# Wavefront analysis of laser beam shaped by birefringent lenses shaping system 

Xiangtong Yang（杨向通）${ }^{1}$ ，Wei Fan（范 薇）${ }^{1}$ ，Xuechun Li（李学春）${ }^{1}$ ， Zunqi Lin（林尊琪）$)^{1}$ ，Shenlei Zhou（周申蕾）${ }^{1}$ ，Zengyun Peng（彭增云）${ }^{1}$ ， Yaping Dai（戴亚平）${ }^{2}$ ，Jian Zhu（朱 俭）${ }^{2}$ ，and Liansheng Zheng（郑连生）${ }^{3}$

${ }^{1}$ National Laboratory of High Power Laser Physics，Shanghai Institute of Optics and Fine Mechanics， Chinese Academy of Sciences，Shanghai 201800
${ }^{2}$ Shanghai Institute of Laser and Plasma，Shanghai 201800
${ }^{3}$ Shanghai Lianneng Photonics Technology Ltd．，Shanghai 201821


#### Abstract

In order to improve the energy efficiency of high power laser system of ICF and use the optical energy sufficiently，converting Gaussian laser beam into uniform beam is important．Meanwhile，the wavefront of the laser beam in the system is also significant because it affects the imaging quality at image surface and the transfer of image in multilevel magnification laser system．Starting from the Jones matrix of spheric lenses，the intensity transmittance distribution of birefringent lenses beam shaping system has been analyzed by transmission matrix．The wavefronts of different polarized states after transmission through beam shaping system are discussed．The effecting factors，such as the distances between lenses， the lens＇mechanical deviation from optical axis of system，have been considered．The uniform laser beam can be obtained．In the ninth beamline of＇SG II＇device，the static beam filling factor of near field can be improved from $66 \%$ to $80 \%$ by using the birefringent lens shaping system．


OCIS codes： $140.0140,140.3300,080.2730,350.5030$.

Uniform intensity distribution of beam is necessary for high power laser facility used for inertial confinement fusion（ICF）．As for high power laser or amplifier，non－ linear effects，such as B integral，would induce the dam－ age of laser material easily as a result of nonuniform distribution of the incident beam．Also，the high energy efficiency cannot be achieved because of low fill factor of beam．In order to improve the whole efficiency of high power laser system and make the best of energy， converting Gaussian beam into uniform distribution is important．In addition，the wavefront of the laser beam， which affects the quality and transfer of image，is also crucial to multilevel amplified high power laser system． So reducing the wavefront aberration as small as possible is a significant work in high power laser system．

Spatial beam shaping controls and converts distri－ bution of optical field effectively in spatial．Now， there have been developed various spatial beam shap－ ing technologies ${ }^{[1-9]}$ ，such as beam reshaping using holographic filter ${ }^{[1]}$ ，binary rectangular phase gratings applied to spatial shaping of Gaussian beam ${ }^{[2]}$ ，diffrac－ tive phase elements for beam shaping ${ }^{[3]}$ ，etc．．Generally speaking，the methods above have different advantages as well as disadvantages such as low energy efficiency， breakage threshold，concurrence of intensity modulation， and phase aberration etc．，so as not to be used in high power laser system．

We adopt the new spatial beam shaping system used at front end of Lawrence Livermore National Laboratory ${ }^{[10]}$ ． Two pairs of birefringent lenses made of quartz crystal and two polarizers are utilized in system to achieve spa－ tial beam shaping conveniently．Starting from the Jones matrix of spheric lenses in this paper，the intensity trans－ mitting distribution of birefringent lenses shaping system has been analyzed in detail．The wavefronts of different polarized states of the laser beam after transmission through the shaping system are discussed．The effecting factors，such as the distances between lenses，the lens
mechanical deviation from optical axis of system，have also been considered．

