Electromagnetically induced negative refractive index in dense atomic gas

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We propose two dense gas schemes of four-level Λ-type and V-type atoms based on the effect of quantum coherence. It is shown that under certain conditions the dense gas can simultaneously exhibit negative permittivity and negative permeability, and thus become a negative refractive-index material. Furthermore, by analyzing the absorption property of the left-handed materials, we find that the absoption can be reduced via choosing appropriate parameters. Such schemes might be used to fabricate isotropic and homogeneous left-handed material with vanishing absorption in a wider optical frequency band.

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Recently, left-handed materials (LHMs)[1] have attracted more attention[2−4] because of their surprising and often counterintuitive electromagnetic properties. The materials have a negative refractive index when the permittivity and permeability are negative simultaneously[1]. In LHMs, the wave vector is opposite to the direction of energy propagation. Although naturally occurring materials with both negative permittivity and permeability simultaneously are not available, the LHMs have been artificially realized[5,6]. Interestingly, a number of alternative theoretical schemes bearing left-handed electromagnetic behavior have been proposed recently, including resistive inductive capacitive (RIC) transmission lines[7], photonic circuits[8], and other nanostructures[9]. Furthermore, some applications[10] have been suggested to exploit the singular physical properties of LHMs, such as beam refocusing, inversion of Snell's law[5,11], backward Cerenkov radiation[12], negative Goss-Hänchen shift[13], etc.. However, many left-handed artificial materials with metallic periodic structure work in the microwave region[14,15]. All such systems (metamaterials) require delicate manufacturing of spatially periodic structure. Excitingly, a scheme of dense gas of three-level atoms has been proposed based on the effect of quantum coherence[16,17], which could exhibit negative refractivity in optical frequency range without any spatial periodicity. However, it requires rigorous level condition. Recently left handedness has been analyzed in atomic hydrogen and neon[18], which has improved the demanding level condition of Ref. [16] but the negative refractive frequency band is narrow.

In this letter, we theoretically put forward two dense gas schemes of four-level Λ-type and V-type atoms based on the effect of quantum coherence, in which the local field effect is considered. It is shown that the negative refractivity can be realized and the frequency band is enlarged compared with that in Refs. [16,18]. On the other hand, by analyzing the absorption property of the left-handed material, we find that the absorption can be reduced via choosing appropriate parameters.

As shown in Fig. 1, a four-level Λ-type atomic system is considered. The three lower levels have the same parity, which is opposite to the upper level. The two-fold levels, labeled $|3\rangle$ and $|4\rangle$, are coupled by a constant control field Ω_c with frequency ω_c, which can be a microwave or quasi-static field. Levels $|1\rangle$ and $|2\rangle$, $|2\rangle$ and $|3\rangle$ are coupled by a weak probe laser beam Ω_p with frequency ω_p and a strong coupling laser beam Ω_s with frequency ω_s, respectively.

In the interaction representation, the semiclassical Hamiltonian in a rotating frame is

$$\tilde{H} = -\hbar \begin{bmatrix} 0 & \Omega_p/2 & 0 & 0 \\ \Omega_p/2 & \Delta_p & \Omega_s/2 & 0 \\ 0 & \Omega_s/2 & \delta_1 & \Omega_c/2 \\ 0 & 0 & \Omega_c/2 & (\delta_1 - \Delta_c) \end{bmatrix}. \quad (1)$$

Here, $\Delta_p = \omega_p - \omega_{21}$, $\Delta_s = \omega_s - \omega_{23}$ and $\Delta_c = \omega_c - \omega_{34}$ denote the single-photon detunings of the probe, coupling and the control field, respectively. $\omega_{ij} = \omega_i - \omega_j$ is the transition frequency of level $|i\rangle$ and $|j\rangle$ $(i, j = 1, 2, 3, 4)$. $\delta_1 = \Delta_p - \Delta_s$ means two-photon detuning. Ω_i $(i = p, s, c)$ denotes the Rabi frequency for the corresponding transition.

