Space debris detection with space-based avalanched photodiode laser radar

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Space debris distribute in a large range in the space, and their size are very different. For the purpose of detecting the small space debris, the space-based laser radar is used. The characteristics of the background radiation in space are analyzed at first. Then, according to the analog and digital detection theory of laser radar, with avalanched photodiode as the detector, the minimal detectable laser powers are calculated. Based on the radar equation, the detection ability of laser radar is got. The result indicates that the space-based laser radar is effective to detect the small space debris in a specific range.

OCIS codes: 280.3640, 010.3640, 030.5290.

Along with the progress of the space flight, more and more space debris are made and threatening the spacecraft. The space debris distribute in the large range in the space, and their size are very different. The big space debris are detected by the ground-based detection system, including radar and optical system. And the space-based detection system would be used for the purpose of detecting the small space debris[1−3]. In the article, the detection of small space debris with space-based laser radar is studied. At first, the characteristics of the background radiations in space are analyzed. Then, according to the analog and digital detection theory of laser radar, the minimal detectable laser powers are calculated respectively. At the end, based on the radar equation, the detection ability of laser radar is got. The result indicates that the space-based laser radar is effective to detect small space debris in a specific range.

By setting a spatial filter before the detector, the background radiation to the space-based laser radar only comes from the reflection of the conceivable sources by the space debris. The sources include the sun, the earth, the stars in the deep space and the reflection of the sun radiation by aerosphere and the moon[4]. According to the Plank's formula we know that only the direct and reflected radiation by aerosphere of the sun radiation has to be taken into account if we select near infrared (NIR) detectors, for example, Si-avalanched photodiode (APD), in the laser radar system. The aerosphere reflection of sun radiation is approximately equal to 30% of the sun direct radiation. According to the sun constant, the total background irradiance on space debris is about 1760 W/m^2, and it is far bigger than the irradiance power from the laser radar. So, the optical filter would be used at the receiver of the laser radar. The optical filter can be treated as being set at space debris equivalently. The background irradiance on space debris after optical filtering is showed in Fig. 1, and can be expressed by photon irradiance form as Fig. 2.

From Figs. 1 and 2, we can see that the optical filter with longer central wavelength can restrain the background irradiance more effectively. So the laser with long wavelength, for example, NA:YAG laser, would be used in the laser radar.

Space debris can be seen as a Lombard body, so its laser radar scattering section is

$$\sigma = \rho A_0/(2\pi), \tag{1}$$

where ρ is space debris' reflectance, and A_0 is its area. The background irradiance E_b and photon irradiance E_pb at the laser radar receiver are calculated as

$$\begin{aligned} E_\mathrm{b} &= \frac{E_\mathrm{b0}\sigma}{4\pi R^2}\eta_\mathrm{r} = \frac{E_\mathrm{b0}\rho A_0}{2\pi R^2}\eta_\mathrm{r}, \\ E_\mathrm{pb} &= \frac{E_\mathrm{pb0}\sigma}{4\pi R^2}\eta_\mathrm{r} = \frac{E_\mathrm{pb0}\rho A_0}{2\pi R^2}\eta_\mathrm{r}, \end{aligned} \tag{2}$$

where R is the distance between space debris and laser radar, η_r is the optical transmission coefficient of laser

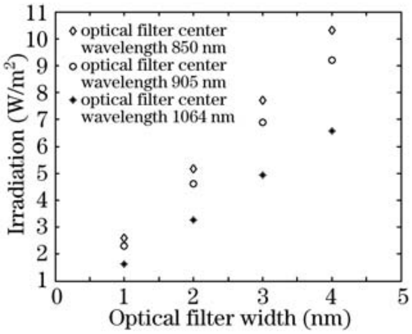

Fig. 1. Background irradiation on space debris after optical filtering.

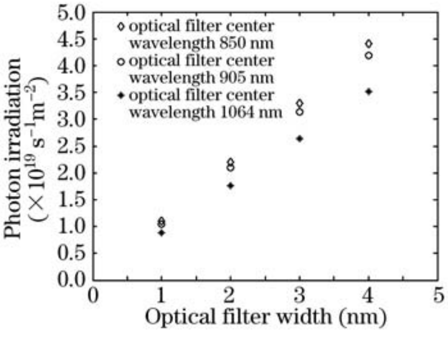

Fig. 2. Background photon irradiation on space debris after optical filtering.

