Measurement of Gouy phase shift by use of supercontinuum spectral interference

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We present a simple method of the spectral interference between two white light continuum spectra to measure the Gouy phase shift of the focused femtosecond laser pulses. From the spectral interference signal, the relative phase is extracted by the Fourier transform method. The minimum spot size (beam waist) of the laser pulses after the lens (focal length $f = 200$ mm) is measured by imaging method. According to the beam radius, the curves of nonlinear fitting are drawn for comparison, which fit the data of experiments very well. The quantity of phase shift is also discussed. The result shows that this method of measuring Gouy phase shifts is available and stable.

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It is known that an electromagnetic beam propagating through a focus experiences an additional π phase shift with respect to a plane wave. This phase anomaly was discovered by Gouy in 1890[1] and has since been referred to as the Gouy phase shift. In the field of high field laser physics, because of the high power density in the interaction area, the beam must be focused and the interaction between laser pulses and matters happens round of the focus point. So, the Gouy phase shift, the extra axial phase increment of focused beams, plays an important role in the modern optics. It accounts for the different frequencies of transverse modes in laser resonators[2]. In nonlinear optics the efficiency of high-order harmonic generation with focused beams is affected by the Gouy phase shift as well[3].

Intuitive explanations of the observed anomaly have been proposed[4,5], i.e., Chow *et al.* employed spatial mode interference frequency-locking error signals to measure the Gouy phase evolution of first-order Hermite-Gaussian spatial modes[6]. Recently, two groups measured the carrier-envelope phase (CEP) shift of laser pulses evolving through a focus induced by the geometrical Gouy phase[7,8]. From their results, we can find that, thanks to the spatial confinement to the laser beam, the CEP changes when the laser pulses produced by a phase-stabilized laser amplifier system propagate from $-\infty$ through the focus point ($z = 0$) to $+\infty$. This variation is of critical importance for any application of ultrashort laser pulses, including high harmonic and attosecond pulse generation[9], as well as phase-dependent effects.

In principle, the Gouy phase shift of a TEM_{00} wave can be described by a simple formula depending on the focusing geometry and wavelength,

$$\phi(z,\lambda) = -\arctan\left[\frac{z}{z_{\text{R}}(\lambda)}\right], \tag{1}$$

where the beam is travelling in the $+z$ direction and z_{R} is the Rayleigh distance (dependent on the wavelength λ). As few-cycle pulses consist of broad spectra with the wavelengths spanning about one octave, the different spectral component corresponds to different Rayleigh distance. So, round the focus point, the variation of the entire pulse could be very complex.

In this letter we present a new experimental method to measure the Gouy phase shift. From the spectral interference (SI) of the supercontinuum spectrum which was produced by the sapphire, we got the phase shift of laser pulses propagating through the focus point. In the experiment, we measured the radius of the laser beam at the focused point, the fitting curve of the Gouy phase shift proved the stabilization and feasibility of this method.

The experimental layout is shown in Fig. 1. The energy of 50-fs full width at half maximum (FWHM) laser pulses at 800 nm with a 1-kHz repetition rate was adjusted by the adjuster consisting of a half-wave plate at 800 nm and an polarizer. About 10 μJ of the beam was split to A and B ways. Both of them were focused to 3-mm-thick sapphire crystals by two 200-mm focal-length lenses. As the density of energy was higher than the self phase modulation (SPM) threshold of sapphire crystals, we got the supercontinuum spectrum shown as Fig. 2, the width was 450−900 nm. The output pulses were collected by 100-mm focal-length lenses. The SI signal was measured by a spectrometer (Acton Research, 300i) with an intensified charged-coupled device detector (Princeton Instruments).

In the A way, the 3-mm-thick sapphire crystal was fixed on a three-dimensional adjustment frame, which had 0.001-mm adjustment precision. We continuously shifted the location of the sapphire crystal from the front of the focus point to the back by adjusting frame in the z direction.

In the A way, the electromagnetic field of the output

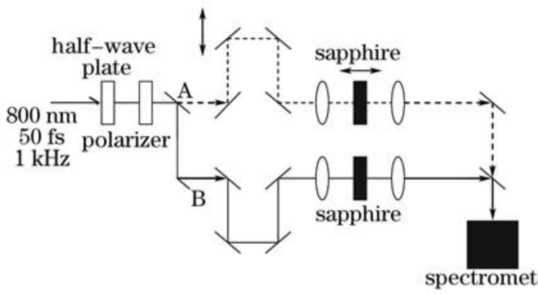

Fig. 1. Experimental setup.

laser from the sapphire crystal could be shown as

$$E_1(t) = A_1(t)\cos[\omega_1 t - kz_1 + \Delta\phi_1 + \phi(z,\lambda)]. \tag{2}$$

In the B way, the electromagnetic field of the same frequency could be shown as

$$E_2(t) = A_2(t)\cos(\omega_1 t - kz_2 + \Delta\phi_2). \tag{3}$$

$A_1(t)$ and $A_2(t)$ were the amplitudes of the A and B ways, ω_1 was the angle frequency, k was the wave vector; z_1 and z_2 were the optical distances of the A and B ways, and $\phi(z,\lambda)$ was the Gouy phase shift. $\Delta\phi_1$ and $\Delta\phi_2$ came from other transmit effects, so they could be treated as constants. The SI signal which was measured by the spectrometer could be shown as

$$\begin{aligned} I &= |E_1(t) + E_2(t)|^2 \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos[k(z_2 - z_1) + \Delta\phi_1 \\ &\quad -\Delta\phi_2 + \phi(z,\lambda)], \end{aligned} \tag{4}$$

I_1, I_2 were the light intensities of A and B ways. When the distance between the sapphire and the 200-mm focal-length lens was a fixed value, $\phi(z,\lambda)$ was a constant. Adjusting the delay, we got the interference signal on the spectrometer, as shown in Fig. 3.