The dynamics of the system is described using density-matrix approach as

$$\frac{d\tilde{\rho}}{dt} = -\frac{i}{\hbar}[\tilde{H}, \tilde{\rho}] + \Lambda\tilde{\rho}, \quad (2)$$

where $\Lambda\tilde{\rho}$ represents the irreversible decay part in the system. For simplicity, we here only consider the spontaneous decay term. Assuming the atoms are initially in the ground level $|1\rangle$ and $\Omega_p \ll \Omega_s, \Omega_c$, it is easy to

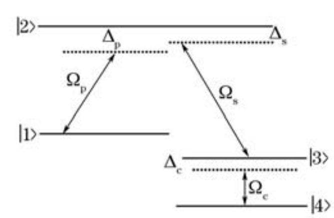

Fig. 1. Scheme of a four-level atom interacting with the probe, coupling and control fields.

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solve Eq. (2) in the steady state:

$$\tilde{\rho}_{21} = -\frac{i\Omega_{\rm p}}{2}\frac{\Omega_{\rm s}^2(-i\delta_2+\Gamma_4)-Z}{(-i\Delta_{\rm p}+\Gamma_2+\Gamma_4)Z}, \quad (3a)$$

$$\tilde{\rho}_{41} = -\frac{i}{2}\frac{\Omega_{\rm p}\Omega_{\rm c}\Omega_{\rm s}}{Z}, \quad (3b)$$

where

$$Z = 4(-i\Delta_{\rm p}+\Gamma_2+\Gamma_4)(-i\delta_1+\Gamma_3+\Gamma_4)(-i\delta_2+\Gamma_4) +\Omega_{\rm c}^2(-i\Delta_{\rm p}+\Gamma_2+\Gamma_4)+\Omega_{\rm s}^2(-i\delta_2+\Gamma_4), \quad (4)$$

$$\delta_2 = \Delta_{\rm p}-\Delta_{\rm s}-\Delta_{\rm c}, \Gamma_2 = \gamma_2/2, \Gamma_3 = \gamma_3/3, \Gamma_4 = \gamma_4/2. \quad (5)$$

γ_i $(i = 2,3,4)$ is the spontaneous decay rate of level $|i\rangle$.

The electric response of the medium to the probe field is relative to the coherence term ρ_{21}, which drives an electric dipole oscillating at frequency $\omega_{\rm p}$. According to the classical electromagnetic theory, the electric dipole moment of the atom is given by $p_i = \alpha_{ij}E_{{\rm p}j} = d_{12}^i\rho_{21}$, where $\vec{d}_{21} = e\langle 2|\vec{r}|1\rangle$ is the electric dipole operator and the polarizability tensor α becomes a scalar if we choose probe field $\vec{E}_{\rm p}$ parallel to the atomic dipole $\vec{d}_{21}$. The atomic polarizability for the probe field is given as

$$\alpha = -\frac{i}{2}\frac{|d_{12}|^2}{\hbar}\frac{\Omega_{\rm s}^2(-i\delta_2+\Gamma_4)-Z}{(-i\Delta_{\rm p}+\Gamma_2+\Gamma_4)Z}. \quad (6)$$

Here, we take into account the local field effect, then the relationship between the polarizability α and the susceptibility $\chi_{\rm e}$ is given by the Clausius-Mossotti relation[19]

$$\chi_{\rm e}\varepsilon_0 = N\alpha\left(1-\frac{N\alpha}{3\varepsilon_0}\right)^{-1}, \quad (7)$$

where N is the atomic density.

In the same way, the induced magnetic dipole is given as

$$\langle\vec{\mu}\rangle = \rho_{41}\vec{\mu}_{14} = -\frac{i}{2}\frac{(-i\delta_2+\Gamma_4)\Omega_{\rm c}\Omega_{\rm s}\Omega_{\rm p}\vec{\mu}_{14}}{Z} \times\exp[-i(\omega_{\rm p}-\omega_{\rm s}-\omega_{\rm c})t]+{\rm c.c.} \quad (8)$$

It is well known that the atomic response to the magnetic component of the probe field would reach maximum if the induced magnetic dipole of atom oscillate in phase with the probe beam[16]. Then the in-phase condition is obtained to be $\omega_{24} = 2\omega_{21}$.

