

# Fundamental properties of high-order harmonic generation from field-free aligned molecules with intense femtosecond laser pulses

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This paper reports fundamental aspects of high-order harmonic generation from N<sub>2</sub>, O<sub>2</sub> and CO<sub>2</sub> molecules spatially aligned with intense femtosecond laser pulses. The time-dependent harmonic signal observed with a pump and probe technique has been found to include several sets of beat frequency for pairs of coherently populated rotational states. We have shown the origin of each frequency component and its effect on the revival signal. The results demonstrate that the high-order harmonic generation provides a sensitive and useful way for an extensive study of dynamic processes in the field-free alignment of molecules.

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Intense ultrashort laser pulses are able to non-adiabatically induce alignment of molecules that is recurrent under field-free conditions<sup>[1,2]</sup>. There has been much interest in this field-free alignment of molecules, since it provides a promising approach to control molecules with an external field for a variety of applications<sup>[3]</sup>. Recently it has been demonstrated that the dynamic alignment of molecules can be detected with high-order harmonic generation (HHG) by intense femtosecond laser pulses<sup>[4,5]</sup>. The characteristic HHG observed demonstrates an effective approach to extensively study wave packet dynamics and also suggests a possible new area of nonlinear optics which uses the molecular wave packet to control the strong-field interaction. This paper reports the fundamental aspect of the HHG from field-free aligned molecules in time and frequency domains, where we focus our attention on the temporal evolution of harmonic yield from the wave packet in N<sub>2</sub>, O<sub>2</sub> and CO<sub>2</sub> molecules and their frequency spectra.

We consider a pump-probe experiment using nonresonant, linearly polarized ultrashort laser pulses for a simple linear molecule. The pump pulse forms a rotational wave packet,

$$\Psi_g(t) = \sum_J a_J \psi_{JM} \exp(-iE_J t/\hbar), \quad (1)$$

where  $\psi_{JM}$  is the field-free rotor wave function, pertaining to the eigenenergy  $E_J = 2\pi\hbar BcJ(J+1)$ ,  $c$  is the light speed in vacuum, for the rotational state with the angular momentum  $J$  and its projection  $M$  on the field direction, and the coefficient  $a_J$  is almost independent on time after interaction<sup>[1,2]</sup>. Equation (1) brings about molecular alignment and its field-free revivals at a period of  $T_{\text{rev}} = 1/(2Bc)$  with the rotational constant  $B$ . It is well known that the degree of alignment is characterized by the expectation value  $\langle \cos^2 \theta \rangle$  calculated with  $\Psi_g(t)$ , where  $\theta$  is the angle between the molecular axis and the field direction. The time-dependent behavior of  $\langle \cos^2 \theta \rangle$  is dominated by beats at frequency  $\omega_1 = 2\pi Bc(4J+6)$  between any pair of rotational states  $J$  and  $J \pm 2$  that are populated through the transition,  $\Delta J \equiv J - J' = 0, \pm 2$  with  $\Delta M = 0$ . The delayed probe pulse generates high

harmonic radiation from the wave packet. The HHG from a single molecule is essentially a single-cycle event in the laser field<sup>[6]</sup>, while the temporal harmonic signal observed as a function of time delay  $\Delta t$  between pump and probe pulses is very slow and would predominantly be governed by the beat at  $\omega_1$  as for  $\langle \cos^2 \theta \rangle$ .

In the experiment, we used a Ti:sapphire laser to produce pulse energy of 40 mJ in 40-fs pulses at 800 nm. The linearly polarized output was split into two beams to produce a variable time delay  $\Delta t$  between pump and probe pulses. The two beams were recombined collinearly and focused with a 50-cm focal-length lens into a pulsed molecular beam jetted from a 1-mm-diameter nozzle. The gas pressure in the jet was typically 10 Torr. The pump pulse intensity was in a range of  $(4-8) \times 10^{13}$  W/cm<sup>2</sup>, and the probe was slightly more intense than the pump. The harmonic radiation was detected by an electron multiplier mounted on a vacuum ultraviolet monochromator, and the signal processed by a boxcar averager was stored on a personal computer. In the preliminary experiment we observed the time-dependent signal for all orders of harmonic higher than the 15th for which the semi-classical model<sup>[6]</sup> would be valid. Since their revival structures represented no fundamental difference, we selected the 19th harmonic ( $\lambda \sim 42.1$  nm) for the present study.

