eq. (4) and variational method are acceptable to deal with this complicated problem.

In conclusion, we analyze how elliptical laser beam propagating in air. Using variational method, the process of propagation is reduced to an analogous problem of a particle moving in a two-dimensional potential well. Different initial conditions are discussed to provide direct understand of the relationship between potential well and beam shape variation. Direct simulation is made to validate the feasibility of the trial solution which is used in variational analysis. In addition, group velocity dispersion is ignored in this letter to make easier calculation but it can affect the propagation process to some extent.

Y. Zhang's e-mail address is zhangying99@mails.tsinghua.edu.cn.

References

1. R. R. Alfano and S. L. Shapiro, Phys. Rev. Lett. **24,** 584 (1970).
2. H. Schillinger and R. Sauerbrey, Appl. Phys. B **68,** 753 (1999).
3. X. M. Zhao, J.-C. Diels, C. V. Wang, and J. M. Elizondo, IEEE J. Quantum Electron. **31,** 599 (1995).
4. A. Brodeur and S. L. Chin, Phys. Rev. Lett. **80,** 4406 (1998).
5. A. Braun, G. Korn, X. Liu, D. Du, J. Squier, and G. Mourou, Opt. Lett. **20,** 62 (1995).
6. A. Brodeur, C. Y. Chien, F. A. Ilkov, S. L. Chin, O. G. Kosareva, and V. P. Kandidov, Opt. Lett. **22,** 304 (1997).
7. M. Mlejnek, E. M. Wright, and J. V. Moloney, Opt. Lett. **23,** 382 (1998).
8. V. P. Kandidov, O. G. Kosareva, I. S. Golubtsov, W. Liu, A. Becker, N. Akozbek, C. M. Bowden, and S. L. Chin, Appl. Phys. B **77,** 149 (2003).
9. M. Kolesik, G. Katona, J. V. Moloney, and E. M. Wright, Appl. Phys. B **77,** 185 (2003).
10. N. Akozbek, C. M. Bowden, A. Talebpour, and S. L. Chin, Phys. Rev. E **61,** 4540 (2000).
11. D. Cheskis, S. Bar-Ad, R. Morandotti, J. S. Aitchison, H. S. Eisenberg, Y. Silberberg, and D. Ross, HAIT J. Science and Engineering **1,** 363 (2004).