Methods for enhancing optical heterodyne beat SNR in absolute frequency measurement using optical frequency comb

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The signal-to-noise ratio (SNR) of the optical heterodyne beat between a low-power laser and an optical frequency comb with a relatively low mode power is often insufficient to exclude the possibility of frequency counter error. We develop some methods to enhance this beat SNR including the adoption of a fiber coupler, the suppression of the shot noise, and the utilization of Ti:sapphire laser output. We obtain the highest SNR ever reported with similar laser systems by these methods. With sufficiently enhanced SNR, we measure the absolute frequency of some optical frequency standards without counter error. We give some experimental criteria of beat SNR for the correct frequency count.

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Recent development of the octave-spanning femtosecond optical frequency comb, which can be directly linked to the microwave frequency standard, has made the direct one-step measurement of optical frequency possible without the employment of the complicated traditional frequency chains^[1,2]. However, due to the low power</sup> level of the single mode of the spectrally broadened optical comb (typically, less than several tens of nanowatt), the signal-to-noise ratio (SNR) of the optical heterodyne beat with low-power lasers is insufficient for the correct frequency counting in some cases. The absolute frequency measurements of these lasers, accordingly, were performed by use of rather indirect experimental methods; the tracking oscillator^[3] and the high-power buffer laser^[4], for example. The frequency measuring system becomes complex with these indirect methods. Moreover, we must check if the additional frequency noise, drift or offset does not take place. The tracking oscillator is hard to track a large modulation in some cases and also it must be checked if there is no cycle slip. If we enhance the SNR of beat signal, the frequency counter error can be removed without the need of the complicated indirect measurement system.

Several optical frequency standards are being operated in the Korea Research Institute of Standards and Science (KRISS) including an iodine-stabilized He-Ne laser at 633 nm (KRISS-R061), an acetylene-stabilized diode laser at 1542 nm (KRISS-Ace1), and a rubidium-twophoton-stabilized diode laser at 778 nm for the national length standards. There had been a problem in measuring the absolute frequencies of these standards, because the SNR of the beat signal between these and the optical comb mode had been limited to 20 - 25 dB, which had been rather insufficient to confirm that there was no frequency counter error. Thus, we have developed some experimental methods to enhance the SNR of the heterodyne beat signal. For the first method, we have adopted a fiber coupler to ensure the spatial overlap and the wave front matching. As the second method, we have detected the beat signal at a sufficiently long distance from a grating for spectral filtering to reduce the shot noise induced by the adjacent comb modes. As the final method, we have used the Ti:sapphire laser output for the optical heterodyne beat to make use of the higher power per comb mode.

We could obtain the beat signal with an enhanced SNR approaching 40 dB, which is quite sufficient for the direct frequency counting without any counter error. The SNR is improved by more than 15 dB compared with that of conventional method. We have successfully measured the absolute frequencies of the KRISS-R061 and the KRISS-Ace1 utilizing the enhanced SNR. Experimental criteria of the SNR for the correct frequency counting are analyzed.

Several national optical frequency standards have been developed in KRISS, which are recommended by the International Committee of Weights and Measures (CIPM) as practical means to realize the meter^[5]. These include iodine-stabilized He-Ne lasers at 633 nm, acetylene-stabilized diode lasers at 1542 nm, and rubidium-two-photon-stabilized diode lasers at 778 nm.

The He-Ne laser, stabilized to a saturated absorption line of iodine molecule at 633 nm, is the most widely used optical frequency standard for the practical meter realization in many national metrological institutes and is frequently used to calibrate laser interferometers for length measurement. The CIPM recommended the frequency of the *f*-component of the ¹²⁷I₂ 11-5, R(127) line to be 473 612 353 604(10) kHz^[5]. This laser has the low output power (typically, less than 100 μ W) and the large frequency modulation (6 MHz). Thus, the SNR of the beat signal with a comb mode has been limited to about 20 dB in a 300-kHz bandwidth, which can lead to a frequency counter error.

The two-photon transition line at 778 nm of rubidium atom (⁸⁵Rb), $5S_{1/2}(F_{\rm g}=3) - 5D_{5/2}(F_{\rm e}=5)$ was recommended by the CIPM to have the frequency of 385 285 142 375(5) kHz^[5]. This wavelength standard has importance in length metrology as well as in the optical communication band via the link of second harmonic generation. The frequency standards around 1.5

 μm have attracted strong interests as a frequency reference values in the optical communication band. Thus, in 2001, the CIPM recommended the transition frequency of the P(16) line of acetylene molecule $({}^{13}C_2H_2)$ at 194 369 569.4(1) MHz as a method for the practical realization of the definition of the meter^[5]. This recommended value has been revised to be $194\ 369\ 569\ 384(10)$ kHz in $2003^{[6]}$. The absolute frequency of a diode laser system, stabilized to a saturated absorption line of acetylene molecule at 1542 nm, can be measured by a Ti:sapphire-laser-based comb utilizing the second harmonic generation (771 nm). But, an octave-spanning comb has relatively low output power around 771 nm, because a large part of the optical power is transferred to the 532 nm and the 1064 nm ranges for the measurement of the carrier envelope offset frequency. The absolute frequency measurement at 778 nm has the similar problem. Thus, the SNR of the beat signal with a comb mode has been limited to about 25 dB in a 300-kHz bandwidth, which is somewhat insufficient to confirm that there was no frequency counter error.