Figure 1 shows a typical example of the harmonic signal observed for N<sub>2</sub> as a function of  $\Delta t$  and its frequency spectrum. We have confirmed that the observed signal shown in Fig. 1(a) is well reproduced by the simulated result of  $\langle \cos^2 \theta \rangle$ , except for the signal at  $\Delta t \sim 0$ <sup>[5]</sup>. This indicates that the 19th harmonic is most efficiently produced with N<sub>2</sub> molecules aligned parallel to the probe pulse field and suppressed with those aligned perpendicularly, as reported so far<sup>[4,5]</sup>. The detailed revival structure has been discussed by comparing the observed signal with the simulated  $\langle \cos^2 \theta \rangle$ <sup>[5]</sup>. Briefly, the initial signal drop at  $\Delta t \sim 0$  is due to strong ionization induced by the intense superimposed pump and probe pulses. The onset of alignment is shown by the first peak at  $\Delta t \sim 0.2$  ps after the pump pulse interaction, as suggested by our previous study<sup>[7]</sup>. This signal peak rapidly

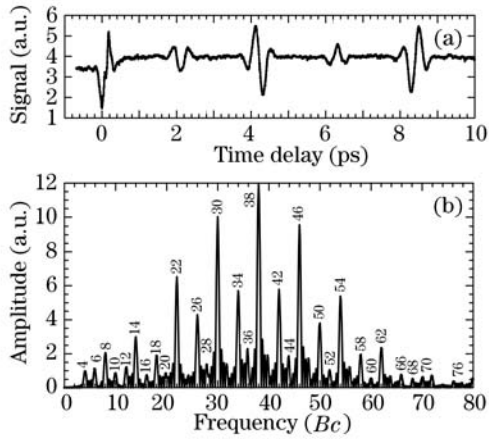


Fig. 1. (a) 19th harmonic signal observed for  $N_2$  as a function of time delay between the pump and probe pulses at the intensities of  $0.8 \times 10^{14}$  and  $1.7 \times 10^{14}$   $W/cm^2$ , respectively, and (b) its frequency spectrum in which the number on each spectral peak denotes the frequency at  $\omega_1$ ,  $\omega_2$  and  $\omega_3$  in  $Bc$ .

decreases due to dephasing of rotational states in the wave packet. The full revival of alignment is observed at  $\Delta t \sim 8.5$  ps, corresponding to  $T_{rev} = 8.3$  ps with  $B = 2$   $cm^{-1}$  for  $N_2$ , where the rapid signal modulation is due to the rotation of aligned molecules.

The frequency spectrum shown in Fig. 1(b) represents the structure of the rotational wave packet. The spectrum is mainly composed of strong beat frequencies (6, 10, 14, 18, ...) at  $\omega_1$  with a separation  $\Delta\omega_1/2\pi = 4Bc$ , as expected from  $\langle \cos^2 \theta \rangle$ . The spectral amplitude in this *allowed series* is larger for the even  $J$  and weaker for the odd  $J$ . This intensity alternation originates from the population ratio 2:1 between the even- and odd- $J$  states of  $N_2$ <sup>[8]</sup>.

The spectrum shown in Fig. 1(b) includes an additional set of frequency component (20, 28, 36, 44, ...) at  $\omega_2 = (E_{J+4} - E_J)/\hbar = 2\pi Bc(8J + 20)$  with a separation of  $8Bc$  for a pair of rotational states  $J$  and  $J \pm 4$ . As discussed in our recent paper<sup>[5]</sup>, this *Raman-forbidden series* arises from the expectation value of  $\langle \cos^4 \theta \rangle$ , and the coherence between the rotational states for  $\Delta J = \pm 4$  would be created by multi-step Raman transitions during the wave packet formation process. Then, the spectral amplitude at  $\omega_2$  should be much weaker than that at  $\omega_1$ . We notice that the signal amplitude at  $\omega_2$  is peaked for  $J = 1 - 3$ . This suggests that the high population of low  $J$  states at the initial low temperature in the gas jet is responsible for the high-order coherence to be detected with the HHG. The weaker frequency component at  $\omega_2$  has also been observed in the high-frequency region for  $J = 8 - 13$  due most likely to the redistribution of rotational states through the pump process<sup>[5]</sup>. The time-dependent signal from the  $\omega_2$  component is easily shown to have the revival period of  $T_{rev}/2 = 1(4Bc)$ , as discussed later.