Through the experiment，the spatial uniform distribu－ tion of the laser beam can be obtained．When it is used in the ninth beamline of Shenguang II（SG II）device，the static beam filling factor of near field can be improved from $66 \%$ to $80 \%$ ．
Spatial beam shaping system consists of two pairs of birefringent lenses and two polarizers，as shown in Fig． $1^{[10]}$ ．Two pairs of plano－convex－plano－concave lenses are labeled by $\mathrm{L}_{1}, \mathrm{~L}_{2}, \mathrm{~L}_{3}, \mathrm{~L}_{4}$ respectively，the crystal direc－ tion of principal axis and the vibrating direction of beam passing through the polarizer are orthogonal to optical axis of system． $\mathrm{L}_{1}$ and $\mathrm{L}_{4}$ are the two same plano－convex lenses，whose centers are to be half－wave plates，while the edges of effective apertures to be quarter－wave plates． Meanwhile，both lenses are arrayed symmetrically．Sim－ ilarly， $\mathrm{L}_{2}$ and $\mathrm{L}_{3}$ are the two same plano－concave lenses， whose centers are to be quarter－wave plates，while the edges of effective apertures to be half－wave plates．Also， both lenses are arrayed symmetrically．The principal axises of lenses labeled by $\mathrm{L}_{1}$ and $\mathrm{L}_{4}$ are fixed in the $x$ direction，while the principal axes of lenses labeled by


Fig．1．Configuration of spatial beam shaping system．
$\mathrm{L}_{2}$ and $\mathrm{L}_{3}$ are parallel to each other and can be rotated to different angles as a whole.

The incident beam is expanded and collimated, and then polarized by polarizer 1 . Because the two birefringent plano-concave lenses can be rotated, so different positions of beam have different states of polarization after transmitting through the four lenses. Then the laser

$$
M_{i}=\left(\begin{array}{c}
\cos \left[\delta_{i}(r)\right]+i \sin \left[\delta_{i}(r)\right] \cos \left(2 \theta_{i}\right) \\
i \sin \left[\delta_{i}(r)\right] \sin \left(2 \theta_{i}\right)
\end{array}\right.
$$

where $r$ is the radial length from any point in beam aperture to system axis, $2 \delta_{i}(r)$ is the phase delay for ordinary ray and extraordinary ray,

$$
\begin{equation*}
2 \delta_{i}(r)=\frac{2 \pi \Delta n}{\lambda} d_{i}(r), \tag{2}
\end{equation*}
$$

$\Delta n=n_{\mathrm{o}}-n_{\mathrm{e}}$ is the difference of refractive indices between ordinary ray and extraordinary ray, $\lambda$ is the incident wavelength and $d_{i}(r)$ is the thickness of crystal at $r$. Geometrically, one can deduce

$$
\begin{equation*}
d_{i}(r)=d_{i 0}-\rho_{i}\left|1-\sqrt{1-\left(\frac{r}{\rho_{i}}\right)^{2}}\right|, \tag{3}
\end{equation*}
$$

where $\rho_{i}$ is radius of curvature for lens, as for convex lens, $\rho>0$, concave lens, $\rho<0 ; d_{i 0}$ is center thickness of lens. So, the Jones matrix of beam shaping system is given by

$$
\begin{equation*}
M=M_{5} \cdot M_{4} \cdot M_{3} \cdot M_{2} \cdot M_{1} \tag{4}
\end{equation*}
$$

where the Jones matrix of the polarizer is

$$
M_{5}=\left(\begin{array}{ll}
1 & 0 \\
0 & 0
\end{array}\right),
$$

the Jones vector of incident beam after being polarized by polarizer 1 can be written as

$$
\begin{equation*}
E_{\mathrm{in}}=\binom{E_{x}(r)}{0}, \tag{5}
\end{equation*}
$$

the Jones vector of output beam is given by

$$
\begin{equation*}
E_{\text {out }}=M \cdot E_{\text {in }} . \tag{6}
\end{equation*}
$$

Hence the intensity transmissivity of the system can be written as

$$
\begin{equation*}
T(r)=I_{\mathrm{out}} / I_{\mathrm{in}} . \tag{7}
\end{equation*}
$$

