# Optimization of the $Q$－switch in a folded resonator with prism reflectors 

Jun Zhou（周 军）and Weibiao Chen（陈卫标）<br>Shanghai Institute of Optics and Fine Mechanics，Chinese Academy of Sciences，Shanghai 201800


#### Abstract

The $Q$－switching performance in a folded resonator containing prism reflectors is analyzed．It is shown that the performance of the $Q$－switch in a folded cavity is affected by its configuration．A variety of configurations are compared theoretically．The transmissivities are calculated when the $Q$－switch is turned on and off，respectively．The effects of the misalignments of the prisms are also calculated．The $Q$－switch in a folded resonator with prism reflectors is optimized．With the optimal $Q$－switch，the laser system is more compact and stable．The effects of the misalignments of the prisms on the $Q$－switching performance are reduced by using optimal $Q$－switching configuration．


OCIS codes：140．3540，230．2090，230．5480．

Folded cavity is widely used to make the laser system more compact and stable．Prisms instead of mirrors are used in resonator to change the direction of the light beam ${ }^{[1]}$ ．However，when light beam is totally reflected on the prism with non－normal incidence，the two planes of polarization generally have different phase shifts ${ }^{[2]}$ ．This makes the folded resonator laser different to the linear resonator laser on $Q$－switching operation．The perfor－ mance of the $Q$－switch in a folded cavity with prism re－ flectors is affected by its configuration．In this paper， the performances of a variety of $Q$－switch configurations are numerically calculated and compared．The optimal $Q$－switch configuration in a folded resonator with prism reflector is achieved．

In this paper，we ignore the influence of thermally in－ duced birefringence on the performance of the $Q$－switch． The polarizing elements in the resonator include polar－ izer，waveplate，and prism reflectors．The matrices of these elements are given as follows ${ }^{[3,4]}$ ．

The Jones matrix of the polarizer is

$$
M_{\mathrm{p}}=\left[\begin{array}{ll}
1 & 0  \tag{1}\\
0 & 0
\end{array}\right]
$$

To a waveplate with phase retardence $\delta$ ，if its fast axis rotated $\theta$ from the $x$（horizontal）axis，its Jones matrix can be expressed as
$M(\delta, \theta)=\cos \frac{\delta}{2}\left[\begin{array}{cc}1-i \tan \frac{\delta}{2} \cos 2 \theta & -i \tan \frac{\delta}{2} \sin 2 \theta \\ -i \tan \frac{\delta}{2} \sin 2 \theta & 1+i \tan \frac{\delta}{2} \cos 2 \theta\end{array}\right]$.
In Fig．1，the light is totally reflected on the prism． The incident ray and the normal to the surface form the incident plane $P 2$ ．The ray can be resolved into p compo－ nent（parallel to the plane of incidence）and s component （perpendicular to the plane of incidence）．The two planes of polarization generally have different phase shifts when totally reflected．The difference in the shifts is known as phase retardance．It is determined by

$$
\begin{equation*}
\tan \frac{\delta}{2}=\frac{\cos \alpha \sqrt{\sin ^{2} \alpha-n^{2}}}{\sin ^{2} \alpha} \tag{3}
\end{equation*}
$$

where $\alpha$ is the incident angle，$n=n_{2} / n_{1}$ is the relative refractive index．The Jones matrix of the surface of total
reflection is

$$
M_{\mathrm{r}}=\left[\begin{array}{cc}
1 & 0  \tag{4}\\
0 & \exp i \delta
\end{array}\right]
$$

Generally $\alpha=45^{\circ}, n_{1}=1.5, n_{2}=1$ ，by inserting these values into Eq．（3），we obtain $\delta \approx 40.3^{\circ}$ ．

In the same way，when light reflects or refracts at pre－ vious polarizing element there is another incident plane $P 1$ ．$P 1$ and $P 2$ do not coincide with each other due to mechanical misalignment．The angle between $P 1$ and $P 2$ is defined as $\beta$ ．The transformation of coordinates is necessary when the polarization of light is calculated by Jones matrix．Matrix related to the transformation of coordinate can be expressed as

$$
M_{\mathrm{t}}=\left[\begin{array}{cc}
\cos \beta & \sin \beta  \tag{5}\\
-\sin \beta & \cos \beta
\end{array}\right] .
$$