In Fig. 1(b), we see the third set of weak component (4, 8, 12, 16, ...) at  $\omega_3$ . As shown by the recent theory developed<sup>[9]</sup>, this *anomalous series* is originated from the expectation value of the form  $\langle \cos^2 \theta \rangle \langle \cos^2 \theta \rangle$  that leads to  $\omega_3 = 2\pi Bc[4(J + J') + 12]$ , and  $2\pi Bc[4(J - J')]$ . In contrast to the  $\omega_1$  series, this anomalous series appears

to be produced with only adjacent low  $J$  states. This suggests that, as for the Raman-forbidden series at  $\omega_2$ , the high-order coherence is efficiently created with the initial high population of low  $J$  states. We note that the HHG reveals such high-order coherence embedded in the rotational wave packet, demonstrating the high sensitivity for detecting the *rotational coherence* created by the pump pulse.

As shown in Fig. 1(b), the time-dependent harmonic signal is predominantly governed by the frequency component at  $\omega_1$ . To see the contribution of the even- and odd- $J$  components to the revival structure, we reconstructed the temporal evolution of the harmonic signal with the inverse Fourier transform for each component, and the result is shown in Fig. 2. It is clearly shown that the even- and odd- $J$  components contribute in anti-phase to the revival signals at  $T_{rev}/4$  and  $3T_{rev}/4$  to produce the small peak due to the population difference, while the signal in phase at  $T_{rev}/2$  and  $T_{rev}$  creates a large modulation in the harmonic signal.

Figure 3 shows the time-dependent 19th harmonic signal for  $O_2$  and its frequency spectrum. In Fig. 3(a), the full revival is seen at  $T_{rev} \sim 11.6$  ps, corresponding to  $B = 1.44$   $cm^{-1}$  for  $O_2$ . The revival signals at multiples of  $T_{rev}/4$  represent almost the same amplitude, since only odd- $J$  states are populated in the ground state of  $O_2$  having no nuclear spin. The observed harmonic yield as well as the revival signal was much smaller than that for  $N_2$ . The low HHG efficiency would result from the anti-symmetric structure of molecular orbital. We tried to find a different angle  $\alpha$  between the pump and probe field directions to produce a larger revival signal, but the best signal modulation was observed at  $\alpha \sim 0^\circ$ , as well as the highest harmonic yield. This is inconsistent with the previous conclusion<sup>[10]</sup> that the HHG from aligned  $O_2$  is peaked at  $\theta \sim 45^\circ$  and minimized at  $\theta \sim 0^\circ$  and  $90^\circ$ , although  $\theta$  is different from  $\alpha$ . Recently, the  $\alpha$ -dependent HHG observed has been shown to reconcile with the theoretical result.

The frequency spectrum shown in Fig. 3(b) consists

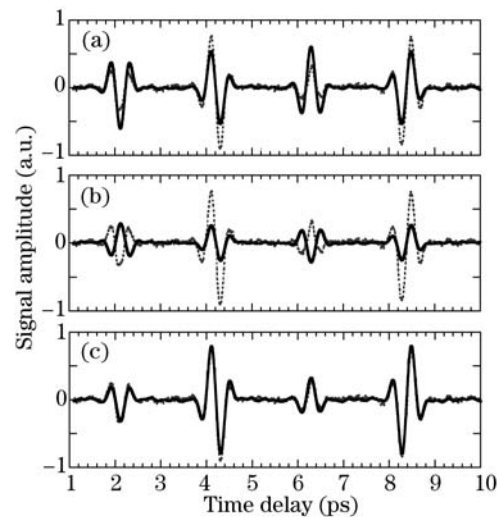


Fig. 2. Time-dependent 19th harmonic signals (solid lines) reproduced with the inverse Fourier transform for (a) the even- $J$  component, (b) odd  $J$ , and (c) both of even and odd  $J$ . For comparison, each trace includes the observed signal (gray dashed lines).