With the location variation of the sapphire crystal in the A way at the z direction, the front and back position of the beam waist interacted with the sapphire crystal, and the Gouy phase shift subjoined on the output pulses. The spectrum of SI signals at 650 nm was shown in Fig. 4.

Continuously adjusting the location of the sapphire crystal, we measured the interference signal with an interval of 0.1 mm around the wave length of 650 nm. The phase fluctuations were analyzed, and the relative phases were extracted from all recorded SI fringes. In the experiment, we made an 1-mm front-back movement and got the Gouy phase shift at the wave length of 630 and 670 nm, shown as Fig. 5.

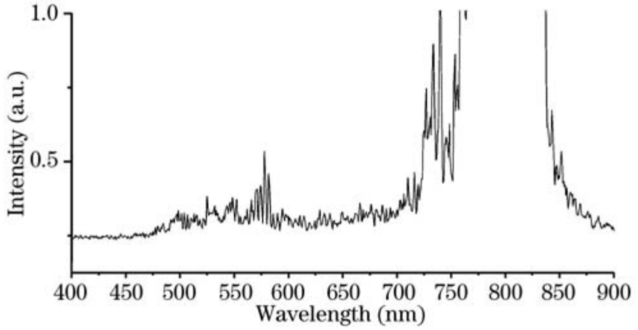

Fig. 2. Spectra of the supercontinuum generated by nonlinear propagation in the sapphire crystal.

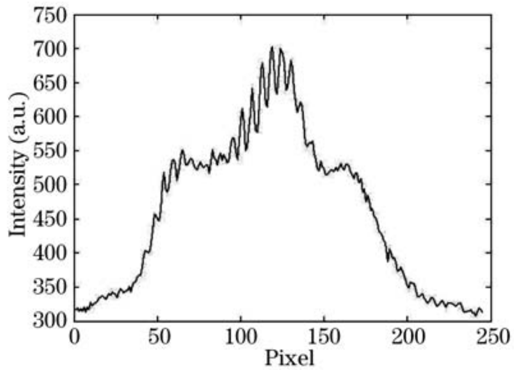

Fig. 3. Interference signal on the spectrometer.

In Fig. 5, the curve was fitted according to $y = -\arctan\left(\dfrac{x}{P_1}\right)$, and $P_1 = \dfrac{\pi\omega_0^2}{\lambda}$, where ω_0 denotes beam waist. In the experiment, we measured the radius of the

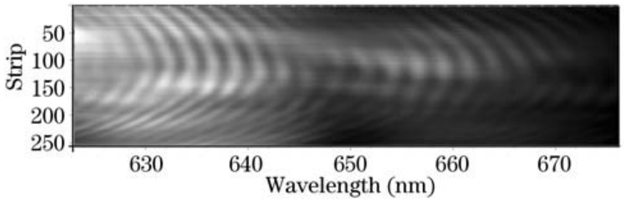

Fig. 4. Spectrum of SI signals round 650 nm.

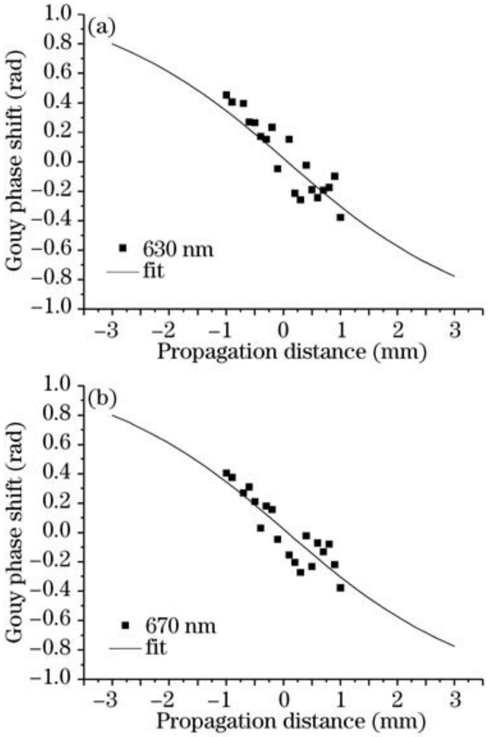

Fig. 5. Gouy phase shift at the wavelengths of 630 and 670 nm.

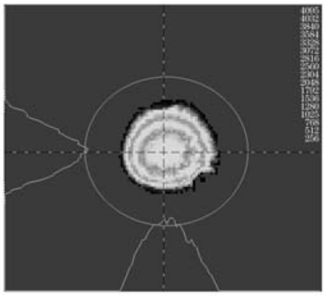

Fig. 6. Image of the focus point on the CCD.

beam at the focus point, which was focused by the 200-mm focal-length lens. As shown in Fig. 6, we got the image zoomed in 20 times. The radius of the image was 0.994 mm, the radius of the beam $2\omega_0 = 49.7$ μm. As shown in Fig. 5, the curve fit the experimental points very well and the phase shift was 0.646 rad.

In conclusion, we have measured the Gouy phase shift which can be described by parameters depending on the focusing geometry and wavelength with a simple method. This result shows the effect of the Gouy phase shift in the interaction between few-cycle laser pulses and matter and provides a new experimental method to measure the Gouy phase shift.

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