The centers of lenses $L_{1}$ and $L_{4}$ are considered to be half-wave plates, while the edges of effective apertures passed by quarter-wave plates, so we have

$$
\left\{\begin{array}{l}
d_{0}=(2 m-1) \lambda /(2 \Delta n)  \tag{8}\\
d_{r_{0}}=d_{0}-\lambda /(4 \Delta n)
\end{array},\right.
$$

geometrically

$$
\Delta n \cdot\left(d_{0}-d_{r_{0}}\right)=\Delta n \cdot\left(\rho-\sqrt{\rho^{2}-r_{0}^{2}}\right)=\lambda / 4,
$$

i.e. $\rho$ can be written as

$$
\begin{equation*}
\rho=2 \Delta n r_{0}^{2} / \lambda+\lambda /(8 \Delta n), \tag{9}
\end{equation*}
$$

beam with different polarization states was polarized by polarizer 2. The spatial distribution of the laser beam was changed.
Suppose the angles between principal axes of lenses $\mathrm{L}_{2}$, $\mathrm{L}_{3}$ and $x$ axis are $\theta$, as shown in Fig. 1, the Jones matrices of transmissivity for single plano-convex lens or plano-concave lens $M_{i}(i=1,2,3,4)^{[11]}$ is given by
$\left.\begin{array}{c}i \sin \left[\delta_{i}(r)\right] \sin \left(2 \theta_{i}\right) \\ \cos \left[\delta_{i}(r)\right]-i \sin \left[\delta_{i}(r)\right] \cos \left(2 \theta_{i}\right)\end{array}\right)$,
similarly, the radii of curvature for lenses $\mathrm{L}_{2}$ and $\mathrm{L}_{3}$ are given by

$$
\begin{equation*}
\rho^{\prime}=-\rho=-2 \Delta n r_{0}^{2} / \lambda-\lambda /(8 \Delta n) . \tag{10}
\end{equation*}
$$

The normalized intensity of incident fundamental transverse mode beam can be written as $I(r)=$ $\exp \left(-2 r^{2} / \omega^{2}\right)$, where $\omega$ is the spot radius. In order to obtain maximal energy efficiency and power, the needed flat top spot radius $r_{0}$ meets

$$
\begin{equation*}
\omega=\sqrt{2} \cdot r_{0} . \tag{11}
\end{equation*}
$$

The wavelength $\lambda$ is 1053 nm , the incident beam aperture is 16 mm and the needed flat top aperture is above 10 mm . By Eq. (11), the flat top beam with the aperture diameter is about 11.3 mm , which meets the demand for project. As for quartz crystal, $\Delta n=0.008747$, the needed radii of curvature for lenses $\mathrm{L}_{1}$ and $\mathrm{L}_{4} \rho=530.36$ mm , center thickness is 3.07 mm at $m=26$ in Eq. (8) with the consideration of fabricating demands, similarly, center thickness of plano-concave lens is 3.04 mm at $m=51$.
From Eqs. (4)-(7), the beam's intensity distribution (see Fig. 2) after being passed by shaping system are simulated numerically by rotating the angle of principal axis for lenses $\mathrm{L}_{2}$ and $\mathrm{L}_{3}$ relative to $x$ axis.
From Fig. 2, it can be seen that the ideal flat top beam (dashed line) is obtained at $\theta=23^{\circ}$, where $\theta$ is the rotated angle of principal axis for lenses $L_{2}$ and $L_{3}$ relative to $x$ axis. The fill factor of beam can reach $94.5 \%$.
By the theory mentioned above, we adopt $1053-\mathrm{nm}$ fiber laser as continuous light source, the experiment setup is shown in Fig. 3.
In the experiment, uniform output beam is obtained at $\theta=31^{\circ}$, compared with theoretic angle $\theta=23^{\circ}$ (see Fig. 4). The larger experimental angle may be caused by less incident beam aperture compared with 16 mm .


Fig. 2. Spatial distribution of normalized output light intensity of the shaped Gaussian beam with different $\theta$.


Fig. 3. Experiment setup.


Fig. 4. One (two) dimension(s) distribution of output light intensity after the spatial filter.

Then, we analyze the wavefront of shaped laser beam using the assistant optical design software ZEMAX ${ }^{[12]}$ developed by Focus Company of USA. Detailedly, different rays are traced to find out their phases delay after passing by the system.