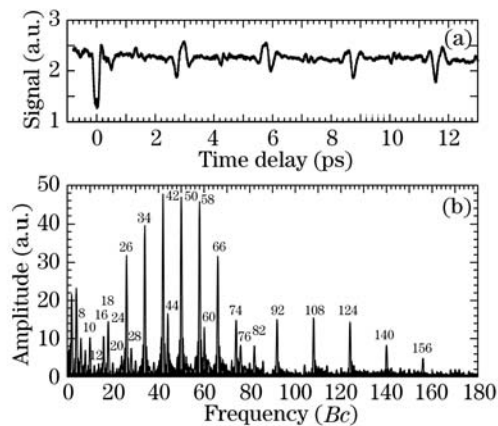


Fig. 3. (a) 19th harmonic signal observed for O<sub>2</sub> as a function of time delay at the pump and probe intensities of  $0.5 \times 10^{14}$  and  $1.2 \times 10^{14}$  W/cm<sup>2</sup>, respectively, and (b) its frequency spectrum in which the number on each spectral peak denotes the frequency at  $\omega_1$ ,  $\omega_2$  and  $\omega_3$  in Bc.

of the strong component at  $\omega_1$  and the weak at  $\omega_2$  and  $\omega_3$  for odd  $J$ . We note that the spectral peaks at  $\omega_1$  (42–58) correspond to those at  $\omega_2$  (92–124) for almost the same values of  $J = 9 - 13$ . This confirms that the  $\omega_2$  component certainly originates from the beat between the rotational states  $J$  and  $J \pm 4$  that are coherently populated in the wave packet. The spectrum shown in Fig. 3(b) involves a set of  $\omega_2$  (28–76) with a sub-peak for  $J = 3$ . As discussed above, the  $\omega_2$  component in the low frequency region accounts for the initial low temperature  $T_{\text{rot}}$  in the supersonic gas jet. Our simulation has shown that the spectral peak for  $J = 3$  corresponds to  $T_{\text{rot}} = 25 - 50$  K.

In Fig. 3(b), the  $\omega_3$  component (8, 12, 16, 20, ...) is also seen in the low frequency regions, as in the N<sub>2</sub> spectrum. The theory has shown that the time-dependent 19th harmonic signal for O<sub>2</sub> are dominated with the expectation values arising from  $\langle \cos^2 \theta \sin^2 \theta \rangle^{2[9]}$ . This term includes the frequencies  $\omega_1$ ,  $\omega_2$  and  $\omega_3$  coming from  $\langle \cos^2 \theta \rangle$ ,  $\langle \cos^4 \theta \rangle$  and  $\langle \cos^2 \theta \rangle \langle \cos^2 \theta \rangle$ , respectively. Since the temporal evolution of  $\langle \cos^2 \theta \rangle$  includes neither  $\omega_2$  and  $\omega_3$  components, the coherence at  $\omega_2$  and  $\omega_3$  embedded in the wave packet is certainly detected through the anisotropic electronic response to the linearly polarized probe pulse used for the HHG.

We tried to reconstruct the time-dependent harmonic signal with the inverse Fourier transform, using two components at  $\omega_1$  and  $\omega_2$ , since the  $\omega_3$  component was very weak. The result presented in Fig. 4 demonstrates that the  $\omega_1$  component certainly creates the revival signals at  $mT_{\text{rev}}/4$  with the positive integer  $m$ , and the  $\omega_2$  component produces the small revival signals at  $mT_{\text{rev}}/8$  with the full revival period of  $T_{\text{rev}}/2$ . Thus the revival signals at  $(2m - 1)T_{\text{rev}}/8$  in Fig. 3(a) predominantly arise from the  $\omega_2$  component. The time-dependent harmonic signal observed is the simple superposition of those arising from the components at  $\omega_1$  and  $\omega_2$ .