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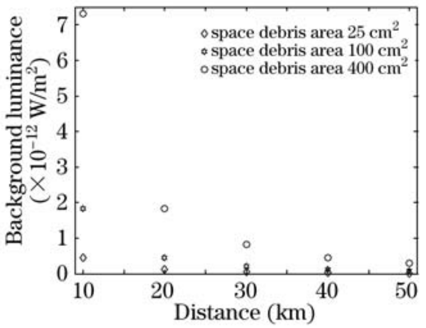

Fig. 3. Background luminance at lidar receiver after filtering.

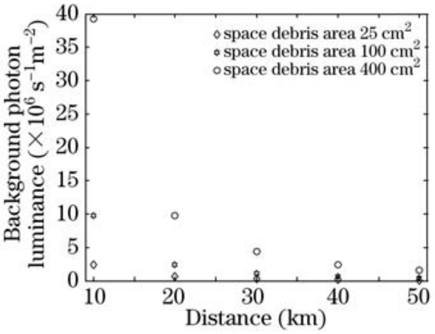

Fig. 4. Background photon luminance at lidar receiver after filtering.

radar receiver, E_{b0} and E_{pb0} are the background irradiance and photon irradiance at space debris respectively. It is provided that the optical filter's central wavelength is 1064 nm, and its full-width at half-maximum (FWHM) is 1 nm. The corresponding background irradiance and photon irradiance at space debris are 1.64 W/m^2 and 8.8×10^{18} s^{-1}m^{-2} respectively. Then according to Eq. (2) we can get the background irradiance and photon irradiance at laser radar's receiver as Figs. 3 and 4.

When the object is detected with laser radar, the false alarm probability and the detection probability are given usually. We can decide the threshold according to the given false alarm probability firstly, and then decide the minimal detectable laser energy according to the threshold and the given detection probability. Because the statistical characteristics of the noise and signal are different between the analog and digital detection modes, the detection is studied respectively.

When the laser radar is working under the analog mode, the noise and signal can be regarded as continuous random variables. The noise current is approximately a Gaussian distributed random variable, and it can be written as[5]

$$P_{\mathrm{n}}(I) \approx \mathrm{Normal}[R_I G E_{\mathrm{b}} + 2I_{\mathrm{d}} e B \left(R_I E_{\mathrm{b}} + I_{\mathrm{d}}\right) G^x + 4kTB/R_{\mathrm{L}}], \quad (3)$$

where "Normal" is the normal distribution function, e is the elementary charge, B is the noise bandwidth of the detection system, R_I is the APD non-gain current responsibility, G is APD mean photoelectron gain, x is a constant related to detector's material, I_{d} is detector dark current, k is Boltzmann constant, T is absolute temperature, and R_{L} is load resistance.

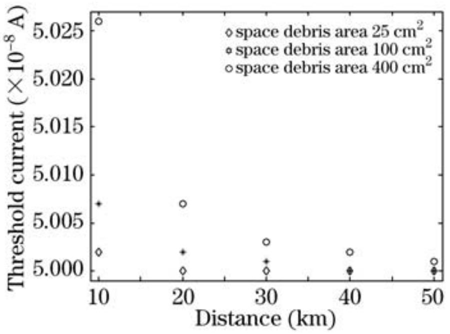

Fig. 5. Threshold current of lidar analog detection mode.

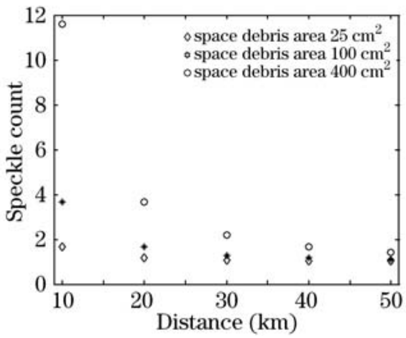

Fig. 6. Number of M within the collecting aperture.

The laser radar false-alarm probability for Gaussian statistics is given by[6]

$$P_{\mathrm{FA}} = \frac{1}{2}\mathrm{erfc}\left(\frac{\mathrm{I_t} - \mathrm{I_n}}{\sqrt{2}\sigma_{\mathrm{n}}}\right), \quad (4)$$

where erfc(x) is the complementary error function, I_{t} is the threshold, I_{n} and σ are the noise mean and variance respectively. Inserting Eqs. (2) and (3) into Eq. (4), assuming $P_{\mathrm{FA}} = 10^{-3}$ and using PerkinElmer C30954E Si-APD, we can get the threshold corresponding to the given space debris distance and size in Fig. 5.