The experimental setup for the absolute frequency measurements is shown in Fig. 1. We have developed three experimental methods of enhancing the beat SNR for the absolute frequency measurement of the KRISS optical frequency standards without counter error. The first one is the adoption of a 2×2 single-mode fiber coupler (FC) (method-I). This automatically ensures the spatial overlap and the wave front matching. The second one is the reduction of the shot noise by adjacent comb modes that does not contribute to the optical heterodyne beat signal (method-II). This is accomplished by detecting the beat signal at a sufficiently long distance from a grating, which acts as a spectral filter with a narrow bandwidth. Figure 2 shows the measured SNR of the heterodyne beat with a comb mode as a function of the distance from the grating to the avalanche photodiode (APD). The beat signal was obtained using an APD with the help of a vertically focusing cylindrical lens (f=4 cm). The SNR was remarkably increased as the distance from grating increased. The power of the laser to be measured and the power of the comb mode were 8.5 μ W and 4.2 nW at the position of APD, respectively. The power of the comb mode was estimated using the total comb power after FC, the transmittance of the intermediate optics, and the spectrum obtained using an optical spectrum analyzer. When the radio frequency power and the SNR were measured, resolution bandwidth (RBW) of the spectrum analyzer was 300 kHz and the video bandwidth (VBW) was 1 MHz



Fig. 1. Schematic of experimental setup for absolute frequency measurement.

throughout all the experiments in this article. The analytical expressions for the dependence of the SNR on the distance from the grating will be reported elsewhere^[7]. The last one is the use of the original Ti:sapphire laser output, which has higher power per comb mode than the spectrally broadened supercontinuum by a photonic crystal fiber (PCF) (method-III). This can be utilized in 771 and 778 nm cases, as shown in Fig. 1, utilizing the spectral coverage of the Ti:sapphire laser output.

The SNR of the beat between a comb mode and an iodine-stabilized He-Ne laser, named KRISS-R061, is improved by more than 15 dB compared with that of conventional method as is shown in Fig. 3. This result was achieved by utilizing method-I and method-II. The distance from the grating was 2600 mm and the beat SNR approaching 35 dB could be obtained. The two laser beams were coupled using FC. All the fiber coupler terminals are angled polish connectors (FC/APC). The polarization of each laser was controlled by a respective half-wave plate. A 40-dB optical isolator is used to prevent the back-reflection from the fiber coupler



Fig. 2. Measured SNR of the heterodyne beat with a comb mode as a function of the distance from the grating to the APD.



Fig. 3. SNR enhancement of 633-nm heterodyne beat. (a) By a conventional method, (b) by method-I and method-II.

to the He-Ne laser, which is very sensitive to the optical feedback. We measured the absolute frequency of KRISS-R061 stabilized on the f-component of the $^{127}I_2$ 11-5, R(127) line utilizing this beat signal with the enhanced SNR. KRISS-R061 has the modulation depth of 6 MHz, the modulation frequency of 8.3 kHz, the cold finger temperature of 15.0 °C, and the one-way intracavity beam power of 10 mW. The comb is provided by a commercial octave-spanning optical frequency generator (Menlosystems GmbH, FC8004) based on a femtosecond Ti:sapphire laser and a PCF. Both the pulse repetition rate (202 MHz) and the carrier-envelope-offset frequency (20 MHz) are phase-locked to a reference signal of 10 MHz from a hydrogen maser, which is linked to the SI second via a global positioning system. The frequency counter (Agilent, 53132A) is referenced to the hydrogen maser. A 70-MHz low-pass filter and a 26-dB low-noise RF amplifier were used before the input of the counter. The absolute frequency of KRISS-R061 could be measured with no counter error. The frequency offset from the value recommended by the CIPM was measured to be 5.90(0.33) kHz, which is well with in the recommended uncertainty of 10 kHz. The Allan deviation of KRISS-R061 was 7.4×10^{-12} at 1 s and was inversely proportional to the square root of average time.

Also we obtained the beat signal between a frequencydoubled acetylene-stabilized laser, named KRISS-Ace1, at 771 nm and a mode of an optical frequency comb with an SNR approaching 40 dB, as shown in Fig. 4. This result was possible by the use of method-I, method-II, and method-III, that is sufficient to ensure that there was no counter error. We measured the absolute frequency of KRISS-Ace1 utilizing this beat signal with the enhanced SNR. The experimental detail for the absolute frequency measurement is nearly same as in the above KRISS-R061



Fig. 4. Optical heterodyne beat signal between acetylenestabilized laser and optical comb. (a) Using photonic crystal fiber output, (b) using Ti:sapphire output.