Combining Eqs．（4）and（5），one surface of total reflec－ tion of the prism can be expressed as

$$
M_{\mathrm{R}}=M_{\mathrm{r}} M_{\mathrm{t}}=\left[\begin{array}{cc}
\cos \beta & \sin \beta  \tag{6}\\
-\mathrm{e}^{i \delta} \sin \beta & \mathrm{e}^{i \delta} \cos \beta
\end{array}\right]
$$

The $Q$－switching configuration shown in Fig． 2 is usu－ ally applied in folded resonator．In this configuration， the performance of the $Q$－switch is the same as that in a linear resonator．The best turning－off performance（with the transmissivity of 0 when turned off）and the best turning－on performance（with the transmissivity of 1 when turned on）can be achieved theoretically．However， in existence of misalignment of the prism，the polarizer has more insertion loss owing to the phase retardance


Fig．1．Total reflection in prism．
between s and p components. When the $Q$-switch in Fig. 2 is turned on, $\mathrm{KD}^{*} \mathrm{P}$ and quarter-wave plate (QWP) equal a full-wave plate which ignored in calculation. The horizontally polarized light has a round-trip transmissivity determined by

$$
\begin{equation*}
t_{0}=M_{\mathrm{p}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{p}} \tag{7}
\end{equation*}
$$

Assume the total reflections at two surfaces in the prism have the same parameters: the incident angle $45^{\circ}$, rotated angle of $p$-polarized plane $\beta$. $t_{0}$ for different values of $\beta$ are calculated. The result is shown in Fig. 3. It can be seen that the turning-on transmissivity is sensitive to misalignment of the prism. In this way the usual $Q$-switching configuration is not ideal especially for the case in which reliable operation under severe environmental conditions is required.

To optimize the $Q$-switch in folded resonator the polarizer must be moved to the upper arm of the cavity. Then the KD*P and the QWP will be placed according to the results of calculation.

The KD*P can be ignored when $Q$-switch is turned off. There are two possible configurations for the $Q$-switch as shown in the left column of Fig. 4. By rotating the QWP and minimizing the round-trip transmissivity the $Q$-switch will be turned off. For Fig. 4(a) the transmissivity is determined as

$$
\begin{equation*}
t_{\mathrm{off}-\mathrm{a}}=M_{\mathrm{p}} M_{\mathrm{W}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{W}} M_{\mathrm{p}} \tag{8a}
\end{equation*}
$$

For Fig. 4(b) shown in Fig. 4 the transmissivity is determined as

$$
\begin{equation*}
t_{\mathrm{off}-\mathrm{b}}=M_{\mathrm{p}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{W}} M_{\mathrm{W}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{p}} \tag{8b}
\end{equation*}
$$

Assuming $\beta$ in Eq. (6) takes the vaules of $0,2^{\circ}, 4^{\circ}$, respectively, the transmissivities for different values of $\theta$ are calculated. The results are shown in the right column of Fig. 4. It can be seen obviously that the $Q$-switch shown


Fig. 2. $Q$-switching configuration in a folded resonator.


Fig. 3. Transmissivity versus different rotated angle of ppolarized plane of a usual $Q$-switching configuration in a folded resonator.
in Fig. 4(a) cannot be turned off completely. So the QWP must be placed in the lower arm of the resonator as shown in Fig. 4(b).

According to the results obtained above, the QWP must be set as shown in Fig. 4(b) showed in Fig. 4. Then


Fig. 4. Two possible configurations for the turn-off $Q$-switch and the corresponding transmissivities.


Fig. 5. Three possible configurations for the turn-off $Q$-switch and the corresponding transmissivities.


