# High-speed detection of terahertz pulse sequence by chirped pulse sampling technology

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The technology of measuring high-speed optical pulse sequence is under fast development for meeting the increasing requirement in optical communication, characterization of optical pulse and other fields. We propose a chirped pulse sampling technology to measure the terahertz (THz) pulse sequence. The principle and detection resolution of this high-speed sampling technology have been discussed and analyzed. The acquisition could measure a THz sequence containing hundreds of pulses at a single-shot. With this method, THz sequence could be measured by low-speed sampling devices.

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Seeking for high-speed optic-electro sampling is an essential goal in today's scientific research as well as optical communication where the characterization of the optical field is needed[1−3]. Demand for higher capacity and higher speed optical networks has promoted rapid developments of both optical transmission and sampling techniques such as the optical wavelength-domain multiplexing (OWDM) and the optical time-domain multiplexing (OTDM) which have increased the bandwidth of the optical network greatly[4,5]. However at the receiver ends, there are still few efficient technologies which could help people to identify the ultra-fast pulses rapidly. The conventional time-domain optic-electro sampling, which is based on the simply repetitive recordings, depends mostly on the recording instruments, and can only reach a speed no higher than 10 GHz. Some latest techniques, namely the FROG and SPIDER, can characterize the shape of the single ultra-fast pulse with resolution as high as 1-fs at a single-shot, but take time much longer than the time duration of the pulse itself to reconstruct the shape[6,7]. However, the sampling speed is quite more desirable than the resolution in these fields.

In this paper, we propose a novel method which could measure the terahertz (THz) pulse sequence at very high rate by using low speed devices. The method could be considered as transplanting the OWDM technology into the measuring section. A linearly chirped pulse is employed as the probe pulse, and the THz pulses sequence is modulated into the chirped pulse through a nonlinear process[8]. With the introduction of this, the time-domain information is transferred into the modulation of spectra in the frequency-domain which can be recorded by a low speed spectrometer. After we finally get the profile of the modulated spectra of the chirped pulse, the time information encoded in it could be extracted at an unprecedented data-acquisition rate[9,10]. And we discuss the resolution of the measurement and sampling speed of the device used in the measuring process.

The principle of the measurement can be understood according to Fig. 1. An unchirped femtosecond laser pulse with duration of tens of fs is stretched into a linearly chirped pulse with duration of as long as several hundred ps by a grating pairs and acts as the probe beam. For a negatively chirped laser pulse (instantaneous frequency in the pulse decreases with time), the blue component with shorter wavelength of the pulse leads to the red component. When the high speed pulses sequence collinearly mixed with this chirped probe beam in a birefringent crystal, different pulses of the pulses sequence modulate the different wavelength component of the chirped probe beam through the optical Kerr effect or Pockels effect. Consequently, the waveform and the information carried by the pulses sequence are encoded on the wavelength spectrum of the probe beam through the alteration in the polarization. Two optical spectrum analyzers (OSAs) are used respectively to measure the spectral distributions with or without considering the polarization. The temporal information carried by the pulses sequence can be extracted by comparing the two distributions.

The reconstruction method of pulses sequence from the measured spectral modulation of the chirped pulse can be analyzed. Assuming the linearly chirped probe pulse has a Gaussian profile and can be written as

$$E_{\mathrm{p}}(t) = \exp\left(-\frac{t^2}{T_{\mathrm{p}}^2} - j\alpha t^2 - j\omega_0 t\right), \qquad (1)$$

where $T_{\mathrm{p}}^2$ is the duration of the chirped pulse, $\omega_0$ is the central frequency of the chirped laser pulse, and $2\alpha$ is the chirp rate. When the linearly chirped pulse co-propagates in the birefringent crystal with the THz pulses sequence, the polarization of the chirped pulse is modulated by the pulses sequence due to the nonlinear effect. After the crystal the chirped pulse is divided into two same pulses by a beam splitter in order to make

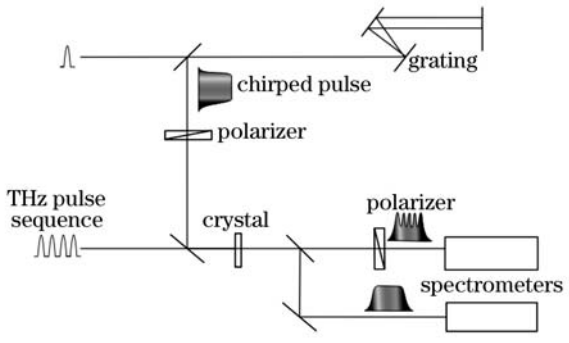


Fig. 1. Layout of the experimental setup.