We use the relation $\langle\vec{\mu}\rangle = \eta(\omega_{\rm p})E_{\rm p}(\hat{x}-i\hat{y})$ to describe magnetic response of atomic system by probe electric field[16]. Assuming the states to be $|1\rangle \equiv |n,l,m\rangle$, $|2\rangle \equiv |n',l+1,m-1\rangle$, $|3\rangle \equiv |n,l,m+1\rangle$, $|4\rangle \equiv |n,l,m-1\rangle$, the coefficient $\eta(\omega_{\rm p})$ can be calculated as

$$\eta(\omega_{\rm p}) = -\frac{i\mu_{\rm B}}{2\hbar}\frac{\Omega_{\rm c}\Omega_{\rm s}}{Z}\langle n',l+1||e\vec{r}||n,l\rangle(l-m+1) \times\sqrt{\frac{(l+m)(l-m+2)}{(l+1)(2l+1)(2l+3)}}, \quad (9)$$

by using the Wingner-Eckart theorem[20], where $\langle n',l+1||e\vec{r}||n,l\rangle = \int_0^\infty {\rm d}rer^3R_{n'l+1}^*(r)R_{nl}(r)$ is the reduced matrix element with $R_{nl}(r)$ being the radial wave function. Combining Clausius-Mossotti equation and Maxwell's equations, we finally get the relative permeability of the atomic system[16]:

$$\mu_{\rm r}(\omega_{\rm p}) = \frac{1}{1+i\mu_0\eta(\omega_{\rm p})c\left[1+\frac{\chi_{\rm e}(\omega_{\rm p})}{3}\right]N}. \quad (10)$$

Now we discuss the negative refraction property of this atomic system through numerical calculation. In our analysis, we consider ^{87}Rb atoms with density of $N = 10^{26}$ m^{-3} to realize the negative refractive index. For simplicity, all the parameters are scaled by $\gamma_2 = 2\pi\times 5.9$ MHz, which is the spontaneous emission rate of level $|2\rangle$[21]. The wavelength of resonant probe transition is 795 nm, corresponding to D1 line. The control field is at resonance and the coupling-field detuning $\Delta_{\rm s} = -0.051\gamma_2$. Typical values for $\Gamma_2 = 0.5\gamma_2$, $\Gamma_3 = \Gamma_4 = 0.002\gamma_2$ are used. The Rabi frequencies of the coupling and control field are $\Omega_{\rm s} = 50\gamma_2$, $\Omega_{\rm c} = 0.1\gamma_2$. The probe-detuning dependence of the relative electric permittivity $\varepsilon_{\rm r}$ and the magnetic permeability $\mu_{\rm r}$ are shown in Fig. 2.

From Fig. 2, one can see that both real parts of the relative dielectric permittivity $\varepsilon_{\rm r}$ and the relative magnetic permeability $\mu_{\rm r}$ become negative over a band of frequency $\sim 0.0025\gamma_2$. In contrast with the case of the three-level atomic system[16], the frequency band of negative refractivity is widened about 2.5 times. In our four-level Λ-type atomic scheme, the coherence terms ρ_{21} and ρ_{41} drive an electric resonator and a magnetic resonator respectively. The large coherence may be achieved by tuning coupling and control fields, which makes it possible to achieve a wide negative refraction frequency band. On the other hand, with the help of the Lorenz-Lorenz local-field contribution, the probe field could maintain strong local currents that could give rise to large enough magnetization. Then the dielectric permittivity and the magnetic permeability are possibly modified to be negative. Therefore, the negative refractive index is realized in a wider optical frequency band in this scheme.

