A new method to generate flattened Gaussian beam by incoherent combination of cosh Gaussian beams

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A new method is proposed which makes it possible to produce flattened irradiance distribution by incoherent combination of cosh Gaussian beams with different parameters. Based on the Collins integral formula, the resulting irradiance distribution is derived and the corresponding beam qualities in terms of M^2 factor and kurtosis parameter are given. The numerical examples illustrate that a flattened intensity distribution of the resulting beam can be obtained in the near filed if appropriate parameters are chosen. Compared with the coherent combination of the Hermite-Gaussian (HG) beams, this new method leads to better beam quality and can be more easily realized experimentally resulting from in-phase-free requirement.

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Flattened Gaussian beam (FGB), which is the so-called supper-Gaussian beam[1], has a nearly uniform irradiance distribution at a certain cross-section[2], so it has been applied in many fields, such as material treating, wielding, drilling, inertial confine fusion as well as optical information processing. By now, there has been much interest in its properties[3−6] and practical produce methods, e.g. vibrating mirror for smoothing and alignment-free resonator methods, have been presented[7,8]. Some theoretical means, such as supposition of multi-Gaussian beams with different waist parameters and elegant Laguerre-Gaussian mode, were put forward to produce FGB[9,10]. Recently, coherent combination of the Hermite-Gaussian (HG) beams has been proved a feasible way to produce FGB in theory[11]. This method, however, faces a critical difficulty due to the fact that keeping the beams in-phase is not easy in practice. To overcome the disadvantage, a new method to produce FGB by incoherent combination of cosh Gaussian (ChG) beams is proposed in this paper. According to the Collins integral formula, the irradiance distribution of incoherent resulting beam and corresponding beam quality factors are derived. One can find that if the appropriate parameters are chosen, the incoherent combination of ChG beams has the flat-top intensity distribution. Compared with the HG coherent combination, this new method does not require all the phases of beams equal, which is desirable for laser engineers. M^2 factor and kurtosis parameter are chosen to characterize the beam quality. The parameters of ChG affecting the resulting beam pattern are investigated by numerical method.

For simplicity, only one dimensional (1D) model is considered since two dimensional (2D) configuration can be generalized from 1D model. Given that the linear laser array consists of N equal elements which are Hermite cosh Gaussian (HChG) mode in the x direction and Gaussian mode in the y direction positioned at the plane $z = 0$, whose waist width of the TEM_{00} is ω_0 and separate distance is x_d, respectively, as shown in Fig. 1. The number N of the array may be odd and even, and positive even is set in this paper. We suppose that the initial phases of the beams are equal to zero, the field distributions of the beams at the input plane can be given by[12]

$$E_{mn}(x,y,0) = H_p\left[\sqrt{2}\frac{(x-mx_\mathrm{d})}{\omega_0}\right] \exp\left[-\frac{(x-mx_\mathrm{d})^2+(y-ny_\mathrm{d})^2}{\omega_0^2}\right] \cosh\left[\Omega_0(x-mx_\mathrm{d})\right], \quad (1)$$

where $m \in [-\frac{M-1}{2}, \frac{M-1}{2}]$, $n \in [-\frac{N-1}{2}, \frac{N-1}{2}]$, Ω_0 is the parameter associated with the cosh part, $b = \Omega_0\omega_0$ is called decentered parameter, H_p denotes the Hermite polynomial of order p. When $p = 0$, the HChG beam mode reduces to the ChG mode[12]. For generalization, in this paper, we adopt the HChG mode instead of ChG mode.

The propagation of the beam described by Eq. (1) passing through a first-order optical system obeys the well-known Collins integral formula[13]

$$E_n(x_1,y_1,z) = \frac{i}{\lambda B}\int\int E_n(x,y,0) \exp\left\{-\frac{i\kappa}{2B}\left[A(x^2+y^2)-2(xx_1+yy_1)+D(x_1^2+y_1^2)\right]\right\} \mathrm{d}x\mathrm{d}y, \quad (2)$$

where κ is the wave number related to the wavelength λ by $\kappa = 2\pi/\lambda$. In fact, for convenience and simplicity a constant phase factor is omitted in Eq. (2) since it has not effect on the relative intensity distribution of the beam. A, B, C and D are the elements of a transfer matrix with the condition that $AD - BC = 1$ for the complete optical system between the input and output planes. When the beams propagate through a thin lens with focal length f, the corresponding $ABCD$ matrix can be expressed as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}. \quad (3)$$