The weak 1/8-revival signals for O<sub>2</sub> was also observed so far with the Coulomb explosion imaging<sup>[11]</sup>, where  $\langle \cos^2 m\theta \rangle$  was used to analyze the fractional revivals. No observation of the  $\omega_3$  component was reported so far. On the other hand, the analysis with  $\langle \sin^2 2\theta \rangle$

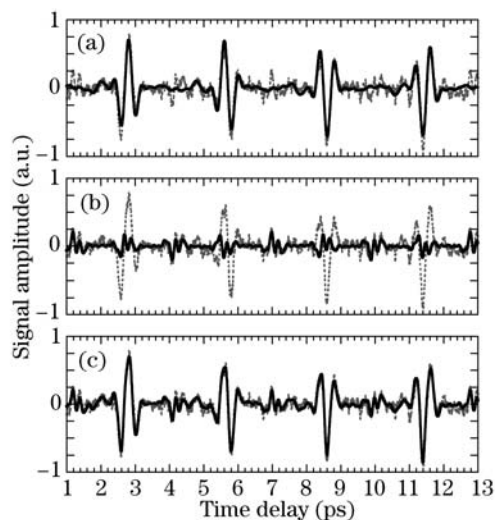


Fig. 4. Time-dependent 19th harmonic signals (solid lines) reproduced for the frequency component at (a)  $\omega_1$ , (b)  $\omega_2$ , and (c)  $\omega_1$  and  $\omega_2$ . For comparison, each trace includes the observed signal (gray dashed lines).

empirically introduced to reproduce the time-dependent harmonic signal for O<sub>2</sub><sup>[10]</sup>. The recent theory has successfully elucidated the origin of the frequency components observed<sup>[9]</sup>.

We have also observed the time-dependent 19th harmonic signal for CO<sub>2</sub>. The initial alignment was induced at  $\Delta t \sim 0.2$  ps after the pump pulse interaction was over, while the harmonic signal was completely reversed to those of N<sub>2</sub> and O<sub>2</sub>. This result suggests that the 19th harmonic is minimized with CO<sub>2</sub> aligned along the pump pulse polarization and peaked with those aligned perpendicularly. The theoretical analysis has shown that this is due to the characteristic electronic response of the highest occupied molecular orbital of the ground state of CO<sub>2</sub> at the 19th harmonic energy, rather than the destructive interference of recombining electron wave proposed so far<sup>[12,13]</sup>. The experimental results on the harmonic signal phase will be presented and discussed in a separate paper.

In summary, we have demonstrated the fundamental aspects of HHG from rotational wave packets created by the intense femtosecond laser pulse. The present results should be useful for controlling rotational wave packets with ultrashort laser pulses and nonlinear optical processes in molecules.

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## References

1. L. Cai, J. Marango, and B. Friedrich, Phys. Rev. Lett. **86**, 775 (2001).
2. T. Seideman, J. Chem. Phys. **115**, 5965 (2001).
3. H. Stapelfeldt and T. Seideman, Rev. Mod. Phys. **75**, 543 (2003), and references therein.
4. M. Kaku, K. Masuda, and K. Miyazaki, Jpn. J. Appl. Phys. **43**, L591 (2004).
5. K. Miyazaki, M. Kaku, G. Miyaji, A. Abdurrouf, and F. H. M. Faisal, Phys. Rev. Lett. **95**, 243903 (2005).

6. P. B. Corkum, Phys. Rev. Lett. **71**, 1994 (1993).
7. K. Miyazaki, T. Shimizu, and D. Normand, J. Phys. B: At. Mol. Opt. Phys. **37**, 753 (2004).
8. G. Herzberg, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules* (Van Nostrand Reinhold, New York, 1950) Chap.III.
9. F. H. M. Faisal, A. Abdurrouf, K. Miyazaki, and G. Miyaji, Phys. Rev. Lett. **98**, 143001 (2007).
10. J. Itatani, D. Zeidler, J. Levesque, M. Spanner, D. M. Villeneuve, and P. B. Corkum, Phys. Rev. Lett. **94**, 123902 (2005).
11. P. W. Dooley, I. V. Litvinyuk, K. F. Lee, D. M. Rayner, M. Spanner, D. M. Villeneuve, and P. B. Corkum, Phys. Rev. A **68**, 023406 (2003).
12. T. Kanai, S. Minemoto, and H. Sakai, Nature **435**, 470 (2005).
13. C. Vozzi, F. Calegari, E. Benedetti, J.-P. Caumes, G. Sansone, S. Stagira, M. Nisoli, R. Torres, E. Heesel, N. Kajumba, J. P. Marangos, C. Altucci, and R. Velotta, Phys. Rev. Lett. **95**, 153902 (2005).