Because space debris belongs to diffuse target, the signal intercepted by the laser radar receiver is a parabolic cylinder random variable immersed in Gaussian noise

$$P_{\mathrm{sn}}(I) = \frac{(M/\alpha)^M}{\sqrt{2\pi}\sigma}\exp\left(-\frac{I^2\alpha^2+2\sigma\alpha MI-M^2\sigma^2}{4\sigma^2\alpha^2}\right) \times D_{-M}\left(-\frac{\alpha I-M\sigma}{\sigma\alpha}\right), \quad (5)$$

where $\alpha = I_{\mathrm{s}}/\sigma$ is the signal-to-noise ratio, M is the number of independent spatial and temporal correlation cells (i.e., speckles) contained within the collecting aperture, and $D_{-M}(x)$ is a parabolic cylinder function of parameter $-M$. M is given by

$$M \cong 1 + \frac{A_{\mathrm{R}} A_{\mathrm{T}}}{\lambda^2 R^2}, \quad (6)$$

where A_{R} is the receiver aperture, A_{T} is space debris area, and λ is laser wavelength. According to Eq. (6) we can get a series of number of M corresponding to the given space debris distance and size as Fig. 6 (assuming $A_{\mathrm{R}} = 0.03$ m^2).

According to the definition of detection probability and

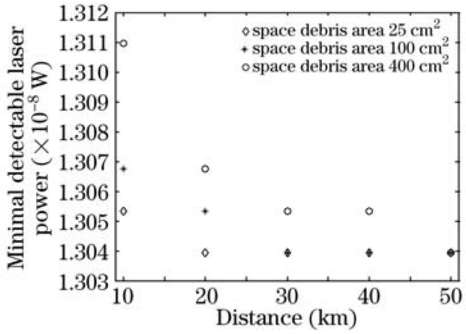

Fig. 7. Minimal detectable laser power of analog detection lidar.

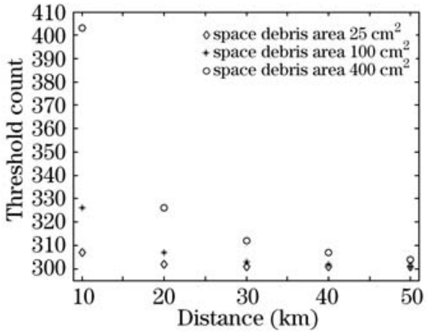

Fig. 8. Threshold of laser radar digital detection mode.

Eq. (5), the laser radar detection probability is

$$P_{\mathrm{D}}=\frac{1}{2}\mathrm{erfc}\left(\beta/\sqrt{2}\right)+\frac{(\mathrm{M}/\alpha)^{\mathrm{M}-1}}{\sqrt{2\pi}}\exp\left(\frac{\mathrm{M}^2}{4\alpha^2}-\frac{M\beta}{2\alpha}-\frac{\beta^2}{4}\right)\cdot\left[D_{-M}\left(M/\alpha-\beta\right)+\sum_{k=1}^{M-1}\left(M/\alpha\right)^{-k}D_{-(M-k)}\left(M/\alpha-\beta\right)\right], \quad (7)$$

where $\beta = I_{\mathrm{t}}/\sigma$ is the threshold-to-noise ratio. According to the gotten I_{t} and M, and assuming $P_{\mathrm{D}} = 0.9$, we can get α and then I_{s}, and the minimal detectable laser power is $P_{\mathrm{s}} = I_{\mathrm{s}}/(R_I G)$. The result is graphed in Fig. 7.

When the laser radar is working under the digital mode, the noise and signal are both discrete random variables. The noise photoelectron is approximately a Poission distributed random variable, and it can be written as

$$P_{\mathrm{n}}(k)=\frac{(N_{\mathrm{n}})^k}{k!}\exp\left(-N_{\mathrm{n}}\right), \quad (8)$$

where N_{n} is the mean of noise photoelectron in the counting interval (i.e., laser pulse width), and it equals background photoelectron if the dark photoelectron is ignored when APD is cooled. The laser radar false-alarm probability is given by[7,8]

$$P_{\mathrm{FA}}=\sum_{k=N_{\mathrm{t}}}^{\infty}P_{\mathrm{n}}(k)=1-\sum_{k=0}^{N_{\mathrm{t}}-1}P_{\mathrm{n}}(k). \quad (9)$$

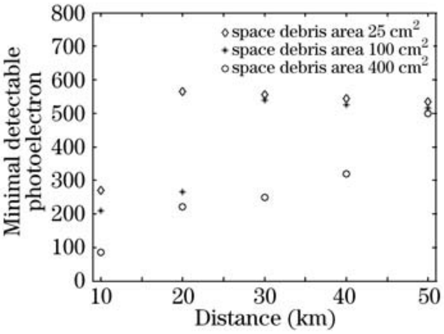

Fig. 9. Minimal detectable photoelectron of laser radar.