For the first plano-convex lens, the incident rays are extraordinary rays (e rays). For the second and third plano-concave lenses, because of the same rotated angles of principal axises, the incident rays can be the combination of ordinary rays (o rays) and ordinary rays as well as extraordinary rays and extraordinary rays. When the rays arrive at the last plano-convex lens, ordinary rays and extraordinary rays exist in crystal simultaneously. In this way, there are four wavefronts of different combined polarization states in system, i.e. $e+e+e+e, e+e+e+o$, e $+o+o+e, e+o+o+o$.

The effect factors on the wavefront of shaped laser beam are discussed in the following. As can be seen in Fig. 5, if the plano-convex and plano-concave lenses are agglutinated with no space between each other, the system can be viewed as two flat plate, which has little influence on the wavefront of the laser beam. But mechanically, it is difficult to achieve. And it was found that the space between plano-convex and plano-concave lenses has great influence on the wavefront of the shaped laser beam. The space between plano-convex and planoconcave lenses in one pair is the same as the other. The space between the two pairs of lenses is fixed. The space between plano-convex and plano-concave lenses in one pair is altered. Then, the marginal ray of meridional plane by ZEMAX is traced, and the changes of phases delay for the four combined polarization states $e+e+e+e$, $\mathrm{e}+\mathrm{e}+\mathrm{e}+\mathrm{o}, \mathrm{e}+\mathrm{o}+\mathrm{o}+\mathrm{e}, \mathrm{e}+\mathrm{o}+\mathrm{o}+\mathrm{o}$ can be achieved by altering the space between lenses in one pair, as shown in Fig. 6.

Compared with the optical path of the ray passing through optical axis of system, the most optical path difference (OPD) values of different combined polarization states wavefronts are negative with the space between plano-convex and plano-concave lenses in one pair changed from 1 to 6 mm . They are all converging wavefronts, whose aberrations become larger with in-
creasing the space. Only the OPD value of the combined polarization state $e+e+e+o$ is positive, the wavefront is divergent, and the aberration of wavefront becomes smaller with increasing the space.
Tracing the rays of combined polarization states $e+e+e+e$ or $e+e+e+o$, we see that the polarized direction of ordinary rays in output beam are vertical to $x$ axis.
When the output laser beam arrives at polarizer 2, the ordinary rays cannot pass it because the optical axis of polarization 2 follows $x$ axis, i.e. the wavefronts of combined polarization states $\mathrm{e}+\mathrm{e}+\mathrm{e}+\mathrm{o}$ and $\mathrm{e}+\mathrm{o}+\mathrm{o}+\mathrm{o}$ make no contribution to the ultimate wavefront. So, we only consider the other two combined polarization states $e+e+e+e$ and $e+o+o+e$. Furthermore, we can obtain the weight factors of energy for the wavefronts $e+e+e+e$ and $\mathrm{e}+\mathrm{o}+\mathrm{o}+\mathrm{e}$, as is $1: 0.03$ by polarization analysis in


Fig. 5. Effect on output beam's wavefronts by spaces between lenses.


Fig. 6. Changes of phase delay for different polarization states.

ZEMAX under the condition of no antireflective coating.
Then the space between plano-convex and planoconcave lenses in one pair is fixed closely, such as 1 mm , while altering the space $L$ between the two plano-concave lenses, it can be seen that the altered space makes little difference to wavefront aberration.