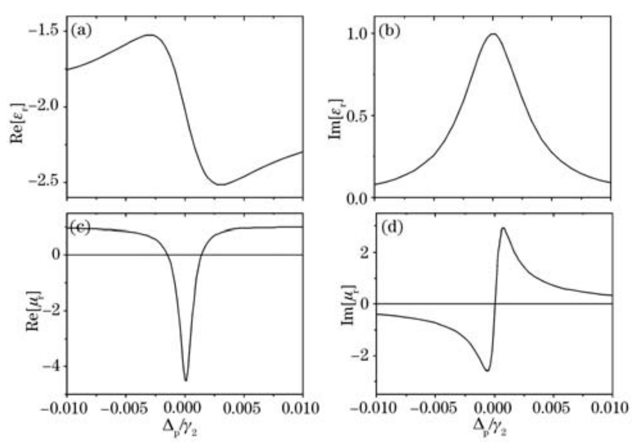

Fig. 2. Frequency dependence of the relative permittivity $\varepsilon_{\rm r}$ and the magnetic permeability $\mu_{\rm r}$ of dense gas of four-level atoms. Parameters are $N = 10^{26}$ m^{-3}, $\gamma_2 = 2\pi\times 5.9$ MHz, $\lambda = 795$ nm, $\Omega_{\rm s} = 50\gamma_2$, $\Omega_{\rm c} = 0.1\gamma_2$, $\Delta_{\rm c} = 0$, $\Delta_{\rm s} = -0.051\gamma_2$, $\Gamma_2 = 0.5\gamma_2$, $\Gamma_3 = \Gamma_4 = 0.002\gamma_2$. All axes are in dimensionless units.

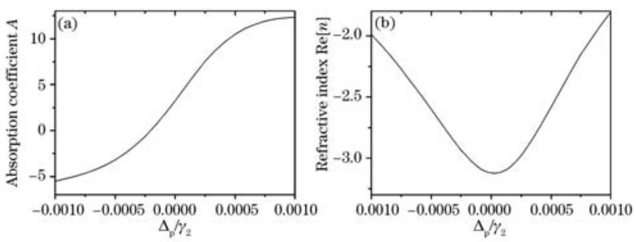

Fig. 3. (a) Absorption coefficient A and (b) refractive index Re$[n]$ dependence on the probe-field detuning Δ_p/γ_2. Parameters are same as those in Fig. 2.

In addition, it should be emphasized that the absorption coefficient of LHMs is expressed as $A = 2\pi\mathrm{Im}[-\sqrt{\varepsilon_r\mu_r}]$[17]. Figure 2 shows when $\Delta_p = 0$, that $\varepsilon_r(0) = -2.0154 + i0.9953$ and $\mu_r(0) = -2.1014 - i0.5752$, i.e., the absorption of the LHM is small at resonance. From Fig. 3(a), it can be found that the absorption is also small in certain band near the resonance region, especially for the case of $\Delta_p = -7509$ Hz, where the absorption is zero. Compared with the other LHMs, in this scheme the absorption can be reduced, even to zero.

In the following we consider a four-level V-type atomic system to realize negative refraction, as shown in Fig. 4. The two upper levels, $|3\rangle$ and $|4\rangle$, have the same parity and $\mu_{34} = \langle 3|\hat{\vec{\mu}}|4\rangle \neq 0$, where $\hat{\vec{\mu}}$ is the magnetic dipole operator. The two lower levels $|1\rangle$ and $|2\rangle$ have opposite parity with $d_{21} = \langle 2|\hat{\vec{d}}|1\rangle$, where $\hat{\vec{d}}$ is the electric dipole operator. Levels $|1\rangle$ and $|2\rangle$ are coupled by a weak probe field Ω_p with frequency ω_p, while levels $|1\rangle$ and $|3\rangle$ are coupled by a strong pump field Ω_s with frequency ω_s. Levels $|2\rangle$ and $|4\rangle$ are coupled by a control field Ω_c with frequency ω_c, whose transition is a two-photon process.

The semiclassical Hamiltonian in a rotating frame is given as

$$H = \hbar \begin{bmatrix} 0 & -\Omega_p & -\Omega_s & 0 \\ -\Omega_p & \Delta_p & 0 & -\Omega_c \\ -\Omega_s & 0 & \Delta_s & 0 \\ 0 & -\Omega_c & 0 & (\Delta_p + \Delta_c) \end{bmatrix}. \quad (11)$$

Here, $\Delta_p = \omega_{21} - \omega_p$, $\Delta_s = \omega_{31} - \omega_s$ and $\Delta_c = \omega_{24} - \omega_c$ denote the detunings of the probe, pump and the control field, respectively. In this scheme, the electromagnetically induced transparency (EIT) approximations are used. These include the atoms are initially in states $|1\rangle$ and $|3\rangle$ and the strong pump approximation $\Omega < \Omega_c \ll \Omega_s$. Using the density-matrix approach, the time evolution of the system is described as Eq. (2). The density matrix elements can be calculated numerically later.