Substituting Eqs. (1), (3), and (4) into Eq. (2) yields

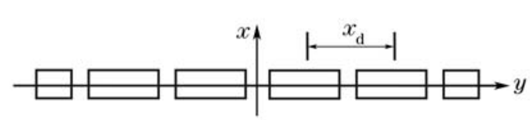

Fig. 1. Schematic illustration of the 1D off-axis HChG.

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$$E_{mn}(x',y',\Delta z)=\frac{i\pi N_f}{1+\Delta z(1-i\pi N_f)}\left[-\frac{1+\Delta z(1+i\pi N_f)}{1+\Delta z(1-i\pi N_f)}\right]^{p/2}H_p\left\{\frac{\sqrt{2}\pi N_f\left(x'+\Delta z m x'_{\mathrm{d}}\right)}{\left[(1+\Delta z)^2+(\pi N_f\Delta z)^2\right]^{1/2}}\right\}$$
$$\times\exp\left\{-\frac{i\pi N_f(1-i\pi N_f)}{1+\Delta z(1-i\pi N_f)}\left[\left(x'+\Delta z m x'_{\mathrm{d}}\right)^2+\left(y'+\Delta z n y'_{\mathrm{d}}\right)^2\right]+i2\pi N_f\left(m x'_{\mathrm{d}}x'+n y'_{\mathrm{d}}y'\right)\right\}$$
$$\times\exp\left\{\left[i\pi N_f\Delta z\left(m x'_{\mathrm{d}}\right)^2+\left(n y'_{\mathrm{d}}\right)^2\right]+\frac{b^2\left(1+\Delta z\right)}{4\left[1+\Delta z(1-i\pi N_f)\right]}\right\}\cosh\left[\frac{i\pi N_f b\left(x'+\Delta z m x'_{\mathrm{d}}\right)}{1+\Delta z\left(1-i\pi N_f\right)}\right],\qquad(4)$$

where

$$x'=\frac{x_1}{\omega_0},\quad y'=\frac{y_1}{\omega_0};\quad d=\frac{x_d}{\omega_0},$$

and the $N_f=\omega_0^2/\lambda f$ is the Fresnel number associated with the beam, and $\Delta z=\frac{(z-f)}{f}$.

Now, the near field distribution of input beam at the propagation plane $\Delta z=-1$ can be obtained from Eq. (4) with the normalized parameters for the incoherent case as

$$I(x',y',-1)=\sum_n E_n(x',y',-1)E_n^*(x',y',-1)\qquad(5)$$

with * being the complex conjugation.

M^2 factor is often used to characterize the resulting beam quality. According to the second-moment definition of the variance σ_x^2 in the spatial domain and the variance σ_{sx}^2 in the spatial-frequency domain[14], the M_x^2 factor for the incoherent combination case was given by[15]

$$M_x^2=\left[\frac{N^2-1}{3}d^2+1+\frac{\left(\omega_0\Omega_0\right)^2}{1+\exp(-\left(\omega_0\Omega_0\right)^2/2)}\right]$$
$$\left[1-\frac{\left(\omega_0\Omega_0\right)^2\exp(-\left(\omega_0\Omega_0\right)^2/2)}{1+\exp(-\left(\omega_0\Omega_0\right)^2/2)}\right].\qquad(6)$$

For flattened beam, the flatness degree is a very important parameter to valuate the beam quality, which is called kurtosis parameter defined as following in one transversal dimension[15]

$$K=\frac{\left\langle x^4\right\rangle}{\left(\left\langle x^2\right\rangle\right)^2},\qquad(7)$$

where $\left\langle x^4\right\rangle$ and $\left\langle x^2\right\rangle$ denote the fourth and second irradiance moments, respectively. From Eqs. (4) and (5) and the moment definition, the kurtosis parameter at the $\Delta z=-1$ plane has been given by Ref. [14].