 

comparison measurement. One pulse transmits a polarization analyzer and is amplitude modulated; its electric field can be described as

$$E_{\rm trans} = 0.5E_{\rm p}(t)[1 - kE_{\rm THz}(t-\tau)], \tag{2}$$

where $E_{\rm THz}(t-\tau)$ is the electric field of the THz pulses sequence, $\tau$ is the time delay between the probe beam and the pulses sequence, and $k$ is a constant which is related to the modulation depth. The modulation depth depends on many factors, notably the electro-optic coefficient, optical bias, scattering, thickness of the crystal, and the group velocity mismatch. Neglecting the diffusions and the absorptions of the crystal, the energy of the probe pulse is unchanged. The other pulse does not transmit a polarization analyzer and can be used as a reference pulse. The electric fields of the both pulses are recorded in the frequency domain by using two OSAs. For an OSA, the measured spectral intensity is proportional to the convolution of the spectral response function of the OSA with the squared amplitude of the Fourier transform of the chirped pulse and has

$$M(\omega) \propto \int_{-\infty}^{+\infty} g(\omega-\omega_1) \times \left| \int_{-\infty}^{+\infty} E_{\rm in}(t)\exp(j\omega_1 t){\rm d}t \right|^2 {\rm d}\omega_1, \tag{3}$$

where $g(\omega-\omega_1)$ is the spectral response function of the OSA, and the $E_{\rm in}(t)$ is the electric field of the measured pulse. Therefore, the measured spectral intensities of these two pulses by the two OSAs can be written respectively as

$$\begin{aligned} M_1(\omega) \propto \frac{1}{2}\int_{-\infty}^{+\infty} g(\omega-\omega_1) \times \Bigg| \int_{-\infty}^{+\infty} \exp \\ \left(-\frac{t^2}{T_{\rm p}^2} - j\alpha t^2 - j\omega_0 t\right)[1 - kE_{\rm THz}(t-\tau)] \\ \times \exp(j\omega_1 t){\rm d}t \Bigg|^2 {\rm d}\omega_1, \end{aligned} \tag{4}$$

$$\begin{aligned} M_2(\omega) \propto \frac{1}{2}\int_{-\infty}^{+\infty} g(\omega-\omega_1) \times \Bigg| \int_{-\infty}^{+\infty} \exp \\ \left(-\frac{t^2}{T_{\rm p}^2} - j\alpha t^2 - j\omega_0 t\right)\exp(j\omega_1 t){\rm d}t \Bigg|^2 {\rm d}\omega_1. \end{aligned} \tag{5}$$

The integrals in Eqs. (4) and (5) can be evaluated by using the method of stationary phase if $\alpha$ is sufficiently large. Since $T_{\rm p}$ and the THz pulse duration are much longer than the oscillation period of the optical beam $2\pi/\omega_0$, the phase factor $j(\alpha t^2 - \omega_0 t)$ can be considered as a fast varying function of time and gives a self-canceling oscillation, which allows the negligence on the contribution of the integrand everywhere except in the vicinity of certain critical point, where the derivatives of the phase term of Eqs. (4) and (5) with respect to t are zero. In this case, it gives $t_\omega = \dfrac{\omega-\omega_0}{2\alpha}$.

The normalized differential intensity is defined as

$$N(\omega) = \frac{M_1(\omega) - M_2(\omega)}{M_2(\omega)}, \tag{6}$$

which can be proved to be proportional to the intensity of THz laser pulses. In the situation where the modulation depth $|k| \ll 1$ for typical crystals, considering only the zero and first order terms, applying the $t_\omega$ to the differential intensity, we can get

$$N(\omega) = \frac{\int_{-\infty}^{+\infty} g(\omega-\omega_1)2kE_{\rm THz}(t_\omega-\tau)\exp(-2t_\omega^2/T_{\rm p}^2){\rm d}\omega_1}{\int_{-\infty}^{+\infty} g(\omega-\omega_1)\exp(-2t_\omega^2/T_{\rm p}^2){\rm d}\omega_1}, \tag{7}$$

which means $N(\omega) \propto 2kE_{\rm THz}(t_\omega - \tau)$ if the OSA has such a high spectral resolution that the spectral response function can be described as a function of $\delta$. Therefore, we can see that the differential intensity in the measurements encodes the information brought by the THz pulses train. And the profile of the pulses train can be extracted if the chirp rate of the probe pulse is large enough and the resolution of the OSAs is sufficiently high.

However, the chirp rate of the probe pulse and the resolution of the OSAs are limited by the apparatus of the experiments, which could result in the distortion of the information and mistakes in the measurement. In order to find the influence brought by the chirp rate and obtain the time resolution of this method, we study the whole procedure for a given input THz pulses train by assuming the THz pulses train composed of a series of Gaussian pulses

$$E_{\rm THz}(t) = \sum_n \exp\left(-\frac{t-2n\Delta T}{\Delta T^2}\right), \tag{8}$$

where $2\Delta T$ is the interval between the two sequential pulses, and the full-width at half-maximum (FWHM) of the pulses is $\sqrt{2\ln 2}\Delta T$. Assuming the resolution of the OSA is high enough for the spectral response function to be $\delta$ function.