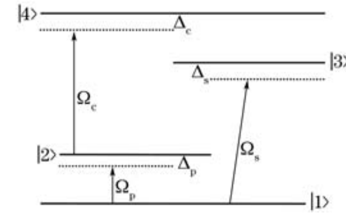

Fig. 4. Scheme of a four-level V-type atom interacting with the probe, pump and control fields.

Using the analysis method of the former scheme, the electric polarizability and the magnetizability for the probe field are expressed as

$$\alpha_e = \frac{d_{21}\rho_{12}}{\varepsilon_0 E_p}, \quad (12)$$

$$\alpha_m = \frac{\mu_0 c \mu_{34}\rho_{43}}{\eta E_p}, \quad (13)$$

where μ_0 is the permeability of vacuum, c is the speed of light in vacuum and η is a unitary complex number depending on the polarization of the probe field E_p. The fields should required $\omega_c = \omega_s$ and the levels satisfy the condition $\omega_{31} = \omega_{42}$, which is looser than the former scheme. The electric susceptibility and the permittivity can be obtained by Eq. (7) and the permeability is given as

$$\mu_r = \frac{1}{1 - N\alpha_m(1 + \chi_e/3)}. \quad (14)$$

Now we investigate the negative refraction property of this atomic system based on the effect of quantum coherence. In order to obtain the relative permittivity and relative permeability, firstly we should get the coherent terms ρ_{12} and ρ_{43} by solving density matrix equations. However the general solutions for the steady state are tedious, we only present the numerical analysis.

In the following, we consider the case of atomic hydrogen to verify our scheme. In this case, the state $|3\rangle$ ($|4\rangle$) is the exited state $^3D_{3/2}$ ($^3D_{5/2}$), while the state

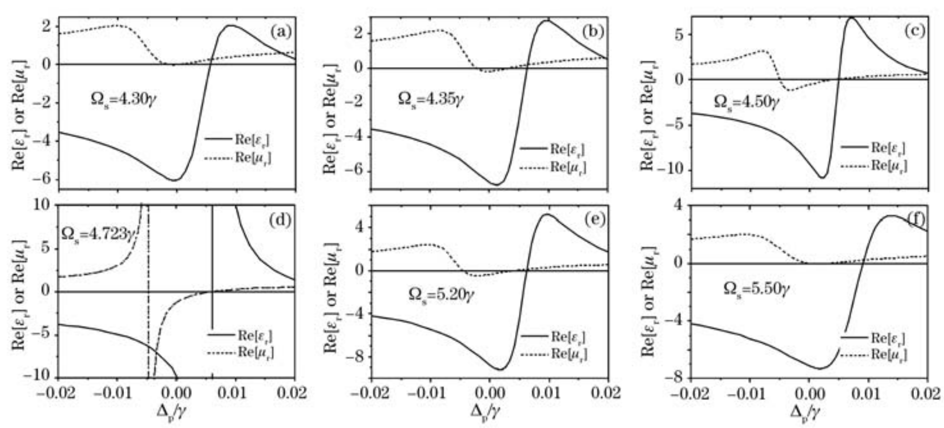

Fig. 5. Re(ε_r) and Re(μ_r) as a function of Δ_p/γ for the parametric conditions: $\gamma_{42} = \gamma_{32} = \gamma_{31} = 0.5\gamma$, $\gamma_{43} = 0.01\gamma$. (a), (b), (c), (d), (e), and (f) are for $\Delta_c = \Delta_s = 0.0065\gamma$, 0.0065γ, 0.0065γ, 0.0075γ, 0.0085γ, 0.01γ, respectively.

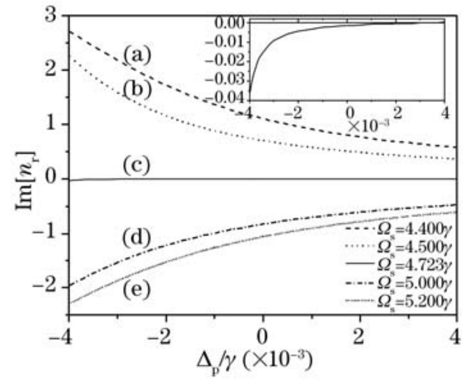

Fig. 6. Imaginary of refractive indexes Im(n_r) as a function of Δ_p/γ for the parametric conditions: $\gamma_{42} = \gamma_{32} = \gamma_{31} = 0.5\gamma$, $\gamma_{43} = 0.01\gamma$. Curves (a), (b), (c), (d), and (e) are for $\Omega_s = 4.40\gamma$, 4.50γ, 4.723γ, 5.00γ, 5.20γ; $\Delta_c = \Delta_s = 0.0065\gamma$, 0.0065γ, 0.0075γ, 0.008γ, 0.0085γ, respectively. The insert shows the less absorption of LHM.