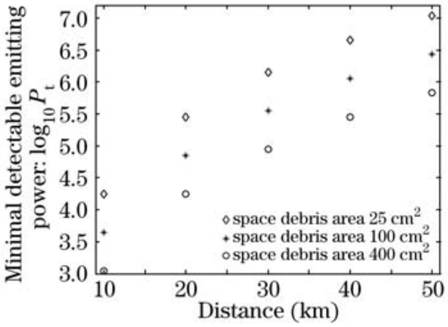

Fig. 10. Minimal detectable laser radar emitting power in watt.

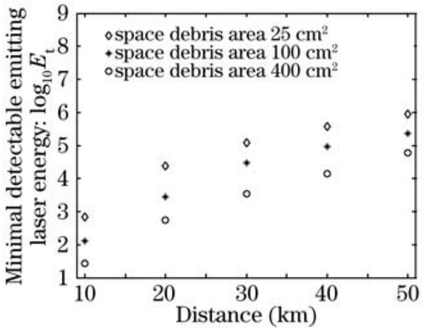

Fig. 11. Minimal detectable emitting energy of laser radar in millijoule.

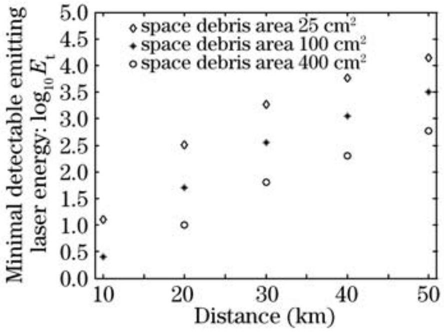

Fig. 12. Minimal detectable emitting laser energy in millijoule under quantum noise limit.

Inserting Eq. (8) into Eq. (9), assuming $P_{\mathrm{FA}} = 10^{-3}$ and using EG&G SILK$^{\mathrm{TM}}$ GAPD, we can get the threshold corresponding to the given space debris distance and size as Fig. 8. The signal plus noise probability distribution is given by

$$P_{\mathrm{s+n}}(k) = \left(\frac{M}{M+N_{\mathrm{s}}}\right)^{M} \frac{\exp(-N_{\mathrm{n}})}{(M-1)!} \times \sum_{j=0}^{k} \frac{(k+M-j-1)!}{j!(k-j)!} (N_{\mathrm{n}})^{j} \left(\frac{N_{\mathrm{s}}}{N_{\mathrm{s}}+N_{\mathrm{n}}}\right)^{k-j}. \quad (10)$$

Inserting Eq. (10) into laser radar detection probability definition (same as the expression of false-alarm probability except for replacing $P_{\mathrm{sn}}(k)$ for $P_{\mathrm{n}}(k)$), and assuming $P_{\mathrm{D}} = 0.9$, we obtain the minimal detectable photoelectron as Fig. 9.

According to the laser radar equation in space, the minimal detectable laser radar emitting power and energy is

$$P_{\mathrm{t}} = \frac{\pi^2 \theta_{\mathrm{t}}^2 R^4}{2\rho A_0 A_{\mathrm{R}} \eta_{\mathrm{s}} \eta_{\mathrm{r}}} P_{\mathrm{s}},$$

$$E_{\mathrm{t}} = \frac{\pi^2 \theta_{\mathrm{t}}^2 R^4 hc}{2\rho A_0 A_{\mathrm{R}} \eta_{\mathrm{s}} \eta_{\mathrm{r}} \lambda} N_{\mathrm{s}}, \quad (11)$$

where θ_{t} is laser divergence angle, and η_{s} is laser radar emitter optical transmission coefficient. Assuming $\theta_{\mathrm{t}} = 100\ \mu\mathrm{rad}$, $\eta_{\mathrm{s}} = 0.7$, and other parameters value as upon, we can get the minimal detectable laser radar emitting power and energy graphed in Figs. 10 and 11.

In conclusion, small space debris detection with space-based APD laser radar is studied under analog and digital modes respectively. The result indicates that space-based laser radar is competent for detecting small space debris of 100 cm^2 within 10 or 20 km under analog and digital modes respectively. But the space background sun radiation limits laser radar to detect smaller space debris. If space debris are in sun shade section, laser radar is working under quantum noise limit, and its detection performance can be improved greatly (see Fig. 12).

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