Fig. 6. Comparison between the optimal and usual configurations.
the $\mathrm{KD}^{*} \mathrm{P}$ has three possible positions as shown in the left column of Fig. 5. When $Q$-switch is turned on the roundtrip transmissivities for three positions are expressed as

$$
\begin{align*}
& t_{\mathrm{on}-\mathrm{a}}=M_{\mathrm{p}} M_{\mathrm{K}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{W}} M_{\mathrm{W}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{K}} M_{\mathrm{p}}  \tag{9a}\\
& t_{\mathrm{on}-\mathrm{c}}=M_{\mathrm{p}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{W}} M_{\mathrm{K}} M_{\mathrm{K}} M_{\mathrm{W}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{p}}  \tag{9b}\\
& t_{\mathrm{on} \_\mathrm{b}}=M_{\mathrm{p}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{K}} M_{\mathrm{W}} M_{\mathrm{W}} M_{\mathrm{K}} M_{\mathrm{R}} M_{\mathrm{R}} M_{\mathrm{p}} \tag{9c}
\end{align*}
$$

where $M_{\mathrm{K}}$ is ideal Jones matrix of the $\mathrm{KD}^{*} \mathrm{P}$,

$$
M_{\mathrm{K}}=M(\pi / 2, \pi / 4)=\frac{1}{\sqrt{2}}\left[\begin{array}{cc}
1 & -i  \tag{10}\\
-i & 1
\end{array}\right]
$$

Similarly, make the values of $\beta$ in Eq. (6) as $0,2^{\circ}, 4^{\circ}$, respectively and calculate the transmissivities of different values of $\theta$. The result is shown in the right column of Fig. 5.

For the $Q$-switch arranged as shown in Fig. 5(a) the turning-on transmissivity have the maximum of 1 when the QWP has a rotated angle of about $45^{\circ}$, which is the same as that for minimum transmissivity when the $Q$ switch is turned off. So this configuration has a good switching performance and is insensitive to the misalignment of the prism. For Figs. 5(b) and (c), it can be seen obviously that the transmissivities are influenced severely by the misalignment of the prism. So Fig. 5(a) should be
the best one for the $Q$-switch in folded resonator.
We compared directly the optimal configuration shown in Fig. 5(a) and the usual configuration shown in Fig. 2. The values of $t_{0}$ in Eq. (7) for $\beta$ of $0,2^{\circ}$, and $4^{\circ}$ are $1,0.99$, and 0.96 , respectively. These values are shown in Fig. 6 to compare with the turning-on transmissivities of configuration Fig. 5(a). It can be seen that the optimal configuration is obviously less sensitive to the misalignment of the prism. So the $Q$-switching performance will be improved markedly by using the optimal configuration. Especially when the unadjustable elements are used in resonator to improve the stability the optimal configuration will reduce the difficulty of process and adjustment. Besides it will go with a shorter assemble time.
In conclusion, the $Q$-switch performances in folded resonator with prism reflectors are theoretically analyzed and calculated. The usual $Q$-switching configuration shown has a high sensitivity to the misalignment of prism. Then the $Q$-switch is optimized by changing its configuration. The transmissivites for various configurations when the $Q$-switch is turned on and off are calculated. The best $Q$-switching configuration in the folded resonator is achieved and compared with the usual one. The result shows that the optimal configuration has better switch performance and lower sensitivity to the misalignment of prism. Benefitting from this merit the difficulties in processing and adjusting are reduced markedly, especially for the case in which unadjustable elements are used. The optimal configuration has higher stability and is more insensitive to shock and vibration in actual application.
J. Zhou's e-mail address is joe427@siom.ac.cn.

## References

1. W. Koechner, Sollid-State Laser Engineering (5th edn.) (Springer-Verlag, Berlin, 1999).
2. M. Born and E. Wolf, Principles of Optics (7th edn.) (Cambridge University Press, Cambridge, 1999).
3. M. Bass, Handbook of Optics (McGraw-Hill, New York, 1995).
4. A. Gerra and J. M. Burch, Introduction to Matrix Methods in Optics (John Wiley \& Sons, New York, 1975).