$|1\rangle$ ($|2\rangle$) is the state $^2P_{1/2}$ ($^2S_{1/2}$). The same optical field is used to drive the $|1\rangle \to |3\rangle$ and $|2\rangle \to |4\rangle$ transitions. Then the condition $\omega_c = \omega_s$ is satisfied. In our numerical calculation, all the parameters are scaled by $\gamma_{21} = \gamma = 100$ MHz, which is the spontaneous emission rate of level $|2\rangle$. The wavelength of resonant probe-field transition is 21 cm, which is in the radio-wave range. The pump-field is resonant at 656.2 nm. Typical values of $\gamma_{42} = \gamma_{32} = \gamma_{31} = 0.5\gamma$ and $\gamma_{43} = 0.01\gamma$ are used, in which γ_{ik} $(i = 1-4)$ is the spontaneous emission decay rate from state $|i\rangle$ to state $|k\rangle$. The atomic density is $N = 2.5 \times 10^{25}/\text{m}^3$. The Rabi frequency of the control field is $\Omega_c = 0.1\gamma$. The control-field detuning Δ_c is set to displace the zero of the imaginary part of μ_r into the area of negative refractive index. The probe-detuning dependence of the real and imaginary parts of ε_r and the real part of μ_r are shown in Figs. 5 and 6.

Figure 5 exhibits that the frequency band of negative refraction varies with the Rabi frequency of the pump field Ω_s. The permittivity and the permeability are simultaneously negative when $4.30\gamma < \Omega_s < 5.55\gamma$. With the increase of pump Rabi frequency, the band would increase in the range of $4.30\gamma < \Omega_s < 4.723\gamma$, while decrease if $4.723\gamma < \Omega_s < 5.55\gamma$. In particular, when $\Omega_s = 4.723\gamma$ (show in Fig. 5(d)), the band of LHM reaches maximum to 1.08 MHz. Compared with Ref. [18], the frequency band is enlarged about five times. In the V-scheme considered here, most of atoms distribute not only in level 1, but also in level 3. This produces large coherence between these two levels, which makes it possible to achieve a wide negative refraction frequency band. Therefore, the negative refractive index can be realized in this scheme and the frequency band of LHM can be manipulated by the pump and control fields.

The absorption property can be manipulated by the pump and control field in our system, which is shown from Fig. 6. We find that the probe filed is absorptive if $4.30\gamma < \Omega_s < 4.723\gamma$ and the absorption would decrease with the increase of the pump Rabi frequency; however, if $4.723\gamma < \Omega_s < 5.55\gamma$, the probe field is enhanced and the enhancement would increase with the increase of pump Rabi frequency. Especially when $\Omega_s = 4.723\gamma$, it is shown from the insert in Fig. 6 that the absorption is almost zero in a wide frequency band. For instance, for $\Delta_p = 299.96$ kHz and $\Delta_c = \Delta_s = 750$ kHz, we obtain $\varepsilon_r = -17.3093 - i0.4245$, $\mu_r = -0.4011 + i0.0098$, and an index of refraction of $n_r = -2.6357 - i7.4328 \times 10^{-9}$. The absorption can be omitted. That is to say the low-loss LHM is possible in this scheme.

In conclusion, two schemes have been proposed to realize negative refraction in four-level atomic system. It was shown that the negative refractive index can be achieved based on the effect of quantum coherence in such systems. On the other hand, by analyzing the absorption property of the LHMs, we find that the absoption can be reduced via choosing appropriate parameters. Such schemes might be used to fabricate isotropic and homogeneous LHMs with reduced absorption in a wider optical frequency band. It will have potential applications in improvements of the perfect lens resolution[5], beam focusing[22] and so on.

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