case. KRISS-Ace1 is an extended cavity laser diode (ECLD) locked to the saturated absorption peak of the P(16) ($\nu_1 + \nu_3$) transition of ${}^{13}C_2H_2$. The power of KRISS-Ace1 was amplified to 100 mW using an erbiumdoped fiber amplifier (EDFA) and the frequency was doubled using a fiber-pigtailed waveguide-periodically-poled lithium niobate (PPLN). The frequency-doubled output from the waveguide-PPLN had a Gaussian-profiled crosssection and a power of approximately 7 mW. The details of KRISS-Ace1 are reported elsewhere^[8]. The frequency counter (from the company of K+K) is referenced to the hydrogen maser. A band pass filter centered at 30 MHz with the bandwidth of 10 MHz was used before the input of the counter. The measured absolute frequency of KRISS-Ace1 is 194 369 569 385.96 kHz with an uncertainty of 0.30 kHz, based on measurements of a single system. Including the uncertainty due to the residual offset, the uncertainty was 1.9 kHz. The frequency offset from the value recommended by the CIPM was 1.96 kHz, which is well within the recommended uncertainty of 10 kHz. The Allan deviation of KRISS-Ace1 was 4.2×10^{-12} at 1 s and was inversely proportional to the square root of average time^[8].

With the sufficiently enhanced beat SNR, we quantitatively analyzed the experimental criteria of the beat SNR for the correct frequency counting. As, in general, the counter performance is a function of SNR, the signal power level, and the integrated noise over the input bandwidth of the frequency counter, these criteria for the correct frequency count can be changed under different experimental conditions. However, the procedure introduced here would be helpful to confirm the correct frequency count. Figure 5 shows the result of the frequency counter reading as a function of beat SNR. The beat frequency was set to be about 51 MHz by controlling the comb repetition rate. The counter model was Agilent 53132A, with a 70-MHz low-pass filter. The SNR was varied with the noise power level maintained at (44 ± 1) dBm at the counter input by adjusting the fiber coupling efficiency of the He-Ne laser and the corresponding half-wave plate. The error bars in Fig. 5 represent the standard deviations of the measured beat frequency at the gate time of 1 s. The counter error starts to appear at the SNR of 28 dB. We conservatively conclude that the SNR should be more than 30 dB in RBW of 300 kHz for the frequency counting without errors in this specific case. Figure 6 shows the result of the frequency counter



Fig. 5. Counter frequency reading as a function of beat SNR (Agilent 53132A, with a 70-MHz low pass filter).



Fig. 6. Counter frequency reading as a function of beat SNR (K+K counter, with a band pass filter centered at 30-MHz).

reading as a function of beat SNR with the counter manufactured by the company of K+K. A band-pass filter centered at 30 MHz with the bandwidth of 10 MHz is used. The beat frequency was set to be about 31 MHz by controlling the comb repetition rate. The SNR was varied with the noise power level maintained nearly the same. The error bars in Fig. 6 represent the standard deviations of the measured beat frequency at the gate time of 1 s. The counter error starts to appear at the SNR of 20 dB in this specific case.

In conclusion, we have developed some methods for enhancing the heterodyne beat SNR to measure the absolute frequency of the optical standard without counter error. With these methods, we could obtain SNR of 34 dB with a low-power iodine-stabilized He-Ne laser and SNR of 38 dB with a frequency-doubled acetylene-stabilized laser. These are the highest SNR ever reported with similar laser systems. Using this enhanced SNR, we successfully measured the absolute frequencies of the optical fre-

quency standards at 633 and 1542 nm. The measured absolute frequencies are well within the uncertainty ranges recommended by the CIPM. Experimental criteria of the SNR for the correct frequency counting are analyzed. Although these criteria for the correct frequency count can be changed under different experimental conditions, the procedure introduced here would be helpful to confirm the correct frequency count. We expect that the experimental method for enhancing beat SNR introduced in this article will be useful in the absolute frequency measurement of a low-power laser by an optical frequency comb with a relatively low mode power.

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References

- Th. Udem, J. Reichert, R. Holzwarth, and T. W. Hänsch, Phys. Rev. Lett. 82, 3568 (1999).
- S. A. Diddams, D. J. Jones, J. Ye, S. T. Cundiff, J. L. Hall, J. K. Ranka, R. S. Windeler, R. Holzwarth, Th. Udem, and T. W. Hänsch, Phys. Rev. Lett. 84, 5102 (2000).
- T. H. Yoon, J. Ye, J. L. Hall, and J. -M. Chartier, Appl. Phys. B 72, 221 (2001).
- L.-S. Ma, M. Zucco, S. Picard, L. Robertsson, and R. S. Windeler, IEEE J. Sel. Top. Quantum Electron. 9, 1066 (2003).
- 5. T. J. Quinn, Metrologia 40, 103 (2003).
- 6. R. Felder, Metrologia 42, 323 (2005).
- W.-K. Lee, D.-S. Yee, and H. S. Suh, Appl. Opt. 46, 930 (2007).
- H. S. Moon, W. K. Lee, and H. S. Suh, IEEE Trans. Instr. Meas. 56, 509 (2007).