In this part, firstly we will give some numerical examples to indicate the flattened distribution of the resulting beam. First of all $d=1.5$ and $b=1.04$ are assumed, according to Eqs. (4) and (5), the irradiance of the resulting beam as a function of normal waist width for different beam numbers N is shown in Fig. 2. It illustrates that if the parameters are chosen suitably the incoherent combination of the ChG beams can yield flattened pattern and the width of the resulting beam is directly proportional to beam number N, which is similar to the properties of coherent of HG beams as Fig. 2 depicted in Ref. [11].

Secondly suppose the separate distance of the beam $d=1$, one can find that the flattened distribution occurs when the N and b change, according to the expressions of M^2 factor and kurtosis parameters the beam quality curves are given by Figs. 3(a) and (b). Figure 3(a) shows that the M^2 factor does not decrease linearly with b, and a best beam quality exists with the increase of b for a given beam number N. It can also be derived from Fig. 3(a) that b value for best beam quality becomes slightly larger with N increasing while the beam quality becomes worse when the beam number increases due to the width increase as shown in Fig. 2. For a given beam number Fig. 3(b) illustrates that the degree of the flatness at first

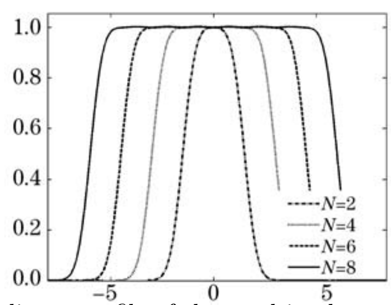

Fig. 2. Irradiance profile of the resulting beam with $d=1.5$ and $b=1.04$ for different values of N.

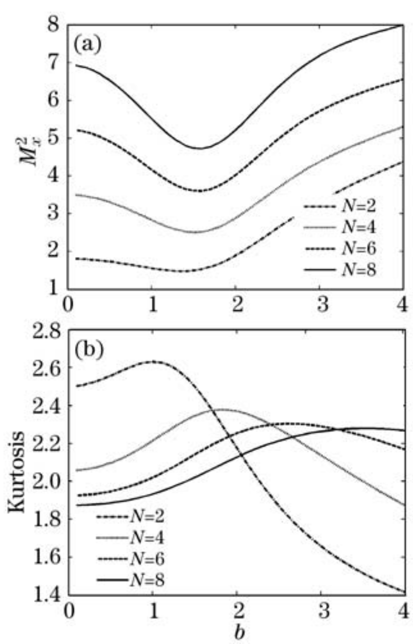

Fig. 3. Beam quality of the resulting beam as a function of b for different values of N. (a) M_x^2 factor, (b) kurtosis parameters.

decreases (kurtosis value rises) and then increases as b increases, which is exact inverse to change tendency of the M^2 factor. The worst flatness degree occurs for the different b with N largening, but the difference of b with worst flat for different N is larger compared with that of the M^2 factor. It also shows that for the small value of b, e.g. b from 0 to 1.7, the kurtosis parameters augment as beam number N increases, which is in agreement with the result in Fig. 2, the trend, however, which is almost reverse for the b value larger than 3.5. At the range of b from 1.7 to 3.5, there exists crossing and the variable trend is not monotonous with N. Considering the two change tendencies, if $N > 2$, it is best to choose $b \approx 1.5$ to obtain the best small M^2 factor and relative small kurtosis parameter value.

Lastly the flattened intensity distribution occurs under the condition of the increases of both b and d, which is a pair of parameters. If the parameters are suitable, e.g., when $d = 1.5$, 1.6, 1.8, 2.0, the corresponding $b = 1.04$, 1.19, 1.10, 1.285, respectively, as shown in Fig. 4(a). Figure 4(a) also shows that the decreases of b and d result in narrowing the width of the resulting beam. Figures 4(b) and (c) illustrate the beam quality with different pairs of parameters. For the M^2 factor, it decreases more slowly when b and d decrease, which is evident for large N value. The degree of flatness rises when the b and d increase, which is not evident for the N larger than 10. Kurtosis parameters value in Ref. [11] is almost beyond 2, which is larger than that of the incoherent combination only if the $N > 3$ is satisfied, as shown in Fig. 4(c).