As shown in Fig. 7, suppose the center axes of collimated beam is reference optical axis, we discuss the effect on wavefront by lens' deviation to optical axis of system in the $x$ direction. It is obvious that output wavefront deviation will deteriorate as a result of lens' deviation, and the focus will shift in the $x$ direction. Generally, beam deviation angle is often used to define the deviating degree to optical axis of system. If $h$ is the deviating distance of the focus to optical axis of system, $f$ is the focal length, and $\theta$ is the beam deviation angle, then we have $\tan \theta=\frac{h}{f}$. Suppose one lens deviates to optical axis of system while other lenses have no deviations. The beam deviation angle will alter with the lens' deviating distance $\delta$ to optical axis of system, as shown in Fig. 8. From Fig. 8 we can see that the beam deviates upwards relative to optical axis of system with plano-convex lens' deviating in positive direction of $x$ axis, and the beam deviation angle alters $10 \mu \mathrm{rad}$ when plano-convex lens' deviating distance changes every 0.01 mm , contrarily, an opposite situation happens to plano-concave lens. In addition, the focal lengths of output wavefronts keep unchangeable when the deviating distances $\delta$ of lenses become larger. So, there are more needs in mechanism for this beam shaping system. The space between lenses in


Fig. 7. Effect on output wavefront by lens deviation to optical axis of system.


Fig. 8. Beam deviation angle alters with lens' deviating distance to optical axis of system.


Fig. 9. Transmitted wavefront.
one pair of system and the deviating distance to optical axis of system in $x$ direction for different lenses have great effect on aberrations of different wavefronts, contrarily, the rotated angle of plano-concave lens has no effect on aberrations of wavefronts.

Through the theoretical analysis above, the shaping system was designed compactly. The distance between plano-convex and plano-concave lenses is fixed at 1 mm and the distance between two plano-concave lenses at 20 mm . After being measured by collimating device, the change of beam deviation angle is $9^{\prime \prime}$. The $\mathrm{P}-\mathrm{V}$ value of wavefront aberration is $0.080 \lambda$ after testing the transmitted wavefront by ZYGO interferometer, as shown in Fig. 9.

In conclusion, in order to improve the energy efficiency of ICF high power laser system and use the optical energy sufficiently, converting Gaussian laser beam into uniform beam is important. The output wavefront of system is also significant, it affects the imaging quality at image surface and the transfer of image at multilevel magnification laser system. Starting from the Jones matrix of spheric lenses, the intensity transmittance distribution of birefringent beam shaping system has been analyzed in detail. The wavefronts of different polarized states after passing by the system are discussed, the effecting factors on the output wavefronts have been considered. We obtained the uniform beam by using this birefringent lenses beam shaping system. When used in the ninth beamline of 'SG II' device, the static beam filling factor of near field can be improved from $66 \%$ to $80 \%$. Furthermore, this system has more practical value in project because of its convenient design, close configuration, and low expenditure. But it also has more needs in mechanical fixing of lens.

The authors acknowledge the supports of Professor Xijie Cai and Engineer Songqing Xu for this work. X. Yang's e-mail address is yxt1108@163.com.

## References

1. C.-Y. Han, Y. Ishii, and K. Murata, Appl. Opt. 22, 3644 (1983).
2. X. Zhang, L. Dong, and B. Zhang, Laser Technology (in Chinese) 19, 74 (1995).
3. X. Tan, B.-Y. Gu, G.-Z. Yang, and B.-Z. Dong, Appl. Opt. 34, 1314 (1995).
4. J. Liu and B.-Y. Gu, Appl. Opt. 39, 3089 (2000).
5. P. W. Rhodes and D. L. Shealy, Appl. Opt. 19, 3545 (1980).
6. C. Wang and D. L. Shealy, Appl. Opt. 32, 4763 (1993).
7. X. Tan, B.-Y. Gu, G.-Z. Yang, and B.-Z. Dong, Appl. Opt. 37, 747 (1998).
8. W. W. Simmons, G. W. Leppelmeier, and B. C. Johnson, Appl. Opt. 13, 1629 (1974).
9. H. Chen, Z. Sui, Z. Chen, B. An, and M. Li, Acta Opt. Sin. (in Chinese) 21, 1107 (2001).
10. B. M. Van Wonterghem, J. T. Salmon, and R. W. Wilcox, "Beamlet pulse-generation and wavefront-control system" UCRL-LR-105821-95-1.
11. Y. Ye, B. Lü, and B. Cai, Laser Technology (in Chinese) 20, 324 (1996).
12. J. M. Geary, Introduction to Lens Design with Practical ZEMAX Examples (Willmann-Bell Inc., Virginia, 2002).