Substituting the Eq. (8) into (6), we can get the time-resolved differential intensity written as

$$N(t) \propto \sum_n \exp\left(-\frac{(t-2n\Delta T)^2}{\Delta T^2\left(1 + \dfrac{T_{\rm p}^2}{\alpha^2\Delta T^4 T_{\rm p}^2 + \Delta T^2}\right)}\right). \tag{9}$$

Here, we can find that the differential intensity $N(t)$ is similar to a Gaussian function such as $E_{\rm THz}(t)$. And the duration of the pulses increases by a factor of $\sqrt{1 + \dfrac{T_{\rm p}^2}{\alpha^2\Delta T^4 T_{\rm p}^2 + \Delta T^2}}$. As the chirp rate $\alpha$ increases, $\sqrt{1 + \dfrac{T_{\rm p}^2}{\alpha^2\Delta T^4 T_{\rm p}^2 + \Delta T^2}}$ decreases, this means that the duration of the pulse become shorter. The narrower the pulse is, the more easily the different pulses could be distinguished. The minimum broadening factor of the

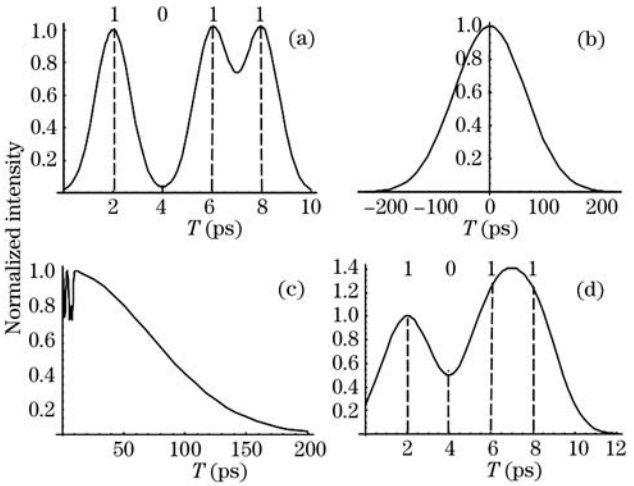


Fig. 2. Profiles of (a) the original pulse sequence, (b) the chirped laser pulse, (c) the transmitted laser after the polarizer, (d) time-resolved difference intensity.

transforming is not an exact value for different original intervals. At no pulse position, the intensity maybe gets greater because of the broadening of the contiguous pulses. However, as far as the intensity induced by the broadening does not exceed half of the maximum intensity of single broadened pulse; it would not be mistaken as a pulse. Therefore, the original half width of the pulse should meet the requirement $\Delta T \geq \sqrt{0.72/\alpha}$. It is easy to prove that $\alpha$ is determined during the stretch of the pulse by $\alpha \cong 1/T_0 T_{\rm p}$, where $T_0$ is the duration of the unchirped pulse from which the chirped probe pulse is stretched. Consequently, combining the two equations, we have

$$\Delta T_{\rm res} = \sqrt{0.72 T_0 T_{\rm p}}. \tag{10}$$

Figure 2 is an application example of this method. The original THz pulses sequence could be seen in Fig. 2(a). The summation parameter $n$ has the value of (1, 3, 4), which carries the digital signals defined as (1,0,1,1). The FWHM of the pulse and the interval are $\sqrt{2\ln 2}$ and 2 ps, respectively. The shape of the probe chirped pulse which has duration of 100 ps is shown in Fig. 2(b), which stretched from an unchirped 10 fs pulse. After the modulation, the code (1,0,1,1) is encoded on the 100 ps chirped pulse as shown in Fig. 2(c). After reconstruction from the measured spectral modulation, the time resolved differential intensity is finally obtained as shown in Fig. 2(d), from which we can see that although the reconstructed shape is different from the original one, the intensities at different key time positions are still representing the same meanings.

According the Nyquist theorem, the sampling technique should take the samples at a rate higher than twice of the frequency of the signals for determine the profile of the signal. For this reason, the resolution of the OSA should be higher than the half of the minimum spectrum interval between the two pulses, which is determined by $\Delta\lambda = \dfrac{\pi c \alpha \Delta T_{\rm res}}{\omega_0^2}$.

Assuming a 100 ps chirped pulse stretched from an unchirped 10-fs pulse is taken as the reference pulse. In this situation, the resolution caused by the chirped pulse is 0.84 ps and the resolution of the OSA is 0.03 nm which are high enough for measuring the spectra of the modulated pulses. Therefore, we can measure a THz sequence containing at most 59 pluses with the time delay of 1.7 ps between two contiguous pulses at a single-shot. In this way, the repetition frequency of the sequence decreases by a factor of tens or hundreds of times. The intensities of light at different wavelength are measured with an OSA, the constraints on which are relaxed by the low repetition of the chirped pulses.

In conclusion, we have demonstrated the THz sequence measurement with chirped pulse sampling technology. The principle of the method is discussed mathematically, and the temporal resolution is analyzed. By taking advantages of the method, real-time measurement of THz sequence could be realized with low repetition device.

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