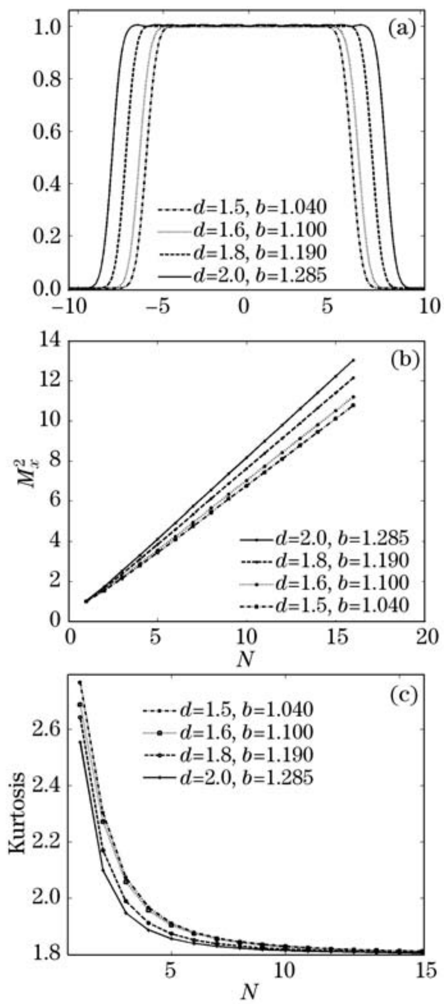

Fig. 4. (a) Irradiance profile of the resulting beam with b and d increasing at the same time, (b) the corresponding M_x^2 factor and (c) kurtosis parameters as a function of beam number N.

In summary, we put forward a new way to produce flattened Gaussian beam by incoherent combination of ChG beams. The combination properties are investigated in detail by numerical simulation. The results prove that this new method is feasible if the appropriate parameters are chosen. They also indicate the M^2 factor and kurtosis parameters of the resulting beam are related to separate distance d of the beams, beam number N and decentered parameter b. The width of resulting beam is direct ratio to N monotonously. When $d = 1$, M^2 factor increases while kurtosis value decreases with b varying from 1 to 1.7, which is reverse for b beyond 3.5. When b is about 1.5, the best M^2 factor can be obtained while the worst kurtosis value is obtained when b within the range from 1.2 to 3. Compared with the coherent combination of HG method, this new way is easy to realize due to in-phase-free need.

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References

1. A. Parent, M. Morin, and P. Lavigne, Opt. Quantum Electron **24,** 1071 (1992).
2. F. Gori, Opt. Commun. **107,** 335 (1994).
3. L. Wang and J. Xue, Chin. Opt. Lett. **1,** 425 (2003).
4. S. R. Luo and B. D. Lü, Chin. J. Lasers (in Chinese) **29,** 425 (2002).
5. S. R. Luo, B. D. Lü, and B. Zhang, Acta Opt. Sin. (in Chinese) **20,** 1213 (2000).
6. T. P. Lu and P. Wu, Chin. J. Lasers (in Chinese) **32,** 908 (2005).
7. G. Z. Zhu, Y. N. Zhang, X. F. Zou, and D. L. Hou, Chin. J. Lasers (in Chinese) **32,** 175 (2005).
8. Y. Cheng, B. Chen, and X. B. Wang, Chin. J. Lasers (in Chinese) **30,** 973 (2003).
9. Y. J. Li, Proc. SPIE **5525,** 128 (2004).
10. R. Borghi, J. Opt. Soc. Am. A **18,** 1627 (2001).
11. P. Wu, J. Zhuang, and B. D. Lü, Chin. J. Lasers (in Chinese) **31,** 48 (2004).
12. B. D. Lü, H. Ma, and B. Zhang, Opt. Commun. **163,** 165 (1999).
13. A. E. Siegman, Proc. SPIE **1224,** 2 (1990).
14. D. M. Deng, Phys. Lett. A **333,** 485 (2004).
15. S. Saghafi, C. J. R. Sheppard, and J. A. Piper, Opt. Commun. **191,** 173 (2001).